

## In-Lecture Assignment #3 on March 4, 2019 with Solutions

Design and analysis of bandpass FIR and IIR filters.

Design bandpass filters to pass the fourth octave of frequencies on the Western music scale that meets the following specification:

- 40 dB stopband attenuation ( $A_{\text{stop1}}$ ) from 0 to 247 Hz,
- 1 dB to 0 dB passband ripple ( $A_{\text{pass}}$ ) from 262 to 494 Hz, and
- 40 dB stopband attenuation ( $A_{\text{stop2}}$ ) at and above 523 Hz.

Please use a sampling rate of 8000 Hz.

Use the filter design and analysis tool (fdatool) in Matlab to complete both designs.

### **(a) FIR Filter Design**

Type fdatool in MATLAB. Select Bandpass for “Response Type”. Select “FIR Filter” for Design Method and then select “Equiripple” for the specific FIR Filter Design Method. Then type in the above design specifications and hit “Design Filter”.

Plot the magnitude response. If the filter does not meet the specification, then redesign the filter by increasing the passband ripple specification until it meets the specification.

*See page 2.*

What is the smallest FIR filter order to meet the specification? (For an FIR filter, “order” means number of zeroes in the transfer function in the  $z$ -domain.)

*Using the above design specifications with 1.5 dB for  $A_{\text{pass}}$ , the equiripple FIR design method (a.k.a. Parks-McClellan, Chebyshev and Remez Exchange) gives a 690<sup>th</sup> order FIR filter.*

How many multiplication operations are needed to compute one output sample?

*The input-output relationship for an FIR filter with  $N$  coefficients is*

$$y[n] = h_0 x[n] + h_1 x[n-1] + \dots + h_{N-1} x[n - (N-1)]$$

*$N$  multiplications are needed to compute one output sample. Here,  $N = 691$ .*

What is the group delay in samples?

*Group delay is the derivative of the phase response with respect to frequency.*

*The filter has linear phase because the impulse response is even symmetric about its midpoint except at the frequencies that do not pass through the filter.*

*For a linear phase FIR filter, the group delay is  $\text{order}/2 = 690/2 = 345$  samples*

### **(b) IIR Filter Design**

Type fdatool in MATLAB. Select Bandpass for “Response Type”. Select “IIR Filter” for Design Method and then select “Elliptic” for the specific IIR Filter Design Method. Then type in the above design specifications and hit “Design Filter”.

Plot the magnitude response to verify that the designed filter meets the filter specification.

*See page 2.*

What is the smallest IIR filter order to meet the specification? (For an IIR filter, “order” means

the number of poles in the transfer function in the  $z$ -domain.)

**12<sup>th</sup> order.**

How many multiplication operations are needed to compute one output sample? Compare this with the result for the FIR filter in part (a).

***A 12<sup>th</sup> order IIR filter has 12 poles and 12 zeros. The transfer function in the  $z$ -domain would have 13 numerator coefficients and 13 denominator coefficients:***

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \dots + b_{12} z^{-12}}{1 - a_1 z^{-1} - \dots - a_{12} z^{-12}}$$

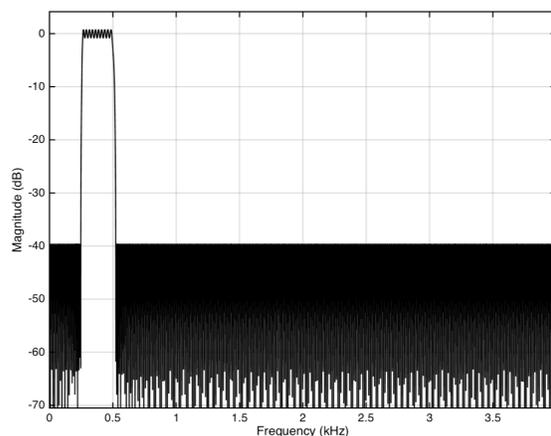
***The difference equation becomes***

$$y[n] = a_1 y[n - 1] + \dots + a_{12} y[n - 12] + b_0 x[n] + b_1 x[n - 1] + \dots + b_{12} x[n - 12]$$

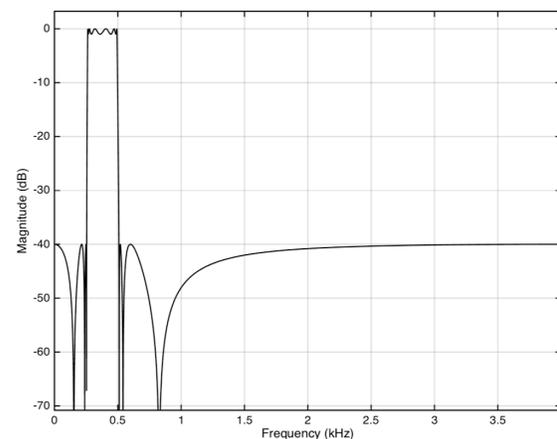
***For an IIR filter with  $N$  poles and  $N$  zeros,  $2N+1$  multiplications are needed to compute one output sample. With  $N=12$ , 25 multiplications per output sample are needed, which is 27 times fewer than for the FIR filter design in part (a).***

What is the minimum and maximum group delay from 270 Hz to 490 Hz in samples? Compare this with the result for the FIR filter in part (a).

***From eyeballing the group delay response plot, the group delay is at its maximum value of about 200 samples at 270 Hz and 490 Hz. Its minimum value is about 30 samples. The average group delay is less than 100 samples, which is about 7 times shorter than the FIR filter in part (a).***



***Magnitude Response for the FIR Bandpass Filter Design***



***Magnitude Response for the IIR Bandpass Filter Design***