## \% In-Lecture Assignment \#4 Related to Homework \#6

\% Consider performing an iterative maximization of
$\% J(x)=8-x^{2}+6 \cos (6 x)$
$\%$ via the steepest descent (ascent) algorithm (JSK equation (6.5) on page 116)
$\%$ with the sign on the update reversed from negative to positive so that $\%$ the algorithm will maximize rather than minimize; i.e.
$\left.\% x[k+1]=x[k]+\mu \frac{d J(x)}{d x}\right]_{x=x[k]}$
\% a. Visualize and analyze the shape of the objective function $J(x)$.
$\% \quad 1)$ Plot $J(x)$ for $-5<x<5$. Give the Matlab code for your answer.
$\mathrm{x}=[-5: 0.01: 5]$;
$J=8-x .^{\wedge} 2+6$ * $\cos \left(6^{*} x\right)$;
plot ( $x, J$ ); $\%$ At end of document

## \% 2) Describe the plot.

\% Sum of concave down parabola and cosine creates many local maxima -OR-
\% Headband-like rainbow shape composed in a parabolic wavy pattern -OR-
\% Comic (graphical novel) sketch of a head with hair or crown
\% 3) How many local maxima do you see?
$\% \quad 11$, which are the 9 peaks with valleys plus the two end points.
\% 4) Of these local maxima, how many are global maxima?
\% Only one, located at $\boldsymbol{x}=0$.
\% b. Derive the steepest descent (ascent) update equation
$\% \quad \mathrm{dJ}(\mathrm{x}) / \mathrm{dx}=-2 \mathrm{x}-36^{*} \sin \left(6^{*} \mathrm{x}\right)$
$\%$ and modify the code below to include the derivative of $\mathrm{dJ}(\mathrm{x}) / \mathrm{dx}$

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% Code below modified from a solution by a Spring 2019 student
% polyconverge.m find the maximum of J(x)=x via steepest descent
N=50; % number of iterations
mu=0.001; % algorithm stepsize
x=zeros(1,N); % initialize sequence of x values to zero
x(1)=0.7; % starting point x(1)
for k=1:N-1
    x(k+1)=x(k) + (-36*sin(6*x(k)) - 2*x(k))*mu; % update equation
end
figure();
stem(x); % to visualize approximation of x
x(N)
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$\% \mathrm{c}$. Implement the steepest descent (ascent) algorithm in Matlab with $x[0]=0.7$.
$\% \quad 1)$ To what value does the steepest descent algorithm converge?
\% $\quad \mathbf{x}=\mathbf{1 . 0 3 7 6}$
\% 2) Is the convergent value of $x$ in the global maximum of $J(x)$ ? Why or why not?
\% No. The only global maximum of $\boldsymbol{J}(\boldsymbol{x})$ occurs at $\boldsymbol{x}=\mathbf{0}$.
\% The objective function $J(x)$ is plotted below vs. $x$

\% The plot below shows the trajectory of $x[k]$ values vs. $k$

\% Below, the objective function $J(x)$ is highlighted with the global maximum at $x=0$,
$\%$ the starting point of the steepest descent (ascent) algorithm at $x=0.7$, and
$\%$ the point where the steepest descent (ascent) algorithm converges at $\boldsymbol{x}=1.0376$.

\% Debugging hint: What happens if one makes a mistake computing \% the derivative? How I can tell that there's a mistake? The steepest $\%$ descent (ascent) will not correctly find the minimum (maximum).

