Spring 2014   EE 445S Real-Time Digital Signal Processing Laboratory   Prof. Evans

Homework #0

Review of Linear Systems and Signals Material

Assigned on Friday, January 17, 2014
Due on Friday, January 24, 2014, by 11:00am sharp in class

Late homework is subject to a penalty of two points per minute late.

Reading: Johnson, Sethares and Klein, chapters 1-2 and Appendices A and F

This assignment is intended to review key concepts from Linear Systems and Signals. Moreover, ***all of the problems on homework #0 directly relate to lab #2.*** This homework will be graded and be counted towards the overall homework grade for the course.

Here are key sections from Lathi's *Linear Systems and Signals* book (2^{nd} ed) and Oppenheim & Willsky's *Signals and Systems* book (2^{nd} ed) with respect to material in EE 445S:

O&W	***Lathi***	***Topic***
1.6	1.7	System properties
1.3 – 1.4	1.4	Basic continuous-time signals
3.2 ##	2.4-4	Fundamental theorem for continuous-time linear systems **
1.3 – 1.4	3.3	Basic discrete-time signals
3.2 ##	3.8-3	Fundamental theorem for discrete-time linear systems **
9.7.2	2.6	Stability of continuous-time filters
10.7.2	3.10	Stability of discrete-time filters
10.1 – 10.3	5.1	*Z* transforms
10.5	5.2	Properties of the *z*-transform
10.7.3 – 10.7.4	5.3	Transfer functions
10.8	5.4	Realizations of transfer functions
4.3 – 4.4	7.3	Fourier transform properties
7.1	8.1	Sampling theorem

** Please see Appendix F and slide 5-13 in the course reader for the fundamental theorem.
O&W covers a slightly different version of the fundamental theorem in which a complex exponential is the input to a linear time-invariant system. Lathi also has that version as well.

Other signals and systems textbooks should contain equivalent material.

The MATLAB code in Johnson, Sethares & Klein also runs in LabVIEW Mathscript and GNU Octave (see footnote on JSK page viii about Octave). Feel free to use these environments. ***Please submit any MATLAB code that you have written with the homework solution.*** In the course reader, Appendix D gives a brief introduction to MATLAB. For quick review of commands in MATLAB for generating and plotting signals, please see

http://www.ece.utexas.edu/~bevans/courses/ee313/lectures/02_Signals/lecture2.ppt

As stated on the course descriptor, **"Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."**

Office hours for the teaching assistants and Prof. Evans; **bold** indicates a 30-minute timeslot.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
9:30 am					**Jia (ENS 137)**
10:00 am					**Jia (ENS 137)**
10:30 am					
11:00 am	MLK Holiday		Evans (ETC 5.148)		Evans (ETC 5.148)
12:00 pm	**MLK Holiday**		**Evans (ENS 433B)**		**Evans (cafe)**
12:30 pm				**Evans (ENS 433B)**	**Evans (cafe)**
1:00 pm				Evans (ENS 433B)	Evans (cafe)
2:00 pm				**Evans (ENS 433B)**	
2:30 pm					
3:00 pm			**Sinno (ENS 137)**		
3:30 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
4:00 pm			**Sinno (ENS 137)**	**Jia (ENS 137)**	
4:30 pm				**Jia (ENS 137)**	
5:00 pm				**Jia (ENS 137)**	
5:30 pm				**Sinno (ENS 137)**	
6:00 pm				Sinno (ENS 137)	

1. Continuous-Time Sinusoidal Generation. 27 points.

In practice, we cannot generate a two-sided sinusoid $\cos(2\pi f_c t)$, nor can we wait until the end of time to observe a one-sided sinusoid $\cos(2\pi f_c t)\, u(t)$.

In the lab, we can turn on a signal generator for a short time and observe the output in the time domain on an oscilloscope or in the frequency domain using a spectrum analyzer.

Consider a finite-duration cosine that is on from 0 sec to 1 sec given by the equation

$$c(t) = \cos(2\pi f_c t)\, \text{rect}(t - ½)$$

where f_c is the carrier frequency (in Hz).

(a) Using MATLAB, LabVIEW Mathscript or GNU Octave, plot $c(t)$ for $-0.5 < t < 1.5$ for f_c = 10 Hz. Turn in your code and plot. You may find the rectpuls command useful. 6 points. Give a formula for the Fourier transform of $c(t)$ for a general value of f_c. 6 points.

(b) Sketch by hand the magnitude of the Fourier transform of $c(t)$ for a general value of f_c. Using MATLAB, LabVIEW Mathscript or GNU Octave, plot the magnitude of the Fourier transform of $c(t)$ for f_c = 8 Hz. Turn in your code and plot. 9 points.

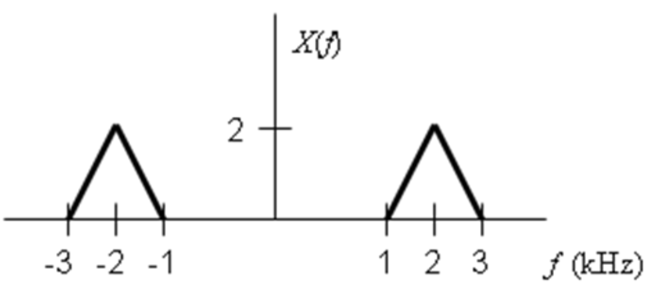
(c) Describe the differences between the magnitude of the Fourier transforms of $c(t)$ and a two-sided cosine of the same frequency. What is the bandwidth of each signal? 6 points.

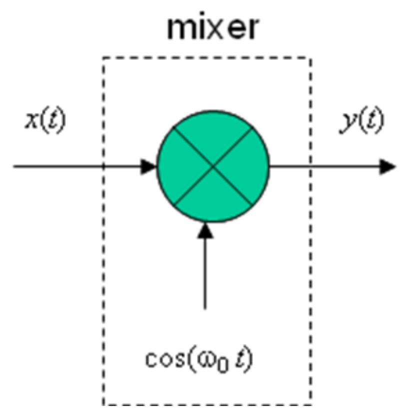
2. Downconversion. 19 points.

A signal $x(t)$ is input to a mixer to produce the output $y(t)$ where

$$y(t) = x(t) \cos(\omega_0 t)$$

where $\omega_0 = 2 \pi f_0$ and f_0 = 2 kHz. A block diagram of the mixer is shown below on the left. The Fourier transform of $x(t)$ is shown below on the right.





(a) Using Fourier transform properties, derive an expression for $Y(f)$ in terms of $X(f)$. 6 points.

(b) Sketch $Y(f)$ vs. f. Label all important points on the horizontal and vertical axes. 6 points.

(c) What operation would you apply to the signal $y(t)$ in part (b) to obtain a baseband signal? The process of extracting the baseband signal from a bandpass signal is known as *downconversion*. 7 points.

3. Sampling in Continuous Time. 24 points.

Sampling the amplitude of an analog, continuous-time signal $f(t)$ every T_s seconds can be modeled in continuous time as

$$y(t) = f(t)\, p(t)$$

where $p(t)$ is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

T_s is known as the sampling duration. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s}\left(1 + 2\cos(\omega_s\, t) + 2\cos(2\,\omega_s\, t) + \ldots\right)$$

where $\omega_s = 2\,\pi\,/\,T_s$ is the sampling rate in units of radians per second.

(a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. 6 points.

(b) Note that in part (a), $p(t)$ is periodic. What is the period? 6 points.

(c) Using the Fourier series representation of $p(t)$ given above, please give a formula for $P(\omega)$, which is the Fourier transform of $p(t)$. Express your answer for $P(\omega)$ as an impulse train in the Fourier domain. 6 points.

(d) What is the spacing of adjacent impulses in the impulse train in $P(\omega)$ with respect to frequency ω in rad/s? 6 points.

4. Discrete-Time Sinusoidal Generation. 30 points.

Consider a causal discrete-time linear time-invariant system with input $x[n]$ and output $y[n]$ being governed by the following difference equation:

$$y[n] = (2\cos\omega_0)\, y[n-1] - y[n-2] + (\sin\omega_0)\, x[n-1]$$

The impulse response of the above system is a ***causal sinusoid*** with discrete-time frequency ω_0 in units of rad/sample. The value of ω_0 would normally be in the interval $[-\pi, \pi)$. You will be implementing the above difference equation in C in lab #2 on a programmable digital signal processor for real-time sinusoidal generation.

(a) Draw the block diagram for this system using add (or summation), multiplication (or gain), and delay blocks. Please label delay blocks with the text z^{-M} to denote a delay of M samples. Use arrowheads to indicate direction of the flow of signals. 6 points.

(b) Please state all initial conditions. Please give values for the initial conditions to satisfy the stated system properties. 6 points.

(c) Find the equation for the transfer function in the z-domain, including the region of convergence. 6 points.

(d) Compute the inverse z-transform of the transfer function in part (c) to find the impulse response of the system. 6 points.

(e) Using MATLAB, LabVIEW Mathscript or GNU Octave, plot the impulse response obtained in part (d) for ω_0 equal to 0, π, and a value in the interval $(0, \pi)$ of your choosing. Turn in your code and plots. 6 points.