Discrete-Time Periodicity

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A discrete-time signal $x[n]$ is periodic if

$$x[n + N_0] = x[n] \text{ for all } n. \quad N_0 \text{ is positive.}$$

The smallest value of N_0 is the fundamental period. For a two-sided cosine signal,

$$x[n] = \cos(w_0 n) \text{ where } w_0 = 2\pi \frac{f_0}{f_s} = 2\pi \frac{N}{L}$$

where N and L are relatively prime integers; and f_0 is the continuous-time frequency and f_s is the sampling rate.

$$x[n + N_0] = \cos\left(2\pi \frac{N}{L}(n + N_0)\right)$$

$$= \cos\left(2\pi \frac{N}{L}n + 2\pi \frac{N}{L}N_0\right)$$

$$= \cos\left(2\pi \frac{N}{L}n\right) = x[n]$$

if $2\pi \frac{N}{L}N_0$ is an integer multiple of 2π, i.e.

if $\frac{N}{L}N_0$ is an integer. The smallest value of N_0 is $N_0 = L$. Fundamental period is L.

There are N continuous-time periods in the fundamental discrete-time period.

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