The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: March 8, 2013 Course: EE 445S Evans

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Last, First

- The exam is scheduled to last 50 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network. ***Please disable all wireless connections on your computer system(s).***
- Please turn off all cell phones.
- No headphones allowed.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	*Point Value*	*Your score*	*Topic*
1	28		Filter Analysis
2	24		Filter Implementation
3	24		Filter Design
4	24		Potpourri
Total	100		

Problem 1.1 *Discrete-Time Filter Analysis.* 28 points.

A causal stable discrete-time linear time-invariant filter with input $x[n]$ and output $y[n]$ is governed by the following transfer function:

$$H(z) = 1 - z^{-3}$$

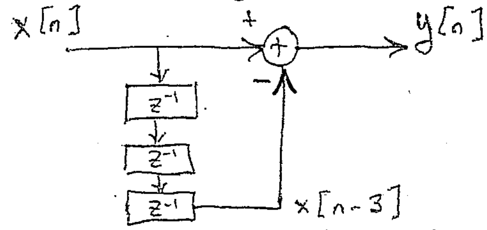
for $|z| \neq 0$.

(a) From the transfer function, derive the difference equation relating input $x[n]$ and output $y[n]$. *6 points.*

$$H(z) = \frac{Y(z)}{X(z)} = 1 - z^{-3}$$

$$Y(z) = (1 - z^{-3})\, X(z) = X(z) - z^{-3} X(z) \overset{\mathcal{Z}^{-1}}{\Rightarrow} y[n] = x[n] - x[n-3]$$

(b) Give the block diagram for the filter. *3 points.*



(c) What are the initial conditions? What values should they be assigned and why? *4 points.*

From the block diagram, there are three initial conditions $x[-1], x[-2], x[-3]$.
They must be zero to satisfy linearity and time-invariant properties.

Alternate Solution for First Step	$y[0] = x[0] - x[-3]$ $y[1] = x[1] - x[-2]$ initial conditions are $x[-1]$, $x[-2]$ and $x[-3]$.	$y[2] = x[2] - x[-1]$ $y[3] = x[3] - x[0]$

(d) Find the equation for the frequency response of the filter. Justify your approach. *6 points.*

The transfer function has a region of convergence of $z \neq 0$, which includes the unit circle: $H_{freq}(\omega) = H(z)\Big|_{z = e^{j\omega}} = 1 - e^{-3j\omega}$

Alternate Justification: LTI system is bounded-input bounded-output stable.

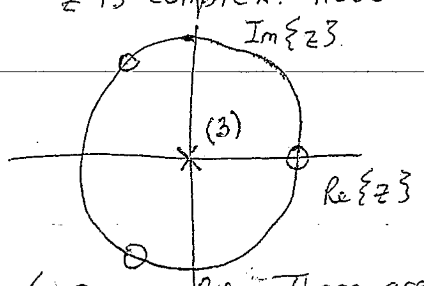
(e) What is the group delay through the filter? *3 points.* $N = 4$ coefficients.

Equal to midpoint of impulse response for linear phase FIR filter: $\frac{3}{2}$ samples

(f) Draw the pole-zero diagram. Is the filter lowpass, highpass, bandpass, bandstop, allpass or notch? *6 points.* For $z \neq 0$,

$H(z) = 1 - z^{-3} = 0 \Rightarrow 1 = z^{-3} \Rightarrow z^3 = 1$

z is complex. Roots (zeros) of $z^3 - 1 = 0$ are 1, $e^{j\frac{2}{3}\pi}$ and $e^{-j\frac{2}{3}\pi}$.



Notch filter.
Eliminates frequencies at $\omega = 0$ and $\omega = \frac{2}{3}\pi$.

(Optional: There are three poles at origin.)

$H_{freq}(\omega) = e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega} - e^{-j\frac{3}{2}\omega}\right)$

$= \underbrace{2\sin\left(\tfrac{3}{2}\omega\right)}_{\text{amplitude}}\ \underbrace{j\, e^{-j\frac{3}{2}\omega}}_{\text{phase}}$

Note: $j = e^{j\frac{\pi}{2}}$

$gd(\omega) = -\frac{d}{d\omega} \angle H_{freq}(\omega)$

$= \frac{3}{2} = \frac{N-1}{2}$

Problem 1.2 *Discrete-Time Filter Implementation.* 24 points.

Consider a causal fourth-order discrete-time infinite impulse response (IIR) filter with transfer function $H(z)$. A filter is a bounded-input bounded-output stable linear time-invariant system.

Input $x[n]$ and output $y[n]$ are real-valued.

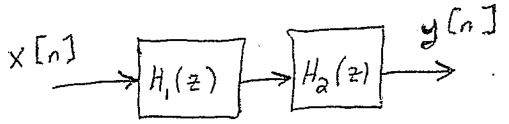
Cascade of biquads. We factor H(z) into a product of two second-order sections (biquads)

$$H(z) = H_1(z)\, H_2(z)$$

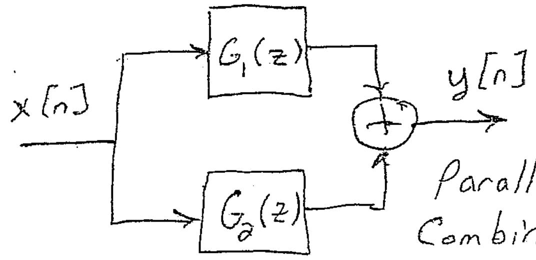
Parallel combination of biquads. We perform partial fraction decomposition on $H(z)$ to write it as a sum of two second-order sections (biquads)

$$H(z) = G_1(z) + G_2(z)$$

(a) Draw the block diagrams for the *cascade of biquads* and the *parallel combination of biquads*. Each block in the block diagram would correspond to a biquad. *6 points.*



Cascade of Biquads



Parallel Combination

(b) Consider the implementation of the two filter structures on the TI TMS320C6700 DSP.
Assuming that data values and filter coefficients are in 32-bit floating point.

i. Compare the memory usage for the two structures. *3 points.*

	Cascade	Parallel
words	10 coefficients	10 coefficients
	12 current & previous inputs/outputs	12 current & previous inputs/outputs
		1 $y[n]$
	22 ~~total~~ (3 outputs of $H_1(z)$ can be shared)	23 total (input can be shared between $G_1(z)$ and $G_2(z)$)

ii. Compare the execution cycles for the two structures. *6 points.*

	Cascade	Parallel
worst case	33 cycles + 33 cycles = 66 cycles	33 cycles + 33 cycles + 4 cycles = 70 cycles
overlapping	(4 + 5 + 28) cycles + 4 cycles = 41	(5 + 5 + 28) cycles + 4 cycles = 42 cycles

(c) Consider the implementation of the two filter structures on a processor with two TI TMS 320C6700 DSP cores (CPUs). The cores share the same on-chip memory.
To communicate data from one core to the other through shared memory takes 10 cycles (1 store + 1 load inst.)

i. Compare the memory usage for the two structures. *3 points.*

Same as (b)i. because of shared on-chip memory

ii. Compare the execution cycles for the two structures. *6 points.*

Implement cascade of biquads on a single core due to the high 10-cycle inter-core communication costs
Same as (b) ii.

Implement parallel combination by placing $G_1(z)$ on one core and $G_2(z)$ on other. They can run in parallel. Also, need 10 cycles to communicate result and 4 cycles for adding the two branches. 47 cycles total.

Problem 1.3 *Filter Design.* 24 points.

In North America, there is a narrowband WWVB timing signal being broadcast at 60 kHz.

The G.hnem powerline communication standard uses a sampling rate 800 kHz and operates in the 34.4 kHz to 478.1 kHz band.

G.hnem receivers experience in-band interference from the WWVB signal.

(a) Design a discrete-time second-order infinite impulse response (IIR) filter for a G.hnem transceiver to remove the 60 kHz WWVB interferer. Give poles, zeros and gain. *12 points.*

Notch filter at $\omega_0 = 2\pi \frac{60 \text{ kHz}}{800 \text{ kHz}} = 2\pi \frac{3}{40}$ rad/sample.

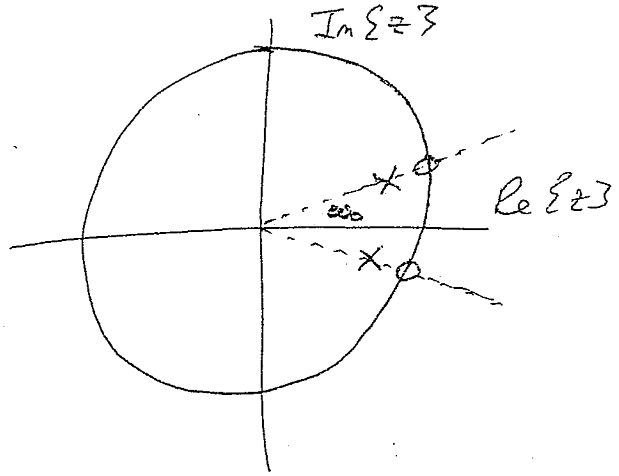
Poles at $0.9 e^{j\omega_0}$ and $0.9 e^{-j\omega_0}$

Zeros at $e^{j\omega_0}$ and $e^{-j\omega_0}$

$$H(z) = C_0 \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})}$$

Set DC gain to be one:

$H(z)\big|_{z=1} = 1$ Solve for C_0.



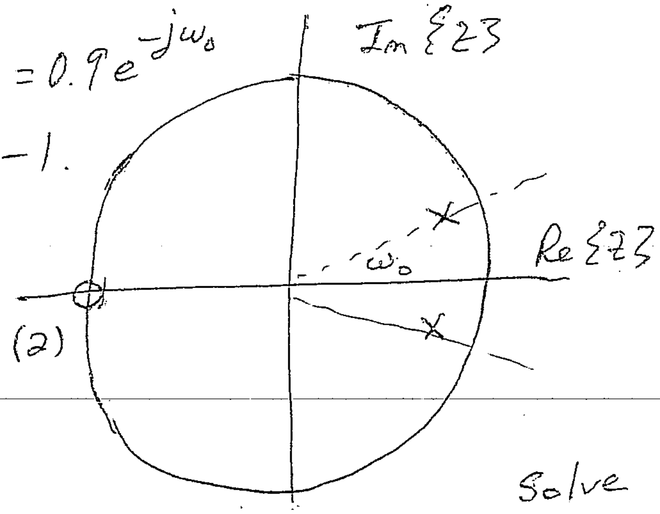
(b) Design a discrete-time second-order infinite impulse response (IIR) filter for a G.hnem transceiver to extract the 60 kHz WWVB signal for use in generating timestamps for power load profiles at the consumer's site. Give poles, zeros and gain. *12 points.*

$\omega_0 = 2\pi \frac{60 \text{ kHz}}{800 \text{ kHz}} = 2\pi \frac{3}{40}$ rad/sample Bandpass filter

Poles at $p_0 = 0.9 e^{j\omega_0}$ and $p_1 = 0.9 e^{-j\omega_0}$

Zeros at $z_0 = -1$ and $z_1 = -1$.

$$G(z) = C_1 \frac{(1 - z_0 z^{-1})(1 - z_1 z^{-1})}{(1 - p_0 z^{-1})(1 - p_1 z^{-1})}$$



Set DC gain to be one:

$G(z)\big|_{z=1} = 1$ Solve for C_1.

(Alternately, set gain at ω_0 to be one: $G(z)\big|_{z=e^{j\omega_0}} = 1$ Solve for C_1)

Problem 1.4. Potpourri. *24 points.*

(a) You want to design a linear phase finite impulse response (FIR) filter with 10,000 coefficients that meets a magnitude specification. Which FIR filter design method would you advocate using? *6 points.* As demonstrated in lecture...

Parks-McClellan (Remez) filter design algorithm is iterative and fails to converge for filter orders ~ 400. Least squares filter design would require inversion of a 10,000 x 10,000 matrix, and may not produce reliable results. Kaiser window design uses formulas for the coefficients, and can design FIR filters with 10,000 coefficients.

(b) Consider a causal first-order IIR filter with non-zero feedback coefficient a_1 and input signal $x[n]$. Output signal is $y[n] = a_1\ y[n-1] + x[n]$. Input data, output data and feedback coefficient are unsigned 16-bit integers. As n increases, does the number of bits needed to keep calculations from losing precision always increase without bound? If yes, show that it is true for all non-zero values of a_1. If no, give a counter-example. *6 points.*

No. Let $x[n] = \delta[n]$ and $a_1 = 1$.

$y[n] = u[n]$.

(Alternate answer from lecture discussion: Let $a_1 = \frac{1}{2}$. $y[n]$ needs 17 bits.)

Alternate answer: Use an IIR filter design method and keep the first 10,000 samples of the impulse response.

(c) Given three reasons why 32-bit floating-point data and arithmetic is better suited for audio processing than 16-bit integer data and arithmetic? *6 points.* "Comparing fixed and floating point DSPs" article in course reader after lecture 2 slides

- 24-bits of mantissa+sign (integer component) gives 144 dB of dynamic range vs. 96 dB of dynamic range for 16-bit integer
- 8-bit of exponent allows wide dynamic range and is very accurate near zero (quiet regions with low accuracy in 16-bit integer)
- Audio uses IIR filters, and 32-bit precision is needed to reduce accumulation of numeric error (truncation/rounding) via feedback

(d) What three instruction set architecture features would accelerate finite impulse response (FIR) filtering? *6 points.*

Instruction set architecture (CPU core)

- Fast multiplier (pipelined)
- Fast adder (pipelined)
- Separate program and data buses
- Multiple data buses and simultaneous load from all buses
- Modulo addressing for circular buffer
- Fast down counting

Other processor enhancements

- Direct memory access controllers for ping-pong buffering + Frame based input/output + Buffer management (on chip)
- Dual-ported on-chip memory for two reads in same cycle
- Autoincrement or autodecrement addressing modes