Homework \#6

## Infinite Impulse Response (IIR) Filters

Assigned on Friday, October 27, 2017
Due on Friday, November 3, 2017, by 12:30 pm via Canvas submission

## Late homework will not be accepted.

Reading: McClellan, Schafer \& Yoder, Signal Processing First, 2003, Ch. 7-8.
Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from DSP First.
Contact information for the teaching assistant, Ms. Ghosh, is available at
https://utexas.instructure.com/files/43676674/download?download frd=1
Office hours for Ms. Ghosh and Prof. Evans follow, as well as Prof. Evans' coffee hours on Friday.

| Time Slot | Monday | Tuesday | Wednesday | Thursday | Friday |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 9:00 am |  |  | Ghosh |  |  |
| 9:30 am |  |  | Ghosh |  |  |
| 10:00 am |  |  | Ghosh |  |  |
| 10:30 am |  |  |  |  |  |
| 11:00 am |  | Ghosh |  | Ghosh |  |
| 11:30 am |  | Ghosh |  | Ghosh |  |
| 12:00 pm |  | Ghosh |  | Ghosh | Evans (EER cafe) |
| 12:30 pm |  | Evans (EER 1.516) |  | Evans (EER 1.516) | Evans (EER cafe) |
| 1:00 pm |  | Evans (EER 1.516) | Evans (EER 6.882) | Evans (EER 1.516) |  |
| 1:30 pm |  | Evans (EER 1.516) | Evans (EER 6.882) | Evans (EER 1.516) |  |
| 2:00 pm |  |  | Evans (EER 6.882) | Evans (EER 6.882) |  |
| 2:30 pm |  |  |  | Evans (EER 6.882) |  |
| 3:00 pm |  |  |  | Evans (EER 6.882) |  |

EE 313 tutoring is available on Mondays through Thursdays from 7:00pm to 10:00pm in ETC 4.150: http://www.ece.utexas.edu/undergraduate/tutoring

## ** Assume that all filters in this homework set are linear and time-invariant. **

## 1. Transfer Function, Difference Equation and Frequency Response Connections. 25 points.

Signal Processing First, problem P-8.13, page 240. Please also answer the following questions.
Plot the frequency response for each filter represented by a pole-zero plot. Once you match the polezero plot to the difference equation, use freqz (numer, denom) where numer is the vector of feedforward coefficients $\left[\begin{array}{llll}b_{0} & b_{1} & \ldots & b_{M}\end{array}\right]$ and denom is the vector of feedback coefficients $\left[\begin{array}{lll}1 & -a_{1} & -a_{2}\end{array} \ldots-\right.$ $\left.a_{N}\right]$. (Note: I had forgotten the leading 1 in denom on lecture slide 11-9.) The numer and denom vectors are also the coefficients in the numerator and denominator of the $z$-domain transfer function.
Indicate each filter's frequency selectivity as lowpass, highpass, bandpass, bandstop, allpass or notch.

## 2. First-Order IIR Filter. 25 points.

Signal Processing First, problem P-8.15, page 241. Also: (c) Draw the block diagram of the IIR filter.

## 3. Second-Order IIR Filter. 25 points.

Signal Processing First, problem P-8.19, page 243. Please complete the following additional part:
(d) Plot the frequency response using freqz. What is the frequency selectivity of the filter? Lowpass, highpass, bandpass, bandstop, allpass, or notch.

## 4. Convolution of Infinite-Length Signals. 25 points.

Sometimes it's easier to work a problem in the time domain, and other tines, it's easier to work the problem in a transform domain.
(a) $y[n]=x_{1}[n] * x_{2}[n]$ where $x_{1}[n]=a^{n} u[n]$ and $x_{2}[n]=b^{n} u[n]$. Handout F "Convolution of Two Causal Exponential Sequences" computes the convolution in the time domain as Case \#1 at http://users.ece.utexas.edu/~bevans/courses/signals/handouts/Appendix\ F\ Convolution\  Exp\%20Sequences.pdf
Find $y[n]$ using $z$-transforms.
Hint: You may need to use partial fractions decomposition of the product $X_{1}(z) X_{2}(z)$. Please see Section 8-7.2 in Signal Processing First.
(b) $y[n]=x_{1}[n] * x_{2}[n]$ where $x_{1}[n]=b^{n} u[n]$ and $x_{2}[n]=b^{n} u[n]$. Handout F "Convolution of Two Causal Exponential Sequences" computes the convolution in the time domain as Case \#2. See the Web address in part (a).
Find $y[n]$ using $z$-transforms. When $x_{1}[n]=x_{2}[n]$, we have resonance.
Hint: You may need to use partial fractions decomposition of the product $X_{1}(z) X_{2}(z)$. Please see Section 8-7.2 in Signal Processing First. The product $X_{1}(z) X_{2}(z)$ has a repeated pole.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.

