Fall 2017 EE 313 Linear Systems and Signals Prof. Evans

Homework #8

Continuous-Time Frequency Response and Fourier Transforms

Assigned on Friday, November 24, 2017
Due on Friday, December 1st, 2017, by 12:30 pm via Canvas submission

Late homework will not be accepted.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Ch. 10 & 11.
Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from *DSP First*.

Contact information and TA office location for Ms. Ghosh is available at

https://utexas.instructure.com/files/44016307/download?download_frd=1

Office hours for Ms. Ghosh and Prof. Evans follow, as well as Prof. Evans' coffee hours on Friday.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
9:00 am			**Ghosh**		
9:30 am			**Ghosh**		
10:00 am			**Ghosh**		
10:30 am					
11:00 am		**Ghosh**		**Ghosh**	
11:30 am		**Ghosh**		**Ghosh**	
12:00 pm		**Ghosh**		**Ghosh**	Evans (EER cafe)
12:30 pm		Evans (EER 1.516)		Evans (EER 1.516)	Evans (EER cafe)
1:00 pm		Evans (EER 1.516)	Evans (EER 6.882)	Evans (EER 1.516)	Evans (EER cafe)
1:30 pm		Evans (EER 1.516)	Evans (EER 6.882)	Evans (EER 1.516)	Evans (EER cafe)
2:00 pm			Evans (EER 6.882)	Evans (EER 6.882)	
2:30 pm				Evans (EER 6.882)	
3:00 pm				Evans (EER 6.882)	

EE 313 tutoring is available on Mondays through Thursdays from 7:00pm to 10:00pm in ETC 4.150:

http://www.ece.utexas.edu/undergraduate/tutoring

1. Continuous-Time Frequency Response. *25 points.*

Signal Processing First, problem P-10.5, page 303.

Also, please complete the following part:

(f) What is the impulse response for the linear time-invariant (LTI) system in part (e)? What is the frequency selectivity— lowpass, highpass, bandpass, bandstop, allpass or notch?

2. Continuous-Time Fourier Transforms. *25 points.*

Signal Processing First, problem P-11.6, page 342. For each part, please plot the signal in the time domain and the frequency domain using MATLAB.

3. Continuous-Time Sinusoidal Signal. *25 points.*

In practice, we cannot generate a two-sided sinusoid $\cos(2\pi f_c t)$, nor can we wait until the end of time to observe a one-sided sinusoid $\cos(2\pi f_c t)\, u(t)$. Instead, we are limited to finite-duration observations.

Consider a finite-duration cosine that is on from 0 sec to 1 sec given by the equation

$$c(t) = \cos(2\pi f_c t)\ \mathrm{rect}(t - \tfrac{1}{2})$$

where f_c is the carrier frequency (in Hz).

(a) Using MATLAB, plot $c(t)$ for $-0.5 < t < 1.5$ for $f_c = 10$ Hz. Turn in your code and plot. You may find the `rectpuls` command useful. *5 points*.

(b) Find the formula for the Fourier transform of $c(t)$ for a general value of f_c. Please use transform pairs and properties. *5 points*.

(c) Using MATLAB, plot the magnitude of the Fourier transform of $c(t)$ for $f_c = 5$ Hz. Turn in your code and plot. *5 points*.

(d) Determine the bandwidth of $c(t)$ based on part (c). Bandwidth is the extent in positive frequencies. There are many ways to measure extent. For this problem, please use the "null bandwidth", which is the distance from the first zero crossing left of f_c to the first zero crossing right of f_c. *5 points*.

(e) Describe the differences between the magnitude of the Fourier transforms of $c(t)$ and a two-sided cosine of the same frequency. What is the bandwidth of a two-sided cosine of the same frequency? *5 points*.

4. Sampling in Continuous Time. *25 points.*

Sampling the amplitude of an analog, continuous-time signal $x(t)$ every T_s seconds can be modeled mathematically in continuous time as

$$y(t) = x(t)\, p(t)$$

where $p(t)$ is the impulse train defined by

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Here, T_s is the sampling time. The Fourier series expansion of the impulse train is

$$p(t) = \frac{1}{T_s}\left(1 + 2\cos(\omega_s t) + 2\cos(2\,\omega_s t) + \ldots\right)$$

where $\omega_s = 2\pi / T_s$ is the sampling rate in units of radians per second.

(a) Plot the impulse train $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. *6 points*.

(b) Note that in part (a), $p(t)$ is periodic. What is the period? *6 points*.

(c) Using the Fourier series representation of $p(t)$ given above, please give a formula for $P(j\omega)$, which is the Fourier transform of $p(t)$. Express your answer for $P(j\omega)$ as an impulse train in the Fourier domain. *6 points*.

(d) What is the spacing of adjacent impulses in the impulse train in $P(j\omega)$ with respect to frequency ω in rad/s? *6 points*.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.