Fall 2018 EE 313 Linear Systems and Signals Prof. Evans

Homework #10

Laplace Transforms

Assigned on Friday, November 30, 2018
Due on Monday, December 10, 2018, by 5:00 pm via Canvas submission
Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Chapter 16:
https://utexas.instructure.com/files/47383586/download?download_frd=1

Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from *DSP First*.

The e-mail address for Mr. Houshang Salimian (TA) is salimian.houshang@gmail.com.
Office hours for Mr. Salimian and Prof. Evans follow. Prof. is holding additional office hours on WF 9:00-10:00am. His Wednesday afternoon office hours have changed to 1:00-2:00pm.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
9:00 am			Evans (EER 6.882)		Evans (EER 6.882)
10:00 am					
11:00 am		Salimian (EER 0.814 Table #4)		Salimian (EER 0.814A)	Salimian (EER 0.814D)
12:00 pm		**Salimian (EER 0.814 Table #4)**		**Salimian (EER 0.814A)**	**Salimian (EER 0.814D)**
12:30 pm		**Evans (EER 1.516)**		**Evans (EER 1.516)**	**Salimian (EER 0.814D)**
1:00 pm		Evans (EER 1.516)	Evans (EER 6.882)	Evans (EER 1.516)	
2:00 pm		Evans (EER 6.882)		Evans (EER 6.882)	
3:00 pm		**Evans (EER 6.882)**	**Salimian (EER 1.810)**	**Evans (EER 6.882)**	
3:30 pm			**Salimian (EER 1.810)**		
4:00 pm			**Salimian (EER 1.810)**		
4:30 pm					

Prof. Evans' coffee hours this week will be 12:00-2:00pm on Friday in the EERC café.

EE 313 tutoring is available on Sundays through Thursdays from 7:00pm to 10:00pm in EER 0.814:
http://www.ece.utexas.edu/undergraduate/tutoring

1. Laplace Transform Using Transforms and Properties. ***24 points.***

The time-shift property states that if $X(s)$ is the bilateral Laplace transform of $x(t)$, then

$$x(t - t_d) \;\leftrightarrow\; e^{-s\,t_d}\,X(s)$$

Use this property to find the bilateral Laplace transforms of the following signals, including their regions of convergence:

(a) $x(t) = u(t) - u(t-1)$

(b) $x(t) = 3\,e^{-3\,t}\,u(t-2)$

(c) $x(t) = 3\,e^{-3\,(t-2)}\,u(t-2)$

(d) $x(t) = 5\sin\big(\pi(t-1)\big)\,u(t-1)$

Inspired by *Signal Processing First*, problem P-16.2, Chapter 16, page 59.

2. Transfer Functions in the Laplace Domain. ***18 points.***

Plot each signal in the time domain for $-1 < t < 1$, compute the Laplace transform including the region of convergence, and sketch the pole-zero plot and region of convergence for the following signals:

(a) $x(t) = \cos(20\,\pi\,t)\,u(t)$

(b) $x(t) = e^{-8t}\,u(t)$

(c) $x(t) = (1 - e^{-8t})\,u(t)$

3. Transfer Function in the Laplace Domain. ***34 points.***

A continuous-time system with input $x(t)$ and output $y(t)$ is described by the following linear constant coefficient differential equation for $t > 0^-$:

$$\frac{d}{dt}y(t) + 2y(t) = \frac{d}{dt}x(t)$$

Initial conditions are set to zero, i.e. $y(0^-) = 0$ and $x(0^-) = 0$, so the system will have linearity and time-invariant properties. (Notice the two different uses of "linear". In the case of linear constant different equation, "linear" refers to "affine" which is a line that does not necessarily go through the origin.)

(a) What is the transfer function $H(s)$ of the system in the Laplace domain including the region of convergence?

(b) What is the impulse response $h(t)$ of the system?

(c) Find the frequency response $H(j\omega)$ of the system from the transfer function. Why is the substitution $s = j\omega$ valid?

(d) From part (c), plot the magnitude response. What is the frequency selectivity— lowpass, highpass, bandpass, bandstop, allpass or notch?

(e) For $x(t) = u(t)$, find $X(s)$ and $Y(s)$.

(f) From part (e), find $y(t)$ by taking the inverse Laplace transform of $Y(s)$.

Note: This problem is a variation on homework problem 9.3 from fall 2017.

4. Another Transfer Function in the Laplace Domain. 24 points.

Signal Processing First, problem P-16.10, page 61.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.

Please read the homework guidelines at
http://users.ece.utexas.edu/~bevans/courses/signals/homework/index.html