Fall 2018 EE 313 Linear Systems and Signals Prof. Evans
Homework \#7

## Infinite Impulse Response (IIR) Filters

Assigned on Saturday, November 3, 2018
Due on Friday, November 9, 2018, by 5:00 pm via Canvas submission
Late homework is subject to a penalty of two points per minute late.
Reading: McClellan, Schafer \& Yoder, Signal Processing First, 2003, Chapter 8. Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/ Web site contains solutions to selected homework problems from DSP First.

The e-mail address for Mr. Houshang Salimian (TA) is salimian.houshang@gmail.com. Office hours for Mr. Salimian and Prof. Evans follow:
$\left.\begin{array}{|c|c|c|c|c|c|}\hline \text { Time Slot } & \text { Monday } & \text { Tuesday } & \text { Wednesday } & \text { Thursday } & \text { Friday } \\ \hline \mathbf{1 1 : 0 0} \mathbf{~ a m} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814 } \\ \text { Table \#4) }\end{array} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814A) }\end{array} & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814D) }\end{array} \\ \hline \mathbf{1 1 : 3 0} \mathbf{~ a m} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814 } \\ \text { Table \#4) }\end{array} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814A) }\end{array} & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814D) }\end{array} \\ \hline \mathbf{1 2 : 0 0 ~ p m ~} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814 } \\ \text { Table \#4) }\end{array} & & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814A) }\end{array} & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814D) }\end{array} \\ \hline \mathbf{1 2 : 3 0 ~ p m ~} & & \begin{array}{c}\text { Evans } \\ \text { (EER 1.516) }\end{array} & & \begin{array}{c}\text { Evans } \\ \text { (EER 1.516) }\end{array} & \begin{array}{c}\text { Salimian } \\ \text { (EER 0.814D) }\end{array} \\ \hline \mathbf{1 : 0 0 ~ p m ~} & & \begin{array}{c}\text { Evans } \\ \text { (EER 1.516) }\end{array} & & \begin{array}{c}\text { Evans } \\ \text { (EER 1.516) }\end{array} & \\ \hline \mathbf{1 : 3 0 ~ p m ~} & & \begin{array}{c}\text { Evans } \\ \text { (EER 1.516) }\end{array} & & \text { Evans } \\ \text { (EER 1.516) }\end{array}\right]$

Prof. Evans' coffee hours this week will be from 12:00-2:00pm on Friday in the EERC café.
EE 313 tutoring is available on Sundays through Thursdays from 7:00pm to 10:00pm in EER 0.814:

## ** Assume that all filters in this homework set are linear and time-invariant. **

## 1. Transfer Function, Difference Equation and Frequency Response Connections. 50 points.

Signal Processing First, problem P-8.14, page 240.
In addition, for each linear time-invariant filter $S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}$, and $S_{7}$,
i. plot the poles and zeros of the transfer function in the $z$-domain, as explained below;
ii. plot the magnitude response, as explained below; and
iii. indicate the frequency selectivity as lowpass, highpass, bandpass, bandstop, allpass or notch.

Plotting poles and zeros: For each filter, plot the poles and zeros for the transfer function in the $z$ domain using Matlab (see below) and try to infer the frequency response from their location. This will help you reinforce what we covered in lecture on Thursday, November $1^{\text {st }}$, on the connections among time, frequency and $z$ domains. Concerning these connections, I recorded a YouTube video in spring 2014 for the Real-Time Digital Signal Processing Laboratory course. Please watch from the 1:29 mark to the 22:25 mark and from 43:01 to the end (50:51 mark) at
https://www.youtube.com/watch?v=WWEKNvvJBvs\&list=PLaJppqXMef2ZHIKM4vpwHIAWyRmw3TtSf
In Matlab, we can use zplane (numer, denom) where numer is a vector of feedforward coefficients $\left[\begin{array}{llll}b_{0} & b_{1} & \ldots & b_{M}\end{array}\right]$ and denom is a vector of feedback coefficients $\left[\begin{array}{llll}1 & -a_{1} & -a_{2} & \ldots\end{array}-a_{N}\right]$. The coefficients can be found from the numerator and denominator of the $z$-domain transfer function:

$$
H(z)=\frac{b_{0}+b_{1} z^{-1}+b_{2} z^{-2}+\cdots}{1-a_{1} z^{-1}-a_{2} z^{-2}-\cdots}
$$

which corresponds to the difference equation

$$
y[n]-a_{1} y[n-1]-a_{2} y[n-2]-\cdots=b_{0} x[n]+b_{1} x[n-1]+b_{2} x[n-2]+\cdots
$$

```
% Filter S1.
% Observation of x[n] and y[n] for }n>=0
y[n] =0.9 y[n-1] +(1/2)x[n] +(1/2)x[n-1]
We can move the term 0.9 y[n-1] to the left-hand side:
y[n] - 0.9 y[n-1]=(1/2) x[n] +(1/2)x[n-1]
We'll take the z-transform of both sides. All initial
conditions are zero as a necessary condition to satisfy
linearity and time-invariance for the system.
Y(z) - 0.9 z^(-1) Y(z) = (1/2) X(z) + (1/2) \mp@subsup{z}{}{\wedge}(-1)X(z)
(1 - 0.9 z-1) Y(z) = (1/2) X(z) + (1/2) z^(-1) X(z)
H(z) = Y(z) / X(z)
    =( (1/2) +(1/2) z^(-1) ) / (1-0.9 z^ (-1) )
i. Plot the poles and zeros in the z domain.
feedforwardCoeffs = [ 1/2 1/2 ];
feedbackCoeffs = [ 1 -0.9 ];
figure;
zplane(feedforwardCoeffs, feedbackCoeffs);
```

Plotting the magnitude response: Plot the magnitude response for each filter $S_{1}, S_{2}, S_{3}, S_{4}, S_{5}, S_{6}$, and $S_{7}$ using freqz (numer, denom). You can use the following code to plot the magnitude response in linear units over $-\pi \leq \omega \leq \pi$ to match the plots in Figure P-8.14:

```
% ii. Plot the magnitude response in linear units
% over the interval -pi <= w <= pi.
W = -pi : 0.001 : pi;
[H, W] = freqz( feedforwardCoeffs, feedbackCoeffs, W );
figure;
plot(W, abs(H));
```


## 2. Equalization. $\mathbf{5 0}$ points.

Signal Processing First, problem P-8.22, page 244.

Although not graded, please review the solution to problem 6.4 from fall 2017 at
http://users.ece.utexas.edu/~bevans/courses/signals/homework/fall2017/solution6.pdf

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.

Please read the homework guidelines at http://users.ece.utexas.edu/~bevans/courses/signals/homework/index.html

