Fall 2021 EE 313 Linear Systems and Signals Prof. Evans

Homework #4 Version 3.0

(with one editorial change to prob. 4.2 and five changes to prob. 4.4 in this color)

Finite Impulse Response (FIR) Filters

Assigned on Saturday, October 2, 2021
Due on Friday, October 8, 2021, by 11:59 pm via Canvas submission

Late homework is subject to a late penalty of two points per minute late.

Reading: McClellan, Schafer and Yoder, *Signal Processing First*, 2003, Chapter 5 (all).
See the book's companion Web site for demos and other supplemental information.
Web site contains solutions to hundreds of worked problems.

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
9:30 am				Evans (Zoom)	
10:00 am				Evans (Zoom)	
10:30 am					
11:00 am		Evans (EER 1.516)		Evans (EER 1.516)	
11:30 am		Evans (EER 1.516)		Evans (EER 1.516)	
12:00 pm		Evans (EER 1.516)		Evans (EER 1.516)	
12:30 pm		Evans (Zoom)			
1:00 pm		Evans (Zoom)			
1:30 pm					
2:00 pm					Evans (Zoom)
2:30 pm					Evans (Zoom)
3:00 pm					Tabbara (Zoom)
3:30 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:00 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:30 pm			Tabbara (Zoom)		

Prof. Evans holds coffee hours on Fridays 12-2pm in the EERC café. Meet others. Any topic.

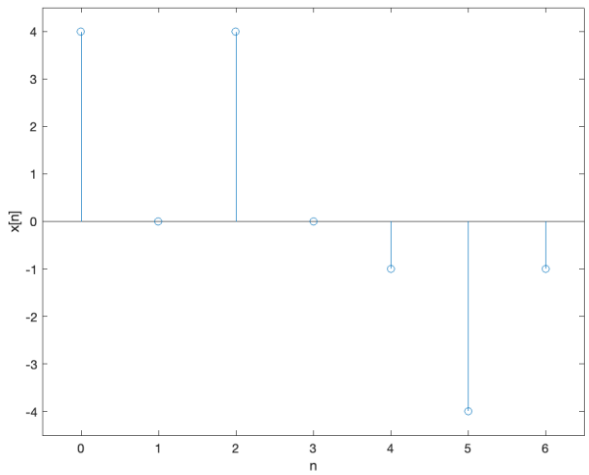
EE 313 tutoring is available 7-10pm on Sundays through Thursdays online.

1. Sampling & Aliasing. 22 points.

Signal Processing First, problem P-4.12, page 98.

2. Finite Impulse Response (FIR) Filter. 27 points.

Signal Processing First, problem P-5.3, page 126, but use the following input signal $x[n]$:




```
x = [ 4 0 4 0 -1 -4 -1 ];
n = 0 : length(x) - 1;
stem(n, x);
xlim( [-0.5, 6.5] );
ylim( [-4.5, 4.5] );
xlabel( 'n' );
ylabel( 'x[n]' );
```

Note that $x[n] = 0$ for $n < 0$ or $n > 6$

3. System Properties. 24 points.

Signal Processing First, problem P-5.9, page 128.

4. System Identification. 27 points.

A common problem that arises in audio and other systems is characterizing an unknown system. Consider an audio system with a speaker transmitting sound in a concert hall that is recorded by a microphone, and we'd like to characterize the acoustic response in the concert hall.

If we model an unknown system as a linear time-invariant finite impulse response (FIR) filter, we can try to infer its FIR filter coefficients b_k (i.e. the impulse response of the FIR filter). In practice, the unknown system will likely not be linear and time-invariant, but sometimes, a linear time-invariant system model captures useful information about the unknown system for the application at hand.

For the concert hall acoustics example, if we know the FIR filter coefficients for the linear time-invariant model of the concert hall acoustics, then we can mimic the effect of the concert hall by using its FIR coefficients to filter a music track and the output will sound as if it was played in the concert hall. Several audio systems with a "concert hall" effect will list several different concert halls.

Here is the input-output relationship for a linear time-invariant FIR filter with N coefficients with input $x[n]$ and output $y[n]$:

$$y[n] = b_0\, x[n] + b_1\, x[n-1] + b_2\, x[n-2] + \cdots + b_{N-1}\, x[n-(N-1)]$$

In advance, we don't know the value of N.

If we could input a known test signal for $x[n]$ and observe $y[n]$, we can attempt to compute the FIR coefficients b_k by *deconvolution*. In the test setup, we would assume that a laptop is sending the test signal to the speakers and the same laptop is receiving the output from the microphone. We'll assume that the test will begin at time $n = 0$ and perform deconvolution in the time domain.

The first output value (i.e. when $n = 0$) is

$$y[0] = b_0\, x[0] + b_1\, x[-1] + b_2\, x[-2] + \cdots + b_{N-1}\, x[-(N-1)]$$

For linear time-invariant systems, it is a necessary (but not sufficient) condition for the system to be "at rest", which means that all the initial conditions $x[-1], x[-2], \ldots, x[-(N-1)]$ must be zero.

$$y[0] = b_0\, x[0] + b_1\, x[-1] + b_2\, x[-2] + \cdots + b_{N-1}\, x[-(N-1)]$$

Since know $x[n]$ and $y[n]$ in our test setup, we have one equation and one unknown at $n = 0$:

$$y[0] = b_0\, x[0]$$

and we can compute

$$b_0 = \frac{y[0]}{x[0]}$$

For this calculation to be valid, the first value of the test signal, $x[0]$, cannot be zero.

(a) Develop an algorithm to compute the remaining values of b_k assuming you know the value of N.

(b) By hand, compute the values of b_k given and

 i. input signal $x[n]$ with non-zero values [1 2 3 4 5]

 ii. output signal $y[n]$ with non-zero values [1 1 1 1 1 -5]

 Use the MATLAB command conv to convolve $x[n]$ and b_k to make sure that the result is $y[n]$.

(c) Write a MATLAB program for your algorithm in (a) and apply it to the signals in part (b) to compute b_k. In your algorithm, stop computing values of b_k when $|b_k - b_{k-1}| \le 10^{-7}|b_k|$. That way, we don't need to know the value of N in advance. I have the stopping criterion as $|b_k - b_{k-1}| \le 10^{-7}|b_k|$ instead of $\left|\frac{b_k - b_{k-1}}{b_k}\right| \le 10^{-7}$ to avoid a possible division by zero error.

Returning to the room acoustics example, the input test signal could be a chirp signal that sweeps all audible frequencies.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.