

Homework #7

Discrete-Time Filtering and Continuous-Time System Properties and Convolution

Assigned on Wednesday, November 3, 2021

Due on Friday, November 12, 2021, by 11:59 pm via Canvas submission

Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Chapters 8 and 9.
 Companion Web site with demos and other supplemental information: <http://dspfirst.gatech.edu/>
 Web site contains solutions to selected homework problems from *DSP First*.

Office hours for Mr. Tabbara and Prof. Evans follow:

<i>Time Slot</i>	<i>Monday</i>	<i>Tuesday</i>	<i>Wednesday</i>	<i>Thursday</i>	<i>Friday</i>
9:30 am				Evans (Zoom)	
10:00 am				Evans (Zoom)	
10:30 am					
11:00 am		Evans (EER 1.516)		Evans (EER 1.516)	
11:30 am		Evans (EER 1.516)		Evans (EER 1.516)	
12:00 pm		Evans (EER 1.516)		Evans (EER 1.516)	
12:30 pm		Evans (Zoom)			
1:00 pm		Evans (Zoom)			
1:30 pm					
2:00 pm					Evans (Zoom)
2:30 pm					Evans (Zoom)
3:00 pm					Tabbara (Zoom)
3:30 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:00 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:30 pm			Tabbara (Zoom)		

Prof. Evans' will not be able to hold office hours or coffee hours on Friday, Nov. 5th.

[EE 313 tutoring](#) is available 7-10pm on Sundays through Thursdays online.

**** ASSUME THAT ALL SYSTEMS IN THIS HOMEWORK SET ARE LINEAR AND TIME-INVARIANT. ****

1. Discrete-Time Averaging Filters. 34 points.

For a discrete-time LTI system with input signal $x[n]$ and impulse response $h[n]$, the output signal $y[n]$ is the convolution of $h[n]$ and $x[n]$:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k] x[n - k]$$

- (a) Compute the output $y[n]$ when the input $x[n]$ is a rectangular pulse of amplitude 1 for $n = 0, 1, \dots, N_x - 1$ and amplitude 0 otherwise, and $x[n]$ is filtered by an LTI unnormalized averaging filter whose impulse response $h[n]$ is a rectangular pulse of amplitude 1 for $n = 0, 1, \dots, N_h - 1$ and amplitude 0 otherwise. Assume $N_x \neq N_h$.
- Write the difference equation relating output $y[n]$ and input $x[n]$. 3 points.
 - What are the initial conditions and what values should they be set to? 3 points.
 - Develop a formula for $y[n] = h[n] * x[n]$ using the convolution definition in terms of N_x and N_h . Show the intermediate steps in computing the convolution. 6 points.
 - Validate the formula for $y[n]$ by using Matlab to compute the convolution for $N_x = 9$ samples and $N_h = 4$ samples. 3 points.
 - Use the z -transform to find $y[n] = h[n] * x[n]$. Track the region of convergence. 3 points.
- (b) An LTI system outputs the weighted average of the previous output value $y[n - 1]$ and the current input value $x[n]$ using the difference equation $y[n] = 0.9 y[n - 1] + 0.1 x[n]$ for $n \geq 0$.
- What are the initial conditions and what value should they be set to? 3 points.
 - Compute a formula for the impulse response $h[n]$ for the system. 3 points.
 - Develop a formula for $y[n] = h[n] * x[n]$ using the convolution definition when the input signal is $x[n] = 0.8^n u[n]$. 6 points.
 - Use the z -transform to find $y[n] = h[n] * x[n]$. Track the region of convergence. 4 points.

2. Transfer Function and Frequency Response Connections. 16 points.

Signal Processing First, problem P-8.16, page 242.

In addition, for each of the four filters,

- give a formula for the transfer function in the z -domain including the region of convergence,
- give a formula for the frequency response from the transfer function in the z -domain in part i. Why can convert
- plot the magnitude response in the frequency domain
- indicate the frequency selectivity as lowpass, highpass, bandpass, or bandstop.

Connecting filter poles/zeros to its frequency selectivity: Please see lecture slide 11-7 and watch a YouTube [video in spring 2014](#) for the Real-Time Digital Signal Processing Lab course from the 1:29 to the 22:25 mark and from 43:01 to the end (50:51 mark). When poles and zeros are separated in angle, the angles of the poles near the unit circle indicate the frequencies in the passband(s), and the angles of the zeros near or on the unit circle indicate the frequencies in the stopband(s). Poles must be inside the unit circle for bounded-input bounded-output (BIBO) stability. Please see lecture slides 11-8 and 11-9.

3. Continuous-Time System Properties. 16 points.

Signal Processing First, problem P-9.2, page 279.

4. Continuous-Time Averaging Filters. 34 points.

For a continuous-time LTI system with input signal $x(t)$ and impulse response $h(t)$, the output signal $y(t)$ is the convolution of $h(t)$ and $x(t)$:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda) x(t - \lambda) d\lambda$$

(a) Compute the output $y(t)$ when the input $x(t)$ is a rectangular pulse of amplitude 1 for $t \in [0, T_x]$ and amplitude 0 otherwise and $x[n]$ is filtered by an LTI unnormalized averaging filter whose impulse response $h(t)$ is a rectangular pulse of amplitude 1 for $t \in [0, T_h]$ and amplitude 0 otherwise. Assume $T_x \neq T_h$.

- i. Write an equation relating output $y(t)$ and input $x(t)$. 4 points
- ii. What is(are) the initial condition(s) and what value should it(they) be set to? 3 points
- iii. Develop a formula for $y(t) = h(t) * x(t)$ using the convolution definition in terms of T_x and T_h . Show the intermediate steps in computing the convolution. 6 points
- iv. Validate the formula for $y(t)$ to compute the convolution for $T_x = 9$ seconds and $T_h = 4$ seconds. 3 points

(b) When an input signal has an average value of zero, i.e. the DC component is zero, an LTI integrator can be used as an averaging filter. The differential equation governing the input-output relationship is

$$y(t) = \int_0^t x(\tau) d\tau \quad \text{for } t \geq 0$$

- i. What is(are) the initial condition(s) and what value should it(they) be set to? 3 points
- ii. What is the impulse response? 3 points
- iii. Develop a formula for $y(t) = h(t) * x(t)$ using the convolution definition when the input signal is $x(t) = u(t)$. Note that $x(t)$ has bounded amplitude. 9 points
- iv. Is the LTI integrator bounded-input bounded-output (BIBO) stable? Your work in part iii might be helpful. 3 points

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

NOTE: In your solutions, please put all work for problem 1 together, then all work for problem 2 together, etc. Please see additional homework guidelines on the homework page.

Please read the [homework guidelines](#).