Fall 2021 EE 313 Linear Systems and Signals Prof. Evans

Homework #9

Fourier and Laplace Transforms

Assigned on Sunday, November 28, 2021
Due on Friday, December 3, 2021, by 11:59 pm via Canvas submission

Late homework is subject to a penalty of two points per minute late.

Reading: McClellan, Schafer & Yoder, *Signal Processing First*, 2003, Ch. 11 and Supplemental Chapter on The Laplace Transform.
Companion Web site with demos and other supplemental information: http://dspfirst.gatech.edu/
Web site contains solutions to selected homework problems from *DSP First*.

Office hours for Mr. Tabbara and Prof. Evans follow:

Time Slot	***Monday***	***Tuesday***	***Wednesday***	***Thursday***	***Friday***
9:30 am				Evans (Zoom)	
10:00 am				Evans (Zoom)	
10:30 am					
11:00 am		Evans (EER 1.516)		Evans (EER 1.516)	
11:30 am		Evans (EER 1.516)		Evans (EER 1.516)	
12:00 pm		Evans (EER 1.516)		Evans (EER 1.516)	**
12:30 pm		Evans (Zoom)			**
1:00 pm		Evans (Zoom)			****
1:30 pm					****
2:00 pm					Evans (Zoom)
2:30 pm					Evans (Zoom)
3:00 pm					Tabbara (Zoom)
3:30 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:00 pm			Tabbara (Zoom)		Tabbara (Zoom)
4:30 pm			Tabbara (Zoom)		

** Prof. Evans holds coffee/advising hours on Fridays 12:00-2:00pm in the EER café.

EE 313 tutoring is available 7-10pm on Sundays through Thursdays online.

Please read the homework guidelines and homework hints.

1. **Continuous-Time Fourier Transforms Using Transform Properties and Pairs.** ***18 points.***

   Signal Processing First, problem P-11.8, page 343.

2. **Transfer Functions in the Laplace Domain.** ***27 points.***

   Plot each signal in the time domain for $-1 < t < 1$, compute the Laplace transform including the region of convergence, and sketch the pole-zero plot and region of convergence for the following signals:

   (a) $x(t) = \cos(20\,\pi\,t)\,u(t)$. *6 points.*

   (b) $x(t) = e^{-8t}\,u(t)$. *6 points.*

   (c) $x(t) = (1 - e^{-8t})\,u(t)$. *6 points.*

   Same as Homework Problem 10.2 from Fall 2018.

   For each part, what is the frequency selectivity— lowpass, highpass, bandpass, bandstop, allpass or notch? *3 points for each part.*

3. **Transfer Function in the Laplace Domain.** ***55 points.***

   A continuous-time system with input $x(t)$ and output $y(t)$ is described by the following linear constant coefficient differential equation for $t > 0^-$:

$$\frac{d}{dt}y(t) + 2y(t) = \frac{d}{dt}x(t)$$

   Initial conditions are set to zero, i.e. $y(0^-) = 0$ and $x(0^-) = 0$, so the system will have linearity and time-invariant properties. (Notice the two different uses of "linear". In the case of linear constant different equation, "linear" refers to "affine" which is a line that does not necessarily go through the origin.)

   (a) What is the transfer function $H(s)$ of the system in the Laplace domain including the region of convergence? *9 points.*

   (b) What is the impulse response $h(t)$ of the system? *9 points.*

   (c) Find the frequency response $H(j\omega)$ of the system from the transfer function. Why is the substitution $s = j\omega$ valid? *9 points.*

   (d) From part (c), plot the magnitude response. What is the frequency selectivity— lowpass, highpass, bandpass, bandstop, allpass or notch? *9 points.*

   (e) For $x(t) = u(t)$, find $X(s)$ and $Y(s)$. *9 points.*

   (f) From part (e), find $y(t)$ by taking the inverse Laplace transform of $Y(s)$. *10 points.*

   Same as Homework Problem 10.3 from Fall 2018.

As stated on the course descriptor, "Discussion of homework questions is encouraged. Please be sure to submit your own independent homework solution."

Please read the homework guidelines and homework hints.