Lecture 13 Continuous-Time Convolution (Part 1)

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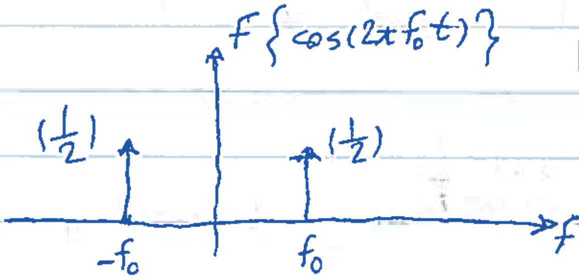
EE 313 Linear Systems and Signals

The University of Texas at Austin



$$\cos(2\pi f_0 t) = \frac{1}{2} e^{j2\pi f_0 t} + \frac{1}{2} e^{-j2\pi f_0 t}$$

$$x(t) = 5\delta(t) \longrightarrow \int_{-\infty}^{\infty} x(t)\,dt = \int_{-\infty}^{\infty} 5\delta(t)\,dt = 5\int_{-\infty}^{\infty} \delta(t)\,dt = 5$$

$$\xrightarrow[a\,x(t)]{x(t)} \boxed{(\cdot)^2} \longrightarrow y(t) = x^2(t)$$

$$y_{scaled}(t) = (a\,x(t))^2 = a^2 x^2(t) \stackrel{?}{=} a\,y(t)$$

NO ×

only works for $a=0$, $a=1$

$$\xrightarrow[x(t)]{x(t-\tau)} \boxed{(\cdot)^2} \longrightarrow y_{shifted}(t) = x^2(t-\tau) \stackrel{?}{=} y(t-\tau) \checkmark$$

$y(t) = x^2(t)$ Yes

observe for $t = t_0$

$$C_0 = \int_{-\infty}^{t_0} x(u)\,du \qquad \xrightarrow[a\,x(t)]{x(t)} \boxed{\int_{t_0}^{t} (\cdot)\,dt + C_0} \xrightarrow[y_{scaled}(t)]{y(t)}$$

$$y(t) = \int_{t_0}^{t} x(t)\,dt + C_0$$

$$y_{scaled}(t) = \int_{t_0}^{t} a\,x(t)\,dt + C_0 = a\int_{t_0}^{t} x(t)\,dt + C_0 \stackrel{?}{=} a\,y(t)$$

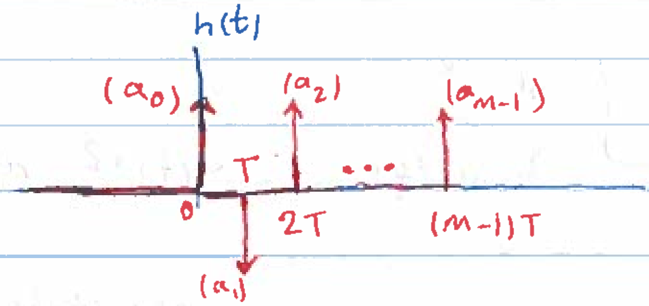
Yes if $C_0 = 0$

$$a\, y(t) = a\left(\int_{t_0}^{t} x(t)\,dt + C_0\right)$$

$$y(t) = a_0 x(t) + a_1 \cancel{x(t)}\, x(t-T) + a_2 x(t-2T) + \dots + a_{M-1} x(t-(M-1)$$

$$y(t) = \sum_{m=0}^{M-1} x(t-mT)$$

$$y[n] = \sum_{m=0}^{M-1} a_m x[n-m]$$



$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y_{impulse}(t) = \int_{-\infty}^{\infty} h(\tau)\, \delta(t-\tau)\, d\tau = h(\tau)\Big|_{\tau=}$$

$$\rightarrow y_{impulse}(t) = h(t)$$