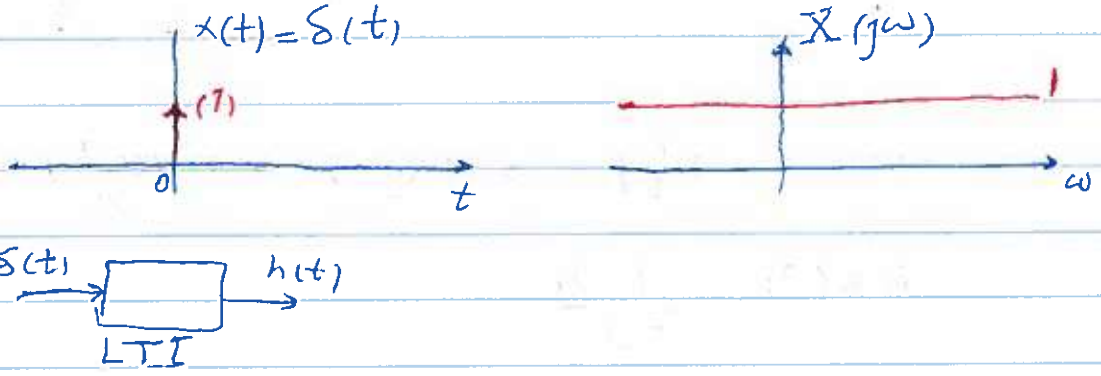
Continuous-Time Fourier Transform

Notes by Mr. Houshang Salimian



Ideal Delay

$$\delta(t) \longrightarrow \boxed{\text{Delay by } T} \longrightarrow h(t) = \delta(t-T)$$

$\delta(t)$	1
$\delta(t-T)$	$e^{-j\omega T}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$ if $\text{Re}\{a\} > 0$

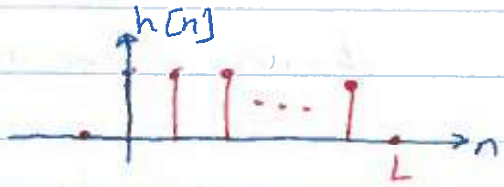
$$x(t) = \delta(t-T) \xrightarrow{F} e^{-j\omega T} \longrightarrow |X(j\omega)| = 1$$

$$\angle X(j\omega) = -\omega T$$

$$\text{Group\_Delay}(\omega) = -\frac{d}{d\omega} \angle H(j\omega) = T$$

in MATLAB $\longrightarrow$ grpdelay

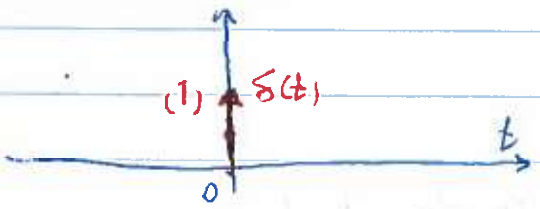
Averaging Filter in discrete time

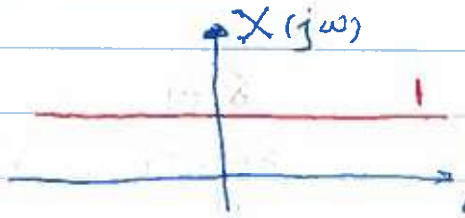


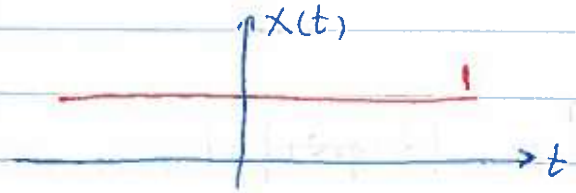
$$x(t) = \text{rect}\left(\frac{t}{T}\right) \longrightarrow X(j\omega) = \text{sinc}\left(\frac{\omega T}{2\pi}\right) = 2\,\frac{\sin\left(\frac{\omega T}{2}\right)}{\omega}$$

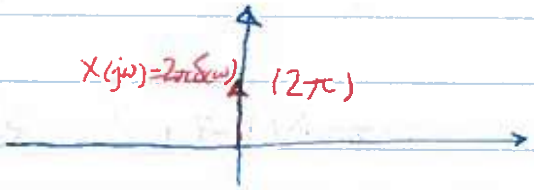
$$\text{sinc}(\theta) = \frac{\sin(\pi\theta)}{\pi\theta}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt \quad , \quad x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$



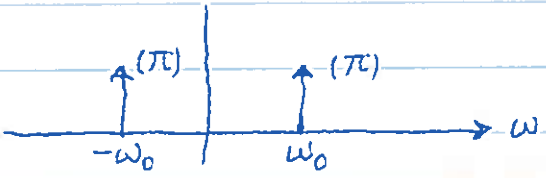






$x(t)$	$X(j\omega)$
$\delta(t)$	1
$\delta(t-T)$	$e^{-j\omega T}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$ if $\text{Re}\{a\}>0$
$\text{rect}\left(\frac{t}{T}\right)$	$\text{sinc}\left(\frac{\omega T}{2\pi}\right) = \frac{2\sin\left(\frac{\omega T}{2}\right)}{\omega}$
$h(t) * x(t)$	$H(j\omega)X(j\omega)$
$x(t)\,y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$

$\cos(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow}$



$\sin(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow}$