Lecture 15 Continuous-Time Fourier Transform, Part 2 Fall 2018

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Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$x(t) = \frac{1}{2\pi j} \oint_{c} X(s)\, e^{st}\, ds$$

- contour integration over the region of convergence
- we'll use transform pairs and properties instead

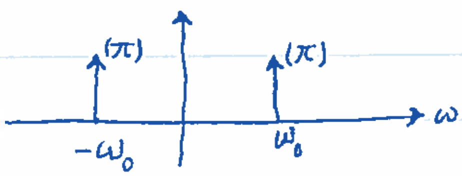
Laplace transforms are defined in lecture 17

Fourier Transform

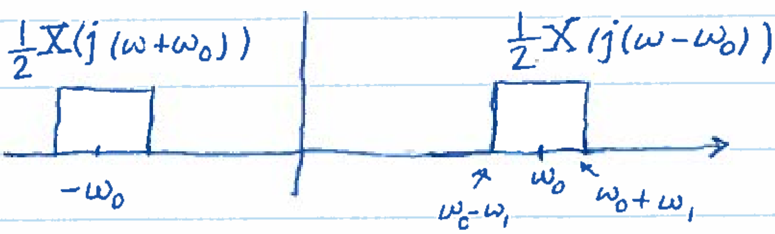
$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

If the Laplace transform of a signal is valid for imaginary values of s, then one can convert a Laplace transform to a Fourier transform by substituting s = j w

$$F\{\cos(\omega_0 t)\} = \pi\delta(\omega+\omega_0) + \pi\delta(\omega-\omega_0)$$

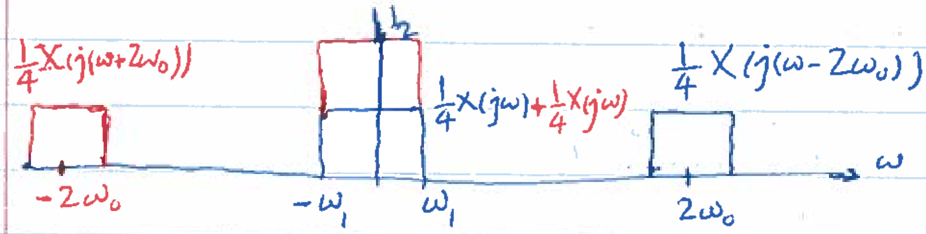


$$\int_{-\infty}^{\infty} \pi\,\delta(\omega+\omega_0)\, d\omega = \pi \int_{-\infty}^{\infty} \delta(\omega+\omega_0)\, d\omega = \pi$$



$$v(t) = y(t)\cos(\omega_0 t)$$

$$V(j\omega) = \frac{1}{2}Y(j(\omega+\omega_0)) + \frac{1}{2}Y(j(\omega-\omega_0))$$



$$a_0 = \frac{1}{T_s}\int_{-\frac{1}{2}T_s}^{\frac{1}{2}T_s} \delta(t)\, e^{-j2\pi k f_s t}\, dt = \frac{1}{T_s}$$

$$x(t) \longrightarrow \boxed{\int_{0^-}^{t} x(\tau)d\tau} \longrightarrow y(t) = h(t) * x(t)$$

$$u(t) = \int_{0^-}^{t} \delta(t)dt = 1 \quad \text{for } t \geq 0^-$$

$$\mathcal{L}\{u(t)\} = \frac{1}{s} \quad \text{if} \quad Re\{s\} > 0$$