

The University of Texas at Austin  
Dept. of Electrical and Computer Engineering  
Final Exam *Solutions*

Date: December 18, 2017

Course: EE 313 Evans

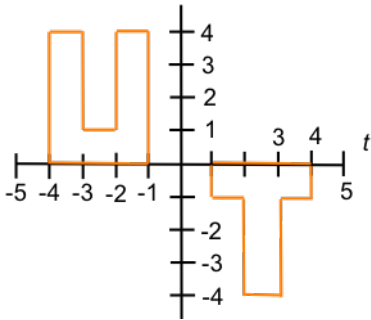
Name: \_\_\_\_\_ **Chronicles** **Kane**  
Last, First

- The exam is scheduled to last three hours.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please turn off all cell phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- **Fully justify your answers.** If you decide to quote text from a source, please give the quote, page number and source citation.

Problem	Point Value	Your score	Topic
1	10		Continuous-Time Fourier Series
2	12		Discrete-Time Convolution
3	9		Discrete-Time Convolution II
4	12		Continuous-Time Convolution
5	12		Discrete-Time FIR Filter Design
6	12		Discrete-Time IIR Filter Design
7	12		Continuous-Time Feedback System
8	9		Continuous-Time Circuit Analysis
9	12		Sinusoidal Amplitude Modulation
Total	100		

**Problem 1.** Continuous-Time Fourier Series. *10 points.*

One period of a periodic “UT” signal is shown below:



HW 8.4  
Lecture slide 3-8  
SPFirst page 55

The fundamental period  $T_0$  is 10 seconds.

(a) Compute the Fourier series coefficients. *9 points.*

**Average value over the fundamental period is**

$$a_0 = \frac{1}{T_0} \int_0^{T_0} x(t) dt = \frac{4 + 1 + 4 + (-1) + (-4) + (-1)}{10} = \frac{3}{10}$$

**The Fourier series integral is “additive”.**

**The “UT” signal is composed of six rectangular pulses of one second in duration.**

**Here is the Fourier series of a rectangular pulse of height 1 lasting from  $t_1$  to  $t_1+1$  seconds:**

$$a_{k,t_1} = \frac{1}{T_0} \int_{t_1}^{t_1+1} e^{-j2\pi(kf_0)t} dt = \frac{1}{T_0} \left[ \frac{e^{-j2\pi(kf_0)t}}{-j2\pi(kf_0)} \right]_{t_1}^{t_1+1} = \frac{1}{T_0} \left( \frac{e^{-j2\pi(kf_0)(t_1+1)} - e^{-j2\pi(kf_0)t_1}}{-j2\pi(kf_0)} \right)$$

$$a_{k,t_1} = -\frac{1}{T_0} \left( \frac{e^{-j2\pi(kf_0)} - 1}{j2\pi(kf_0)} \right) e^{-j2\pi(kf_0)t_1} = \left( \frac{1 - e^{-j2\pi(kf_0)}}{j2\pi k} \right) e^{-j2\pi(kf_0)t_1}$$

$$a_k = 4 a_{k,-4} + a_{k,-3} + 4 a_{k,-2} - a_{k,1} - 4 a_{k,2} - a_{k,3}$$

*See the MATLAB code on the next page.*

(b) If the periodic “UT” signal is synthesized using 100 Fourier series coefficients, will it suffer from Gibbs phenomenon? *1 point.*

**Yes, for any finite number of Fourier series coefficients, Fourier series synthesis will suffer from Gibbs phenomenon at the amplitude discontinuities at  $t \in \{-4, -3, -2, -1, 1, 2, 3, 4\}$ .**

*See the MATLAB code on the next page.*

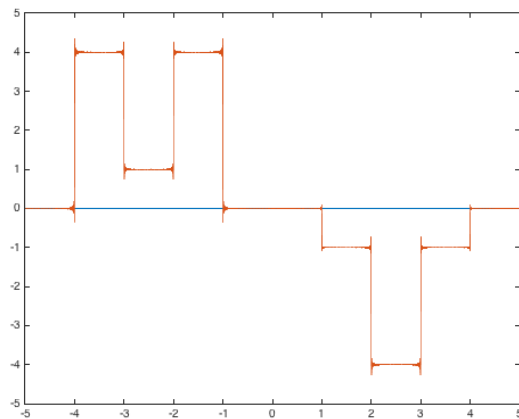
```

% Fourier synthesis to illustrate answers
% for EE 313 Fall 2017 Final Exam Prob 1
% Prof. Brian L. Evans
% The University of Texas at Austin

% Pick a value for the period of x(t)
T0 = 10;
f0 = 1 / T0;
% Pick number Fourier synthesis terms
N = 100;
fmax = N * f0;
% Define a sampling rate for plotting
fs = 24 * fmax;
Ts = 1 / fs;
% Define samples in time for one period
t = -0.5*T0 : Ts : 0.5*T0;
% First Fourier synthesis term
a0 = 0.3;
x = a0 * ones(1, length(t));
figure;
plot(t, x);
hold on;
% Generate each pair of synthesis terms
for k = 1 : N
    % Define Fourier coeffs at k and -k
    akpos = 0;
    akneg = 0;
    t1vec = [ -4 -3 -2  1  2  3 ];
    C0vec = [  4  1  4 -1 -4 -1 ];
    for i = 1 : 6
        t1 = t1vec(i);
        C0 = C0vec(i);
        akpos = akpos + C0 * exp(-j*2*pi*k*f0*t1) * (1 - exp(-j*2*pi*k*f0)) /
(j*2*pi*k);
        akneg = akneg + C0 * exp(-j*2*pi*(-k)*f0*t1) * (1 - exp(-j*2*pi*(-
k)*f0)) / (j*2*pi*(-k));
    end
    theta = j*2*pi*k*f0*t;
    x = x + akpos * exp(theta) + akneg * exp(-theta);
    % Plot Fourier synthesis for indices -k ... k
    plot(t, x);
end
hold off;

close all;
hold on
plot(t, zeros(1, 24001));
plot(t, x)

```



**Problem 2.** Discrete-Time Convolution. 12 points.

Using forward and inverse z-transforms, derive the formula in the time domain for

$$y[n] = x[n] * h[n]$$

where

$$x[n] = na^n u[n] \text{ and } h[n] = b^n u[n]$$

Here,  $a$  and  $b$  are complex-valued constants such that  $a \neq b$ .

$$Y(z) = X(z) H(z) \text{ where } H(z) = \frac{1}{1-bz^{-1}} \text{ for } |z| > |b| \text{ and } X(z) = \frac{az^{-1}}{(1-az^{-1})^2} \text{ for } |z| > |a|$$

Using partial fractions decomposition,

$$Y(z) = \frac{az^{-1}}{(1-az^{-1})^2} \left( \frac{1}{1-bz^{-1}} \right) = \frac{C_1 z^{-1}}{(1-az^{-1})^2} + \frac{C_2}{1-az^{-1}} + \frac{C_3}{1-bz^{-1}} \quad (1)$$

By putting terms over a common denominator, we have 3 equations in 3 unknowns  $C_1, C_2, C_3$ :

$$Y(z) = \frac{(-bC_1 + abC_2 + a^2C_3)z^{-2} + (C_1 - (a+b)C_2 - 2aC_3)z^{-1} + (C_2 + C_3)}{(1-az^{-1})^2(1-bz^{-1})} = \frac{az^{-1}}{(1-az^{-1})^2(1-bz^{-1})} \quad (2)$$

First, isolate  $C_1$  by multiplying both sides of (1) by  $z(1-az^{-1})^2$  and evaluate at  $z = a$ :

$$C_1 = \left[ \frac{a}{1-bz^{-1}} \right]_{z=a} = \frac{a}{1-\frac{b}{a}} = \frac{a^2}{a-b}$$

Second, isolate  $C_3$  by multiplying both sides of (1) by  $1-bz^{-1}$  and evaluate at  $z = b$ :

$$C_3 = \left[ \frac{az^{-1}}{(1-az^{-1})^2} \right]_{z=b} = \frac{\frac{a}{b}}{\left(1-\frac{a}{b}\right)^2} = \frac{ab}{(b-a)^2}$$

Third, from (2), we notice that  $C_2 + C_3 = 0$ , or

$$C_2 = -C_3 = -\frac{ab}{(b-a)^2}$$

$$y[n] = C_1 na^n u[n] + C_2 a^n u[n] + C_3 b^n u[n]$$

Z-transform approach to solving a convolution problem

- Solution to homework 6.4(a)
- Course Handout F “Convolution of Two Causal Exponential Sequences”

Z-transforms

- $x[n]$ : Solution to homework 6.4(b) using Approach #2
- $h[n]$ : SPFirst Table 8-1 on page 217 or lecture slide 11-5

Student used MATLAB command `ztrans` to find z-transform of  $x[n]$  using the Symbolic Toolbox:

```
syms a n z
f = n * (a^n) * heaviside(n);
ztrans(f, n, z)
```

The answer is

$$(a*z)/(a-z)^2$$



**Problem 3.** Discrete-Time Convolution II. 9 points.

Compute the discrete-time convolution

$$y[n] = x[n] * h[n]$$

where

$x[n]$  is a causal rectangular pulse with an amplitude of 1 and a duration of  $N_x$  samples, and  $h[n]$  is a causal rectangular pulse with an amplitude of 1 and a duration of  $N_h$  samples.

(a) Give a formula for  $y[n]$  in terms of  $N_x$  and  $N_h$ . 6 points.

*This is a discrete-time version of Midterm 2.2(a). It is also related to Midterm 2.1(a).*

**Convoluting two causal signals gives a causal signal; hence,  $y[n]$  will be a causal signal.**

**Also,  $y[n]$  will have  $N_x + N_h - 1$  samples starting at  $n = 0$  and ending at  $n = N_x + N_h - 2$ .**

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m] x[n - m] = \sum_{m=0}^{N_h-1} h[m] x[n - m]$$

**We are flipping signal  $x$  in the convolution variable  $m$  and shifting it by  $n$ .**

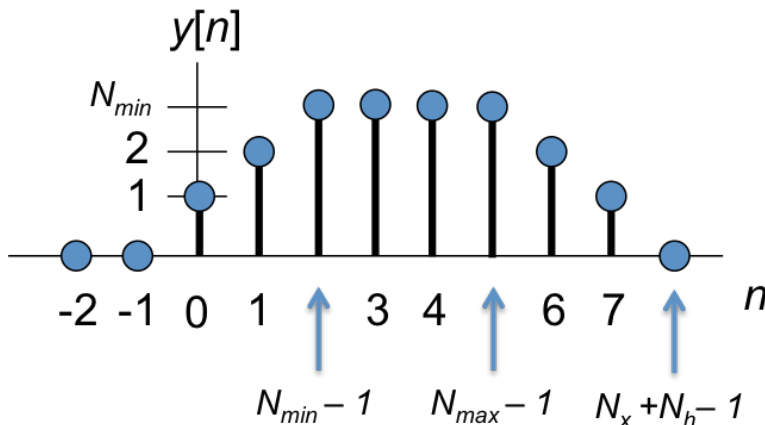
**We will have five intervals of interest: (1) no overlap, (2) partial overlap with increasing amount of overlap as  $n$  increases, (3) complete overlap, (4) partial overlap with decreasing amount of overlap as  $n$  increases, and (5) no overlap. See Handout E “Convolution of Rectangular Pulses”.**

**Convoluting two rectangular pulses of different lengths gives a trapezoid (Midterm 2 Problem 1).**

**Let  $N_{min} = \min\{N_x, N_h\}$  and  $N_{max} = \max\{N_x, N_h\}$  and  $N = N_x + N_h - 1$ .**

$$y[n] = \begin{cases} 0 & \text{for } n < 0 \\ (n + 1) & \text{for } 0 \leq n < N_{min} - 1 \\ N_{min} & \text{for } N_{min} - 1 \leq n \leq N_{max} - 1 \\ N_h + N_x - 1 - n & \text{for } N_{max} - 1 < n < N \\ 0 & \text{for } n \geq N \end{cases}$$

(b) Plot  $y[n]$ . 3 points. Plot shown for  $N_h = 3$  and  $N_x = 6$  which gives  $N_{min} = 3$  and  $N_{max} = 6$  and  $N = 8$ .



```
h = [ 0 1 1 1 0];
x = [ 0 1 1 1 1 1 0];
y = conv(h, x);
n = [-2 -1 0 1 2 3 4 5 6 7 8 9];
stem(n, y);
ylim( [-0.2 3.2] );
```

**Problem 4.** Continuous-Time Convolution. *12 points.*

Convolve the two-sided continuous-time signals

$$x(t) = \cos(\omega_0 t) \text{ and } h(t) = \frac{\sin(\omega_1 t)}{\omega_1 t}$$

Both signals are defined for  $-\infty < t < \infty$ .

**Both signals are two-sided in the time domain. The time-domain approach to computing the convolution integral  $y(t) = x(t) * h(t)$  would be very involved. The Laplace transform approach cannot be used because neither signal has a Laplace transform.**

**However, both signals have continuous-time Fourier transforms. We will compute the product of the continuous-time Fourier transforms of signals  $x(t)$  and  $h(t)$ , i.e.  $Y(j\omega) = X(j\omega)H(j\omega)$ , and then take the inverse continuous-time Fourier transform of  $Y(j\omega)$ . From *SPFirst* page 338,**

$$X(j\omega) = \pi \delta(\omega + \omega_0) + \pi \delta(\omega - \omega_0)$$

$$H(j\omega) = \frac{\pi}{\omega_1} (u(\omega + \omega_1) - u(\omega - \omega_1))$$

**Note that  $H(j\omega)$  is an ideal lowpass filter that passes frequencies  $-\omega_1 \leq \omega \leq \omega_1$  and  $X(j\omega)$  only has frequency components at  $-\omega_0$  and  $\omega_0$  :**

$$y(t) = \begin{cases} 0 & \text{if } \omega_1 < \omega_0 \\ \frac{\pi}{\omega_1} \cos(\omega_0 t) & \text{otherwise} \end{cases}$$

$y(t)$  is defined for  $-\infty < t < \infty$ .

*HW 3.3 & 8.2*

*Lecture slides 14-8 to 14-13*

*Lecture slide 15-9*

*SPFirst Sec 10-2, 10-3 & 10-4*

*SPFirst Ch. 11 pp. 338-339*

**Problem 5. Discrete-Time FIR Filter Design. 12 points.**

Design a discrete-time finite impulse response (FIR) filter that will

- Zero out 0 Hz,
- Zero out all harmonics of 60 Hz, i.e. 60 Hz, 120 Hz, 180 Hz, 240 Hz, 300 Hz, 360 Hz, etc., and
- Pass all other frequencies in the range (-240 Hz, 240 Hz) as much as possible

The FIR filter is linear and time-invariant.

(a) What sampling rate would you use? Why? 3 points.

**The sampling rate  $f_s$  should be chosen so that  $f_s > 2 f_{max}$ .**

**Since we would like to pass frequencies in the range  $-240 \text{ Hz} < f < 240 \text{ Hz}$ , the sampling rate should be  $f_s \geq 480 \text{ Hz}$ .**

**We also want the sampling rate to be a multiple of 60 Hz. This will allow multiples of 60 Hz that are greater than or equal to  $\frac{1}{2} f_s$  to alias down to a multiple of 60 Hz less than  $\frac{1}{2} f_s$  and will hence get filtered out.**

**Let  $f_s = 480 \text{ Hz}$ .**

(b) How many zeros would the discrete-time FIR filter have? Give formulas for them. Plot them on a pole-zero plot. 6 points.

**In the  $z$ -domain, we will place a zero on the unit circle for each frequency to be zeroed out.**

**For  $f_s = 480 \text{ Hz}$ , we would like to zero out continuous-time frequencies of -240 Hz, -180 Hz, -120 Hz, -60 Hz, 0 Hz, 60 Hz, 120 Hz, and 180 Hz.**

**Mapping from continuous-time frequency  $f_0$  to discrete-time frequency  $\hat{\omega}_0$  is  $\hat{\omega}_0 = 2\pi \frac{f_0}{f_s}$ .**

**Due to sampling, a continuous-time frequency of -240 Hz will map to a discrete-time frequency of  $-\pi$  and 240 Hz will map to a discrete-time frequency of  $\pi$ . Since the discrete-time frequency domain is periodic with period  $2\pi$ , eliminating  $-\frac{1}{2} f_s$  will also eliminate  $\frac{1}{2} f_s$ .**

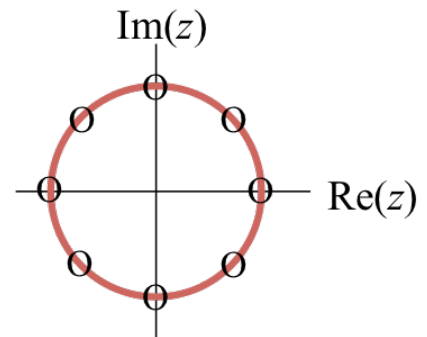
**We need 8 zeros in the  $z$ -domain. Each zero has the form  $z = e^{j\hat{\omega}_i}$  where  $\hat{\omega}_i = 2\pi \frac{f_i}{f_s}$  and  $f_i \in \{-240, -180, -120, -60, 0, 60, 120, 180\}$  and  $f_s = 480 \text{ Hz}$ . Zeros are equally spaced on the unit circle at angles  $-\pi/2, -3\pi/4, -\pi/2, -\pi/4, 0, \pi/4, \pi/2,$  and  $3\pi/4$ , respectively.**

**There are no poles, or one could say that there are eight “artificial” poles at the origin in the  $z$ -domain.**

(c) Give the formula for the impulse response for the discrete-time FIR filter. Please simplify the formula as much as possible. 3 points.

**“Echo” filter from Mini-Project #2:  $h[n] = \delta[n] - \delta[n - 8]$**

**Transfer function in the  $z$ -domain is  $H(z) = 1 - z^{-8}$  and its zeros are eight roots of unity.**



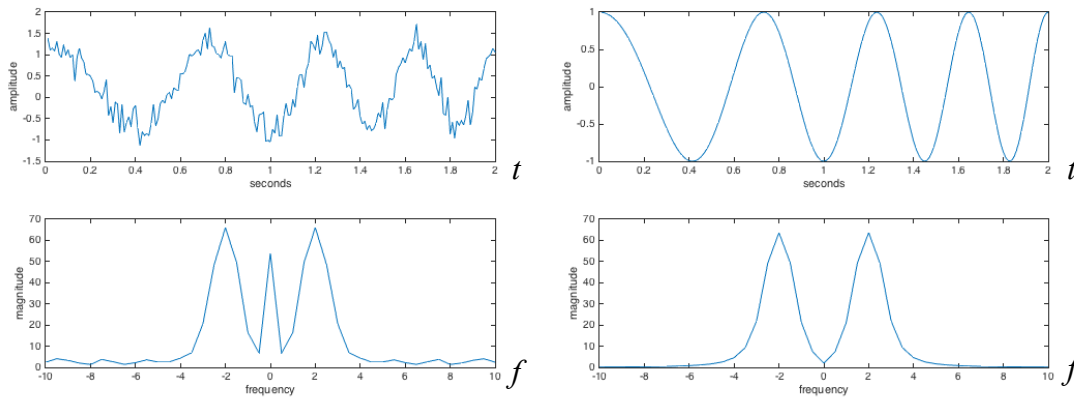
**Problem 6.** Discrete-Time IIR Filter Design. *12 points.*

A sinusoidal signal of interest has a principal frequency that can vary over time in the range 1-3 Hz.

Using a sampling rate of  $f_s = 20$  Hz, a sinusoidal signal was acquired for 2s and shown below on the left in the upper plot. The lower plot is the magnitude of the signal's frequency content.

The acquired signal has interference and other impairments that reduce the signal quality.

The signal shown below on the right is the sinusoidal signal without the impairments.



Design a second-order infinite impulse response (IIR) filter to filter the acquired signal above on the left to give the sinusoidal signal above on the right. **Note: Upper right signal is a chirp signal.**

(a) Give the two poles and the two zeros of the second-order IIR filter. *9 points.*

**Passband: 1-3 Hz with center frequency  $f_c = 2$  Hz.**  $\hat{\omega}_c = 2\pi \frac{f_c}{f_s} = 2\pi \frac{2}{20} = \frac{\pi}{5}$

**Poles at  $p_0 = 0.9e^{j\hat{\omega}_c}$  and  $p_1 = 0.9e^{-j\hat{\omega}_c}$**

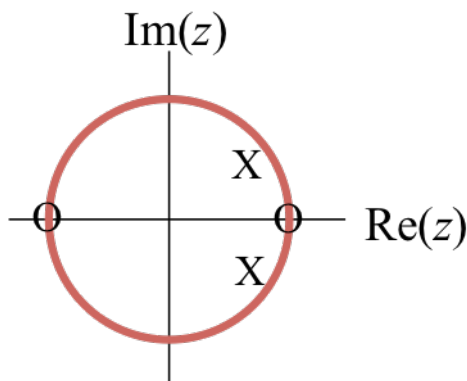
**Stopbands: Remove impairments around 0 Hz and in 4-10 Hz range.**

**Zeros at  $z_0 = e^{j0} = 1$  and  $z_1 = e^{j2\pi \frac{10\text{Hz}}{20\text{Hz}}} = e^{j\pi} = -1$**

**In the time domain plots, signal with interference and filtered signal are real-valued. Hence, filter coefficients must be real-valued, which means that zeros are either real-valued or in a conjugate symmetric pair and poles are either real-valued or in a conjugate symmetric pair.**

(b) Draw the pole-zero diagram for the second-order IIR filter.

*3 points.* Pole-zero plot is drawn by hand below.



```
% zeros on unit circle
z0 = 1;
z1 = -1;
numer = [1 -(z0+z1) z0*z1];
% poles inside unit circle
r = 0.9; poleAngle = pi/5;
p0 = r * exp(j*poleAngle);
p1 = r * exp(-j*poleAngle);
denom = [1 -(p0+p1) p0*p1];
% pole-zero plot
zplane(numer, denom);
% frequency response plot
figure;
freqz(numer, denom);
```

**Problem 7.** Continuous-Time Feedback System. 12 points.

HW 9.3 & 9.4

Consider a linear time-invariant (LTI) system with input signal  $x(t)$  and output signal  $y(t)$  that is governed by the following second-order differential equation for  $t \geq 0$ :

$$y''(t) + 6y'(t) + Ky(t) = x(t)$$

where  $K$  is a real-valued constant.

- (a) Derive the transfer function  $H(s)$  for the system, which will depend on  $K$ . 3 points.

**Because the system is LTI, the initial conditions  $y(0)$  and  $y'(0)$  have to be zero.**

**We take the Laplace transform of both sides of the equation to obtain**

$$s^2 Y(s) + 6s Y(s) + K Y(s) = X(s)$$

Slides 18-5, 18-7 & 18-8

$$(s^2 + 6s + K) Y(s) = X(s)$$

SPFirst Sec. 16-6.2

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 6s + K}$$

- (b) Give the range of values for  $K$  for which the system is bounded-input bounded-output (BIBO) stable. 6 points.

**For a BIBO stable system, the poles must be in the left-hand side of the Laplace domain, i.e. have negative real components.**

**Using the quadratic formula to find the roots of the denominator, poles  $p_0$  and  $p_1$  are at**

$$\frac{-6 \pm \sqrt{6^2 - 4(1)K}}{2} = -3 \pm \sqrt{9 - K}$$

**When  $9 - K \geq 0$  or  $K \leq 9$ , poles are real-valued. For  $K = 0$ , poles are at  $s = 0$  and  $s = -6$ , which is BIBO unstable due to the pole at  $s = 0$ . The system is BIBO stable for  $0 < K \leq 9$ .**

**When  $9 - K < 0$  or when  $K > 9$ , the poles have a real component of  $-3$  and imaginary components of  $\pm\sqrt{K - 9}$ . The system is BIBO stable for  $K > 9$ .**

**The system is BIBO stable for  $K > 0$ .**

Slides 18-9, 18-10 & 18-11

- (c) Describe the possible frequency selectivity (lowpass, highpass, bandpass, bandstop, allpass or notch) that the system could exhibit for different values of  $K$  for which the system is BIBO stable. 3 points.

**The system is BIBO stable for  $K > 0$ . For  $K > 0$ , we can convert the transfer function in the Laplace domain to the Fourier domain by substituting  $s = j\omega$ .**

$$H(j\omega) = \frac{1}{-\omega^2 + 6j\omega + K} = \frac{1}{(j\omega - p_0)(j\omega - p_1)}$$

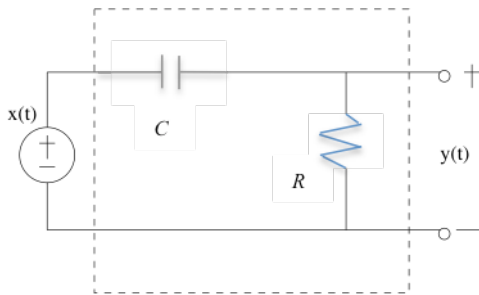
Slides 18-9, 18-10 & 18-11

**For  $0 < K \leq 9$ , the poles are real-valued, which means a lowpass frequency response.**

**As  $K$  increases beyond 9, it will eventually become bandpass in its frequency response with a center frequency in rad/s equal to the imaginary components of the pole locations.**

**Problem 8.** Continuous-Time Circuit Analysis. *9 points.*

Consider the following analog continuous-time circuit with input voltage  $x(t)$  and output voltage  $y(t)$ :



*HW 9.3 & 9.4*

The initial voltage across the capacitor is 0V, and hence, the circuit is a linear time-invariant system.

(a) Using the voltage drop around the loop

*Slides 18-5 & 18-7*

*SPFirst Sec. 16-6.5*

$$x(t) - \frac{1}{C} \int_{0^-}^t i(t) dt - Ri(t) = 0$$

take the Laplace transform of both sides of the equation to find the relationship between  $X(s)$  and  $I(s)$ .  $I(s)$  is the Laplace transform of the current  $i(t)$ . *2 points.*

**Because the system is LTI, the initial current and voltage across the capacitor is zero.**

**Taking the Laplace transform of both sides of the equation gives**

$$X(s) - \frac{1}{Cs} I(s) - R I(s) = 0 \text{ which gives } X(s) = \frac{1}{Cs} I(s) + R I(s) = \left( R + \frac{1}{Cs} \right) I(s)$$

(b) Using the formula for the voltage across the resistor

$$y(t) = R i(t)$$

*Slides 18-5 & 18-7*

take the Laplace transform of both sides and substitute the expression for  $I(s)$  obtained in part (a) to obtain the transfer function  $H(s)$  in the Laplace domain so that  $H(s) = Y(s) / X(s)$ . *2 points.*

$$Y(s) = R I(s) = R \left( \frac{X(s)}{R + \frac{1}{Cs}} \right) = \frac{s}{s + \frac{1}{RC}} X(s) \text{ which means that } H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s + \frac{1}{RC}}$$

(c) Find a formula for the frequency response  $H(j\omega)$  of the circuit. *2 points.*

**The pole in the transfer function in part (b) is at  $s = -\frac{1}{RC}$  which is real-valued and negative.**

**Since pole is negative and real-valued, system is BIBO stable and we can substitute  $s = j\omega$ :**

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$$

*Slides 18-9 & 18-11*

(d) What is the frequency selectivity of the circuit? Lowpass, highpass, bandpass, bandstop, allpass or notch. Why? *3 points.*

**When  $\omega = 0$ ,  $|H(j\omega)| = 0$ .**

**As  $\omega \rightarrow \infty$ ,  $|H(j\omega)| \rightarrow 1$ .**

**Highpass response.**

*Slides 18-9 & 18-11*

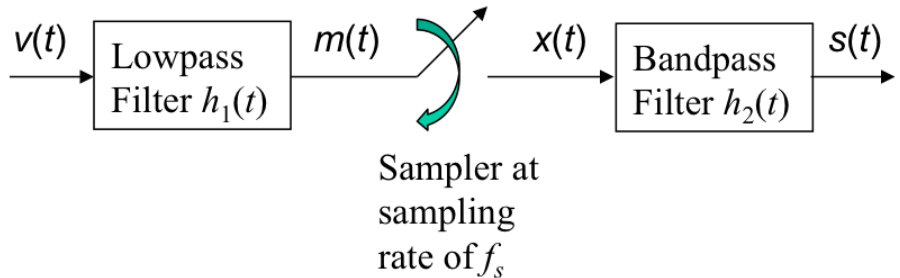
**Problem 9.** Sinusoidal Amplitude Modulation. *12 points.*

Mixing provides an efficient implementation in analog continuous-time circuits for sinusoidal amplitude modulation of the form

$$s(t) = m(t) \cos(2 \pi f_c t)$$

where  $m(t)$  is the baseband message signal with bandwidth  $W$ , and  $f_c$  is the carrier frequency such that  $f_c > W$

Lecture slides 15-12 & 15-13  
Handout R Modulation Example  
SPFirst Sec. 12-2.1



Lecture slide 16-4 on Sampling

**Note:** The units for bandwidth  $W$  are in Hz. This comes from the statement  $f_c > W$  and  $f_c$  is in Hz.

- (a) Assume  $h_1(t)$  is an ideal lowpass filter. Give the range of negative and positive frequencies that it passes. *3 points.*

Lecture slides 14-9 & 14-10  
SPFirst Sec. 10-3.2 pp. 296-297

**A baseband spectrum is a spectrum centered at zero frequency (SPFirst p. 360 left column).  
Baseband message signal  $m(t)$  has frequencies  $-W \leq f \leq W$  (SPFirst Fig. 12-14(a) on p. 360).  
The ideal lowpass filter  $h_1(t)$  passes frequencies  $-W \leq f \leq W$ .**

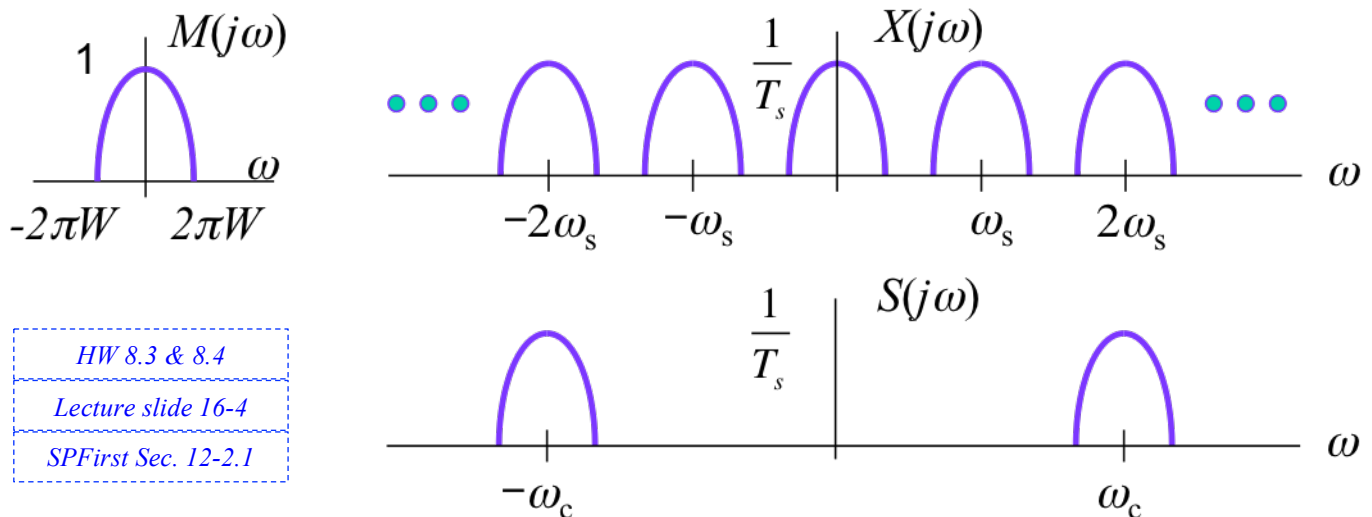
- (b) Assume  $h_2(t)$  is an ideal bandpass filter. Give the range of negative and positive frequencies that it passes. *3 points*

Lecture slides 14-9 & 14-12  
SPFirst Sec. 10-3.4 pp. 297-298

**Bandpass signal  $s(t) = m(t) \cos(2 \pi f_c t)$  has a bandwidth of  $2W$  centered at  $f_c$ .**

**The ideal bandpass filter  $h_2(t)$  passes frequencies  $f_c - W \leq f \leq f_c + W$  and  $-f_c - W \leq f \leq -f_c + W$ .**

- (c) Draw the magnitude of the Fourier transforms of  $m(t)$ ,  $x(t)$ , and  $s(t)$ . You do not need to draw the magnitude of the Fourier transform of  $v(t)$ . *6 points.*



HW 8.3 & 8.4  
Lecture slide 16-4  
SPFirst Sec. 12-2.1