Tune-Up Tuesday for September 14, 2021

% This problem uses Fourier series to synthesize a periodic waveform
% one term at a time. The problem revisits homework problem 2.4(a).

% (a) Use the Matlab code below that generates a cosine signal
% x0(t) = 0.5*cos(2 pi f0 t) with f0 = 440 Hz for 3 seconds at
% a sampling rate of fs = 48000 Hz and play it as a note.
% The gain of 0.5 is to prevent clipping when using the sound command.

```
fs = 48000;             % sampling rate
Ts = 1/fs;              % sampling time
tmax = 3;               % 3 sec
t = 0 : Ts : tmax;
f0 = 440;
x0 = 0.5*cos(2*pi*f0*t);
sound(x0, fs);
pause(tmax+1);
```

% (b) create y(t) = | x0(t) | and play y(t) as sound.
% Describe the difference in the sound in (b) vs. the sound in (a).
% Answer: The sound in (b) has a higher pitch (frequency) than the
% sound in (a). Per part (c), the principal frequency in the sound in (b)
% is twice the principal frequency of the sound in (a), i.e one octave higher.

```
y = abs(x0);
sound(y, fs);
pause(tmax+1);
```

% (c) plot y(t) for 5 periods of x0(t).
% How many periods of y(t) are there?
% Answer: 10 periods of y(t) in the same duration of 5 periods of x0(t).
% The fundamental period of y(t) is half that of x0(t) and hence the
% the fundamental frequency of y(t) is twice that of x0(t).

```
t5periods = 5/f0;
n5periods = round(t5periods/Ts);
figure;
plot( t(1:n5periods), y(1:n5periods) );
```

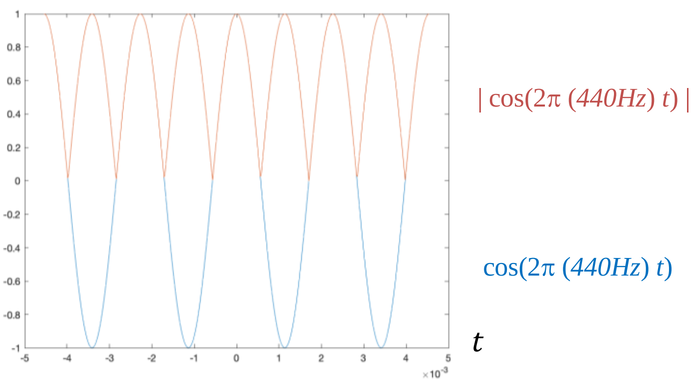
% (d) using Fourier series synthesis, use an increasing number
% of terms N = 1, 2, 3, 4, 5, and play each synthesized sound.
% The Fourier series coefficient formulas from homework
% problem 2.4(a) are the following after accounting for the
% gain of 0.5: A0 = 1 / pi and Ak = cos(pi k) / (pi (1 - 4*k^2)).
% Comment: The values of Ak decay in absolute value at a
% rate of 1/k^2. We won't need many terms for the synthesized
% sound to match the sound in (b).

```
% Create an array to hold the synthesized sound for efficiency
numSamples = length(t);
synthSound = zeros(1, numSamples);
```

```
% Add the first term
A0 = 1/pi;
synthSound = synthSound + A0*ones(1, numSamples);

f0y = 2*f0;
for k = 1:5
   % Add in terms for +k and -k
   Ak = cos(pi*k) / ( pi*(1 - 4*k^2) );
   fk = k * f0y;
   synthSound = synthSound + Ak*exp(j*2*pi*fk*t);
   kneg = -k;
   Akneg = cos(pi*kneg) / ( pi*(1 - 4*kneg^2) );
   fkneg = kneg * f0y;
   synthSound = synthSound + Akneg*exp(j*2*pi*fkneg*t);
   sound(synthSound, fs);
   pause(tmax+1);
end
```

Although not asked, here's a plot of a cosine at 440 Hz and its absolute value to show that the fundamental period for the absolute value is half of the period for the cosine at 440 Hz; in other words, the fundamental frequency doubles.



Although not asked, here's a plot of the Ak coefficients as a function of k. In this case, the values of Ak are real-valued and even symmetric.

```
k = -10 : 10;
Ak = cos(pi*k)./ ( pi*(1 - 4*k.^2) );
stem(k, Ak);
```