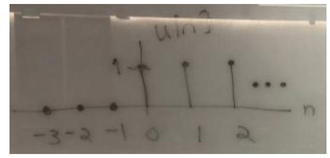
Tune-Up Tuesday #5 for October 14, 2021

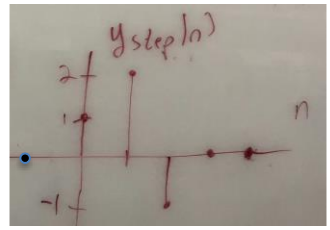
The tuneup is to solve homework problem 5.3(b) and verify the solution.

Intro. A step function $u[n]$ is a function that turns "on" at the origin and stays on, as plotted on the right. Mathematically,



$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

Problem. A linear time-invariant (LTI) system gives the output plotted on the right when the input is $x_1[n] = u[n]$. The output, also called the step response, is



$$y_1[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-2]$$

We'll model the unknown LTI system as a finite impulse response (FIR) filter with input signal $x[n]$ and output signal

$$y[n] = h[0]\,x[n] + h[1]\,x[n-1] + h[2]\,x[n-2] + h[N-1]\,x[n-(N-1)]$$

From this information, compute the filter coefficients $h[n]$ for $n = 0, 1, \ldots, N-1$ and manually verify that the step response of the FIR filter is $y_1[n]$.

Deconvolution. We'll use deconvolution to compute the filter coefficients. We derive the time-domain deconvolution algorithm by evaluating the output at $n = 0$:

$$y[0] = b_0\,x[0] + b_1\,x[-1] + b_2\,x[-2] + \cdots + b_{N-1}\,x[-(N-1)]$$

For LTI systems, it is a necessary (but not sufficient) condition for the system to be "at rest", which means that all initial conditions $x[-1], x[-2], \ldots, x[-(N-1)]$ must be zero. Since we know $x[n]$ and $y[n]$, we have one equation and one unknown at n = 0:

$$y[0] = b_0\,x[0]$$

and we can compute

$$b_0 = \frac{y[0]}{x[0]}$$

For this calculation to be valid, the first value of the test signal, $x[0]$, cannot be zero.

The second output value is: $y[1] = b_0\,x[1] + b_1\,x[0]$, and therefore, $b_1 = \frac{y[1] - b_0 x[1]}{x[0]}$.

The third output value is: $y[2] = b_0\,x[2] + b_1\,x[1] + b_2\,x[0]$ and $b_2 = \frac{y[2] - b_0 x[2] - b_1 x[1]}{x[0]}$.

In general, $b_N = \frac{y[N] - \sum_{i=0}^{N-1} b_i x[N-i]}{x[0]}$.

The MATLAB script utdeconvolve.m implements this algorithm.

Part (a). Give the vectors for x and y that you used when running utdeconvolve.m and the filter coefficients in vector b that the code computes.

x	**y**	**b**
[1 1 1]	[1 2 -1]	[1 1 -3]
[1 1 1 1]	[1 2 -1 0]	[1 1 -3 1]
[1 1 1 1 1]	[1 2 -1 0 0]	[1 1 -3 1 0]
[1 1 1 1 1 1]	[1 2 -1 0 0 0]	[1 1 -3 1 0 0]

Part (b). Verify that the filter coefficients by using them in the difference equation for the LTI FIR filter

$$y[n] = h[0]\,x[n] + h[1]\,x[n-1] + h[2]\,x[n-2] + h[N-1]\,x[n-(N-1)]$$

given that the input signal is $u[n]$ and the output signal is $y_1[n]$

$$y_1[n] = u[n] + u[n-1] - 3\,u[n-2] + \,u[n-3]$$

This is a closed-form solution for $y_1[n]$ which gives the correct output values for $y_1[n]$ for all n. For $n \geq 0$, the values are [1 2 -1 0 0 0 ….].

Alternate solution for part (b). One could simply compute several values of $y[n]$:

$$y[n] = x[n] + \,x[n-1] - 3\,x[n-2] + x[n-3]$$
$$y[0] = x[0] + \,x[-1] - 3\,x[-2] + x[-3] = 1 + 0 + 3 \cdot 0 + 0 = 1$$
$$y[1] = x[1] + \,x[0] - 3\,x[-1] + x[-2] = 1 + 1 - 3 \cdot 0 + 0 = 2$$
$$y[2] = x[2] + \,x[1] - 3\,x[0] + x[-1] = 1 + 1 - 3 \cdot 1 + 0 = -1$$
$$y[3] = x[3] + \,x[2] - 3\,x[1] + x[0] = 1 + 1 - 3 \cdot 1 + 1 = 0$$
$$y[4] = x[4] + \,x[3] - 3\,x[2] + x[1] = 1 + 1 - 3 \cdot 1 + 1 = 0$$

and so forth.