Tune-Up Tuesday #9 Inverse Continuous-Time Fourier Transforms (Nov. 30, 2021)

Warm-up problem. Compute the inverse continuous-time Fourier transform of

$$X(j\omega) = \frac{1}{2 + j\omega}$$

Solution: This is the first entry in the table of Fourier transform pairs on page 338:

$$x(t) = e^{-a\,t}\,u(t) \text{ for } a > 0 \quad \leftrightarrow \quad X(j\omega) = \frac{1}{a + j\omega}$$

Hence, the inverse continuous-time Fourier transform for

$$X(j\omega) = \frac{1}{2 + j\omega}$$

is

$$x(t) = e^{-2\,t}\,u(t)$$

Homework Problem 9.1(a)-(c) which *Signal Processing First* P-11.8(a)-(c) on page 343:

"In the following, the Fourier transform $X(j\,\omega)$ is given. Using the tables of Fourier transforms [page 338] and Fourier transform properties [page 339] to determine the inverse Fourier transform for each case. You may give your answer either as an equation or a carefully labeled plot, whichever is most convenient."

Solution:

Part	$X(j\omega)$	Rewrite $X(j\omega)$	Notes	$x(t)$
(a)	$\frac{e^{-j\omega 3}}{2 + j\omega}$	$\left(\frac{1}{2 + j\omega}\right) \underbrace{\left(e^{-j\omega 3}\right)}_{\text{delay by 3 in time domain}}$	Delay by 3s the result of $F^{-1}\left\{\frac{1}{2+j\omega}\right\}$	$e^{-2\,(t-3)}\,u(t-3)$
(b)	$\frac{j\omega}{2 + j\omega}$	$\left(\frac{1}{2 + j\omega}\right) \underbrace{(j\omega)}_{differentiate}$	Differentiate with respect to t the result of $F^{-1}\left\{\frac{1}{2+j\omega}\right\}$	$\frac{d}{dt}\{e^{-2\,t}\,u(t)\} =$ $e^{-2\,t}\,\delta(t) - 2\,e^{-2\,t}\,u(t)$
(c)	$\frac{(j\omega)}{2 + j\omega}e^{-j\omega 3}$	$\left(\frac{1}{2 + j\omega}\right)(j\omega)\left(e^{-j\omega 3}\right)$	Differentiate with respect to t the result of $F^{-1}\left\{\frac{1}{2+j\omega}\right\}$ and then delay by 3s	We can delay the above result by 3s: $e^{-2\,(t-3)}\,\delta(t-3) -$ $2\,e^{-2\,(t-3)}\,u(t-3)$