OBJECTIVE

• Develop a formal mathematical framework for analysis and design of error diffusion algorithms for digital image halftoning
  - Model halftoning as two-dimensional delta-sigma modulation
  - Derive objective measures for subjective quality of edge sharpening and noise in halftoned images

• Applications:
  - Design of optimal error diffusion filters with respect to subjective quality
  - Optimize quality of halftoned oversampled images

Work supported by a grant from Hewlett-Packard Laboratories
• Assume quantizer adds white noise uncorrelated with input
• Output given by
  \[ Y(z) = X(z) + N(z)(1 - H(z)) \]
• Signal transfer function (STF) is flat, noise transfer function (NTF) is high-pass
• Circuit is equivalent in form to a noise-shaping feedback coder
• Error image is highly correlated with input (Knox, 1992)
• Correlation is higher for larger error filters
• Degree of image sharpening increases with correlation
• Suggests linear gain model for quantizer
• Signal and noise paths modeled separately (Ardalan and Paulos, 1987)
Output given by

\[ Y(z) = \frac{K}{1 + (K - 1)H(z)} X(z) + \frac{1 - H(z)}{1 + (K - 1)H(z)} N(z) \]

- K is measured empirically; varies with image and error filter
- Accounts for image sharpening
- Noise treated separately (K = 1)
Similar results for 1-D delta-sigma modulators

Top: Floyd-Steinberg. Bottom: Jarvis et al.
• Linear gain model accounts for sharpening seen with large error filters

Floyd-Steinberg STF, $K = 2.0$

Jarvis et al. STF, $K = 4.5$
RESULTS OF LINEAR MODEL

- Sharpening is decoupled from noise
- Effect of noise shaping can be quantified
• Narrow histogram at quantizer input leads to higher effective quantizer gain, K
• Quantizer error bounded by ±0.5
Can small error filters be designed to sharpen as much as large filters?

- Design large sharpening filter
- Construct smaller filter whose frequency response is closest to the large filter in a mean square sense (Wong, 1996):

\[ g_n = h_n + \alpha \]

where:

- \( h_n, g_n \) are the coefficients of the large and small error filters, respectively
- \( \alpha \) is a constant chosen to satisfy the gain constraint at DC

Result: sharpening ability falls off linearly as number of filter taps decreases

Degree of sharpening related to bandwidth of error filter
Variation of quantizer gain with filter size

Variation of quantizer gain with filter bandwidth

- Sharpening correlated with bandwidth
VISUAL SYSTEM MODEL

- Noise isolated by subtracting sharpened, noiseless image from halftoned image
- Weighted noise figure computed using visual system model

Sensitivity of human visual system

Radial

Angular
CONCLUSION

• Summary
  - Error diffusion can be modeled as a noise-shaping feedback coder, a form of two-dimensional delta-sigma modulation
  - Quantizer can be modeled by a gain block plus additive noise
  - Objective measures of subjective quality by decoupling edge sharpening and noise effects:
    • Edge sharpening proportional to gain
    • Weight noise by perceptual SNR measure

• Future work
  - Design of optimal error diffusion filters with respect to subjective quality using constrained nonlinear optimization
  - Optimize algorithm complexity and subjective quality for halftoned oversampled images
    • Combine error diffusion (sharpening) with interpolation (smoothing)