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FAST TIME-DOMAIN EQUALIZATION FOR DISCRETE MULTITONE MODULATION SYSTEMS

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ABSTRACT

In discrete multitone receivers, a time-domain equalizer (TEQ) shortens the effective channel impulse response. The effective channel impulse response is ideally nonzero inside a short window and zero elsewhere. The samples outside of the window cause intersymbol interference. The ratio between the samples inside and outside of this window is called the shortening signal-to-noise (SSNR). In this paper, we develop a suboptimal method for maximum SSNR TEQ design that requires two orders of magnitude fewer computations than the original maximum SSNR method. We reduce computation by using a proposed heuristic to estimate the optimal delay of the window and a proposed divide-and-conquer method to compute the TEQ taps. For typical ADSL channels, the tradeoff for the reduction in computation is roughly 4 dB SSNR for two-tap TEQs and less than 1 dB for 17-tap TEQs.

1. INTRODUCTION

A transmitted signal experiences linear, nonlinear, and additive distortion when passing through a channel. A key linear effect is the dispersion of a transmitted pulse due to filtering by the channel, i.e. convolution of the transmitted pulse with the channel impulse response. The dispersion may cause symbols to interfere with adjacent symbols, a.k.a. *intersymbol interference* (ISI).

In a discrete multitone (DMT) system, a guard sequence called the *cyclic prefix*, is prepended to each DMT symbol. The DMT symbol consists of N samples, and the cyclic prefix is a copy of the last ν samples of the DMT symbol. The value of ν must be greater than or equal to the length of the channel impulse response (a.k.a. channel length) to prevent ISI.

At the receiver, the cyclic prefix is discarded, and the received symbol is further processed. Since the ν samples do not convey any new information about the transmitted signal, the efficiency of a DMT transceiver is decreased by a factor of $N/(N+\nu)$. Either N should be large or ν should be small. Memory requirements and computational complexity increase with N. The channel length, however, is generally not known, so it is difficult to choose ν *a priori*.

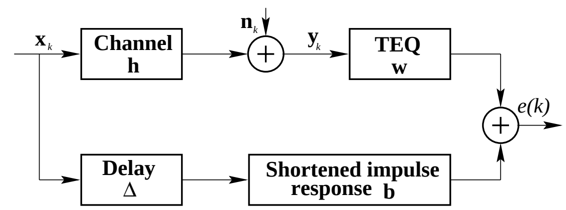


Figure 1: Time-domain equalizer: $\mathbf{x}_k$ is a vector of $(\nu+1)$ input samples; $\mathbf{n}_k$ is a discrete-time additive noise process; $\mathbf{y}_k$ is a vector of the current and previous N_w received samples; $\mathbf{h}$ is the original channel impulse response; $\mathbf{w}$ is a vector of the N_w TEQ taps; $\mathbf{b}$ is a vector of the shortened channel impulse response with $(\nu+1)$ samples; and Δ is the system delay of the overall response from both the channel and TEQ.

Chow and Cioffi [1] propose a finite impulse response TEQ to shorten the channel impulse response, as shown in Fig. 1. The kth error signal is $e(k) = \mathbf{w}^T\mathbf{y}_k - \mathbf{b}^T\mathbf{x}_{\Delta_k}$ where $\mathbf{y}_k = [y(k), \cdots, y(k-N_w+1)]^T$ of the received signal, Δ_k means the kth sample given a fixed Δ, and $\mathbf{x}_{\Delta_k} = [x(\Delta_k), \cdots, x(\Delta_{k-\nu})]^T$ of the input training sequence. Fig. 1 shows that the shortened impulse response (SIR) $\mathbf{b}$ is the convolution of the channel and a TEQ with a delay difference. The SIR length should be less than or equal to $(\nu+1)$. N_w (TEQ length) and ν can be set *a priori* and do not vary with the channel. Therefore, we need to find a filter with N_w taps such that the cascade of the channel $\mathbf{h}$ and the TEQ $\mathbf{w}$ yields a shortened impulse response, $\mathbf{b}$, which has a duration limited to $(\nu+1)$ samples.

The key contributions of the paper are

1. a heuristic to estimate optimal delay Δ (Section 3)
2. a divide-and-conquer real-time TEQ (Section 4)
3. a comparison of TEQ performance for a carrier-serving-area digital subscriber loop 1 (Section 5)

2. BACKGROUND

This section describes the Minimum Mean Squared Error (MMSE) [2] and Shortening SNR (SSNR) [3] methods for TEQ design. Although neither the SSNR nor MMSE methods optimize the channel capacity [4, 5], the SSNR method yields TEQ designs with significantly higher channel capacity than those from the MMSE method [6]. The optimal MMSE and SSNR design methods are computationally intensive and not cost effective for a real-time implementation.

2.1. MSE Method for TEQ Design

The mean squared error is defined as $\text{MSE} = E\left\{e^2(k)\right\}$. where $E\{\cdot\}$ expectation. Substituting $e(k)$ to obtain

$$\text{MSE} = E\left\{\left(\mathbf{w}^T\mathbf{y}_k - \mathbf{b}^T\mathbf{x}_{k-\Delta}\right)^2\right\} \tag{1}$$

To avoid the trivial solution of $\mathbf{w} = \mathbf{0}_{N_w\times 1}$ and $\mathbf{b} = \mathbf{0}_{(\nu+1)\times 1}$ when minimizing the MSE in (1), one may apply either

1. unit tap constraint: one element of $\mathbf{b}$ is set to 1, or
2. unit norm constraint: either $||\mathbf{b}|| = 1$ or $||\mathbf{w}|| = 1$.

Minimizing the MSE in (1) gives an analytically tractable way to find both the TEQ and SIR. The goal in TEQ training is to minimize the MSE as the system delay Δ varies. In deriving the MMSE TEQ problem, the solution involves the minimum eigenvalue of a matrix that is dependent on the channel and noise models [1, 4, 7, 8, 9] All the MMSE methods treat the samples inside and outside $(\nu+1)$ samples equally likely although the samples outside $(\nu+1)$ samples cause ISI and should be minimized to zero.

2.2. Maximum SSNR Method for TEQ Design

Melsa, Younce, and Rohrs [3] define the effective channel impulse response as $h_{\text{eff}}(k) = h(k) * w(k)$, which in vector form becomes

$$\mathbf{h}_{\text{eff}} = [h_{\text{eff}}(1), h_{\text{eff}}(2), \cdots, h_{\text{eff}}(L_h + N_w - 1)].$$

If all of the samples of $\mathbf{h}_{\text{eff}}$ outside of the window of $(\nu+1)$ samples are negligible, then the impulse response of the cascade of the channel and TEQ is effectively shortened. Splitting $\mathbf{h}_{\text{eff}}$ into two parts, $\mathbf{h}_{\text{win}}$ and $\mathbf{h}_{\text{wall}}$, emphasizes the samples inside and outside of the window [3]:

$$\begin{aligned}\mathbf{h}_{\text{win}} &= [h_{\text{eff}}(\Delta+1), h_{\text{eff}}(\Delta+2), \cdots, h_{\text{eff}}(\Delta+\nu+1)]\\ \mathbf{h}_{\text{wall}} &= [h_{\text{eff}}(1), \cdots, h_{\text{eff}}(\Delta), h_{\text{eff}}(\Delta+\nu+2), \cdots,\\ &\quad h_{\text{eff}}(L_h + N_w - 1)]\end{aligned} \tag{2}$$

The samples in $\mathbf{h}_{\text{wall}}$ include the samples before the window and the samples after the window (a.k.a. the tail). The SSNR objective function [3] is

$$\text{SSNR} = 10\log_{10}\frac{\text{Energy in }\mathbf{h}_{\text{win}}}{\text{Energy in }\mathbf{h}_{\text{wall}}} \tag{3}$$

We write $\mathbf{h}_{\text{win}}$ and $\mathbf{h}_{\text{wall}}$ in (2) in matrix form in Fig. 2. The energy of $\mathbf{h}_{\text{win}}$ and $\mathbf{h}_{\text{wall}}$ in (3) can be written as

$$\begin{aligned}\mathbf{h}_{\text{wall}}^T\mathbf{h}_{\text{wall}} &= \mathbf{w}^T\mathbf{H}_{\text{wall}}^T\mathbf{H}_{\text{wall}}\mathbf{w} = \mathbf{w}^T\mathbf{A}\mathbf{w}\\ \mathbf{h}_{\text{win}}^T\mathbf{h}_{\text{win}} &= \mathbf{w}^T\mathbf{H}_{\text{win}}^T\mathbf{H}_{\text{win}}\mathbf{w} = \mathbf{w}^T\mathbf{B}\mathbf{w}\end{aligned} \tag{6}$$

where

$$\mathbf{A}_{N_w\times N_w} = \mathbf{H}_{\text{wall}}^T\mathbf{H}_{\text{wall}}, \text{ and } \mathbf{B}_{N_w\times N_w} = \mathbf{H}_{\text{win}}^T\mathbf{H}_{\text{win}}.$$

The optimal shortening method would find $\mathbf{w}$ to minimize $\mathbf{w}^T\mathbf{A}\mathbf{w}$ while satisfying $\mathbf{w}^T\mathbf{B}\mathbf{w} = 1$ [3]. By assuming that $\mathbf{B}$ is positive definite, $\mathbf{B} = \sqrt{\mathbf{B}}\sqrt{\mathbf{B}}^T$ by Cholesky decomposition [3]. Then, $\mathbf{l}_{\min}$ is computed as the eigenvector associated with the smallest eigenvalue of the matrix $\left(\sqrt{\mathbf{B}}\right)^{-1}\mathbf{A}\left(\sqrt{\mathbf{B}^T}\right)^{-1}$. Finally, $\mathbf{w}_{\text{opt}} = \left(\sqrt{\mathbf{B}}^T\right)^{-1}\mathbf{l}_{\min}$. Table 1 summarizes the method and its computational cost.

In order to prevent $\mathbf{B}$ from being singular, Yin and Yue [11] suggest an objective function to maximize $\mathbf{w}^T\mathbf{B}\mathbf{w}$ while satisfying the constraint $\mathbf{w}^T\mathbf{A}\mathbf{w} = 1$. Both implementations [3, 11] require a Cholesky decomposition and an eigendecomposition of an $N_w \times N_w$ matrix to find $\mathbf{w}_{\text{opt}}$.

3. HEURISTIC SEARCH OF DELAY Δ

The original maximum SSNR method varies the delay Δ to maximize $\text{SSNR} = J_1 = \dfrac{\mathbf{w}^T\mathbf{B}\mathbf{w}}{\mathbf{w}^T\mathbf{A}\mathbf{w}}$ subject to the constraint $\mathbf{w}^T\mathbf{A}\mathbf{w} = 1$. Instead of performing the optimization for each candidate value of Δ, we estimate the optimal delay Δ first and then solve for the TEQ taps once. We use the following heuristic for Δ_{ratio} to estimate the optimal delay Δ_{opt} based on a window of the channel impulse response:

$$\Delta_{\text{ratio}} = \arg\max_{\Delta}\frac{\text{energy inside a window}}{\text{energy outside a window}} \tag{7}$$

Here, the window is of length $(\nu+1)$ samples beginning at index $(\Delta+1)$. The calculation of (7) requires L_h multiplications, $L_h - 2$ additions, and one division per value of Δ. The multiplications and additions can be reused if additional Δ values are considered. If Δ is enumerated from 0 to ν, then the calculation of (7) would require L_h multiplications, $2(\nu+1) + L_h - 2$

$$\underbrace{\begin{bmatrix} h_{\text{eff}}(\Delta+1) \\ h_{\text{eff}}(\Delta+2) \\ \vdots \\ h_{\text{eff}}(\Delta+\nu+1) \end{bmatrix}}_{\mathbf{h}_{\text{win}}} = \underbrace{\begin{bmatrix} h(\Delta+1) & h(\Delta) & \cdots & h(\Delta-N_w+2) \\ h(\Delta+2) & h(\Delta+1) & \cdots & h(\Delta-N_w+3) \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta+\nu+1) & h(\Delta+\nu) & \cdots & h(\Delta+\nu-N_w+2) \end{bmatrix}}_{\mathbf{H}_{\text{win}}} \underbrace{\begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N_w-1) \end{bmatrix}}_{\mathbf{w}} \tag{4}$$

$$\underbrace{\begin{bmatrix} h_{\text{eff}}(1) \\ \vdots \\ h_{\text{eff}}(\Delta) \\ h_{\text{eff}}(\Delta+\nu+2) \\ \vdots \\ h_{\text{eff}}(L_h+N_w-1) \end{bmatrix}}_{\mathbf{h}_{\text{wall}}} = \underbrace{\begin{bmatrix} h(1) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ h(\Delta) & h(\Delta-1) & \cdots & h(\Delta-N_w+1) \\ h(\Delta+\nu+2) & h(\Delta+\nu+1) & \cdots & h(\Delta+\nu-N_w+3) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & h(L_h-1) \end{bmatrix}}_{\mathbf{H}_{\text{wall}}} \underbrace{\begin{bmatrix} w(0) \\ w(1) \\ \vdots \\ w(N_w-1) \end{bmatrix}}_{\mathbf{w}} \tag{5}$$

Figure 2: Matrix form of the equalized channel impulse response inside and outside of a window of interest.

additions, and $(\nu+1)$ divisions. The calculation of (7) does not depend on N_w.

For each value of Δ considered, the full SSNR algorithm requires $\left(\frac{7}{6}+L_h\right) N_w + \frac{5}{2} N_w^2 + \frac{25}{3} N_w^3$ multiplications, $4N_w^2 + 2N_w$ fewer additions than multiplications, and N_w^2 divisions, assuming that N_w iterations are used for steps 6.3–6.6 in Table 1. The heuristic always requires fewer multiplications, additions, and divisions if more than one Δ value is considered by the full SSNR algorithm. For example, with $N_w = 10$, $L_h = 512$, and $\nu = 32$, the heuristic requires 512 multiplications to find Δ and 37,295 multiplications to compute the TEQ taps for the estimated Δ. The full SSNR algorithm requires $33 \times 37,295 = 1,230,735$ multiplications, if we search $\Delta \in [0, \nu]$.

4. DIVIDE-AND-CONQUER TEQ

We propose a low-complexity sub-optimal *Divide-and-Conquer TEQ* (DC-TEQ) design method in this section. The DC-TEQ method divides the N_w taps of the TEQ filter into $(N_w - 1)$ two-tap filters and iteratively designs each two-tap filter to maximize the SSNR. Maximizing the SSNR in (3) is equivalent to minimizing

$$\frac{1}{J_1} = J_2 = \frac{\mathbf{w}^T\mathbf{A}\mathbf{w}}{\mathbf{w}^T\mathbf{B}\mathbf{w}} \tag{8}$$

We apply divide-and-conquer directly to the channel impulse response to cancel the tail. For each iteration i of the *DC-TEQ-cancellation* method, $\tilde{\mathbf{h}}_{\text{wall}_i}$ is computed similarly to $\mathbf{h}_{\text{wall}}$ in (5). The equalized channel impulse response at the output of the ith filter is denoted as $\tilde{\mathbf{h}}_i$, which is the convolution of $\tilde{\mathbf{h}}_{i-1}$ and $\mathbf{w}_i$ where $\tilde{\mathbf{h}}_0$ is the channel impulse response

$$\tilde{\mathbf{h}}_0 = [h(1), h(2), \ldots, h(L_h-1)].$$

We set the first tap of each filter to one to prevent the trivial solution. For the ith two-tap filter, the coefficients are $\mathbf{w}_i = [1, g]^T$. In deriving the closed-form solution for g, we first define

$$\tilde{\mathbf{h}}_{\text{wall}_i} = \begin{bmatrix} \tilde{h}_{i-1}(1) & 0 \\ \tilde{h}_{i-1}(2) & \tilde{h}_{i-1}(1) \\ \vdots & \vdots \\ \tilde{h}_{i-1}(\Delta) & \tilde{h}_{i-1}(\Delta-1) \\ \tilde{h}_{i-1}(\Delta+\nu+2) & \tilde{h}_{i-1}(\Delta+\nu+1) \\ \vdots & \vdots \\ \tilde{h}_{i-1}(L_{h_{i-1}}) & \tilde{h}_{i-1}(L_{h_{i-1}}-1) \end{bmatrix} \begin{bmatrix} 1 \\ g \end{bmatrix} \tag{9}$$

Here, $L_{h_{i-1}}$ is the length of $\tilde{\mathbf{h}}_{i-1}$ at the ith iteration. Then, the tail energy can be expressed as

$$\begin{aligned} \tilde{\mathbf{h}}_{\text{wall}_i}^T \tilde{\mathbf{h}}_{\text{wall}_i} &= \textstyle\sum_{k\in S} \left(\tilde{h}_{i-1}(k) + g\,\tilde{h}_{i-1}(k-1)\right)^2, \\ S &= \{1, 2, \ldots, \Delta, \Delta+\nu+2, \ldots, L_{h_{i-1}}\} \end{aligned} \tag{10}$$

We find the minimum of the quadratic function of g in (10) by differentiating with respect to g, setting the derivative to zero, and solving for g yields

$$g = -\frac{\displaystyle\sum_{k\in S} \tilde{h}_{i-1}(k-1)\tilde{h}_{i-1}(k)}{\displaystyle\sum_{k\in S} \tilde{h}_{i-1}^2(k-1)} \tag{11}$$

Step	*Description*	$\times$	$+$	$\div$
1	Fix a Δ, Compute $\mathbf{A}_{(N_w \times N_w)}$	$(L_h - \nu)N_w$	$(L_h - \nu)N_w$	0
2	Compute $\mathbf{B}_{(N_w \times N_w)}$	$\nu N_w + (N_w+1)N_w/2$	$\nu N_w + (N_w+1)N_w/2$	0
3	Take Cholesky decomposition of $\mathbf{B}$	N_w^3	N_w^3	0
4	Calculate $\left(\sqrt{\mathbf{B}}\right)^{-1}$ [10]	$(5N_w^3 + N_w)/3$	$(5N_w^3 + N_w)/3$	0
5	Calculate $\mathbf{C} = \left(\sqrt{\mathbf{B}}\right)^{-1} \mathbf{A} \left(\sqrt{\mathbf{B}^T}\right)^{-1}$	$2N_w^3$	$2N_w^2(N_w - 1)$	0
6	Use power method to find eigenvector corresponding to the minimum eigenvalue of $\mathbf{C}$			
6.1	Calculate $\mathbf{C}^{-1}$ [10]	$(5N_w^3 + N_w)/3$	$(5N_w^3 + N_w)/3$	0
6.2	Initialize $\mathbf{l}^{(0)}$			
6.3	$\mathbf{z}^{(k)} = \mathbf{C}^{-1}\mathbf{l}^{(k-1)}$	N_w^2	$(N_w - 1)N_w$	0
6.4	$\mathbf{l}_{\text{opt}}^{(k)} = \mathbf{z}^{(k)} / \parallel \mathbf{z}^{(k)} \parallel$	N_w	$N_w - 1$	N_w
6.5	$\lambda^{(k)} = \left[\mathbf{l}^{(k)}\right]^T \mathbf{C}^{-1}\mathbf{l}^{(k)}$	$(N_w+1)N_w$	$N_w^2 - 1$	0
6.6	if $\lvert\lambda^{(k)} - \lambda^{(k-1)}\rvert >$ threshold, go to Step 6.3			
7	$\mathbf{w}_{\text{opt}} = \left(\sqrt{\mathbf{B}^T}\right)^{-1} \mathbf{l}_{\text{opt}}^{(k)}$	N_w^2	$(N_w - 1)N_w$	0

Table 1: Implementation and computational cost of the maximum SSNR method to find $\mathbf{w}_{\text{opt}}$ for a fixed Δ. The algorithm requires $\left(\frac{7}{6} + L_h\right) N_w + \frac{5}{2} N_w^2 + \frac{25}{3} N_w^3$ multiplications, $4N_w^2 + 2N_w$ fewer additions than multiplications, and N_w^2 divisions, assuming that N_w iterations are needed for steps 6.3–6.6.

This calculation requires one scalar division and two vector multiplications.

One iteration of the DC-TEQ-cancellation method is given in Table 2. The DC-TEQ-cancellation method does not require Cholesky decomposition, eigenvalue decomposition, or matrix inversion, unlike the maximum SSNR method in [3]. For a fair comparison, we replace the eigenvalue decomposition in the maximum SSNR method by the iterative power method [12] since only the minimum eigenvalue and its corresponding eigenvector are needed. For the power method, we used 10 iterations to find the eigenvector corresponding to the minimum eigenvalue. Table 3 compares the number of multiplications, additions, and divisions for a fixed value of Δ for the design of a 21-tap TEQ ($N_w = 21$) with $N = 512$, $\nu = 32$, and $L_h = 512$ for the SSNR and the Divide-and-Conquer-cancellation method.

5. SIMULATION RESULTS

We test the heuristic search method for Δ and the proposed DC-TEQ-cancellation method on carrier-serving-area digital subscriber loop 1. The sampling rate is 2.208 MHz, the number of the samples per symbol is 512, and the cyclic prefix is 32 [13]. Fig. 3(a) shows the optimum delay Δ obtained by the maximum SSNR method in [3] and our proposed DC-TEQ-cancellation method in Section 4, and the delay Δ calculated by the proposed heuristic search method. Fig. 3(b) shows the SSNR from the maximum SSNR method and DC-TEQ-cancellation method. We also provide the results of SSNR with the heuristic search method of Δ first and then the corresponding methods. From Fig. 3(b), the maximum SSNR method modified to use the heuristic search for Δ can obtain an SSNR close to the optimum maximum SSNR method. The second tap g of the DC-TEQ-cancellation method decreases while the number of taps increases. Therefore, we set the stopping criterion in Table 2, and the SSNR does not change after 4 taps. The DC-TEQ-cancellation with heuristic search yields SSNR values of about 1 dB lower than the DC-TEQ-cancellation method.

6. CONCLUSION

In discrete multitone modulation, the TEQ shortens the effective impulse response of the cascade of the channel and the TEQ. This paper proposes a suboptimum method, DC-TEQ-cancellation, to maximize the SSNR by cancelling the tail of the effective channel impulse response iteratively, which shortens the channel impulse response. The computational cost for the DC-TEQ-cancellation is about one-third of the computational cost of the he original maximum SSNR method. We also propose a heuristic search method to further reduce the computation cost by as much as two orders of magnitude. We have tested our proposed method on all eight CSA DSL channels, and show the results for CSA DSL channel 1. For the other seven CSA DSL channels, the difference in SSNR performance between

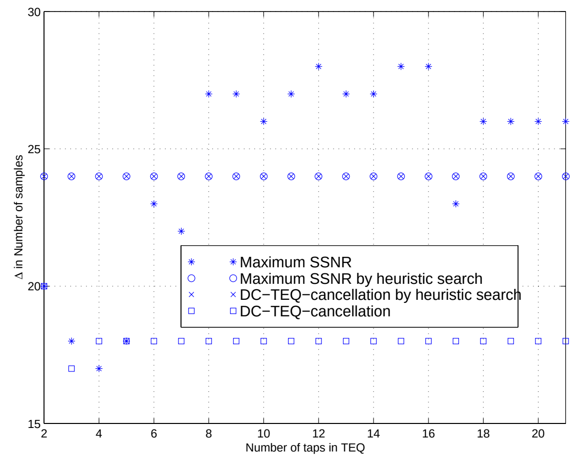
Step	*Description*	$\times$	$\div$
1	Set $\mathbf{w}_{\mathrm{TEQ}} = [1]$		
2	Set $\mathbf{h}_0 = \mathbf{h}$		
3	Fix Δ. For $i = 1 \ldots N_w - 1$		
3.1	Calculate g from (11)	$2(L_{h_i} - \nu - 1)$	1
3.2	If $g > 10^{-5}$, calculate $\mathbf{w}_{\mathrm{TEQ}} = \mathbf{w}_{\mathrm{TEQ}} * \mathbf{w}_i$	$2(i+2)$	0
3.3	$\tilde{\mathbf{h}}_i = \tilde{\mathbf{h}}_{i-1} * \mathbf{w}_i$	$2(L_h + i)$	0

Table 2: Implementation and computational cost in the Divide-and-Conquer TEQ for a fixed Δ where $L_{h_i} = L_h + i - 1 \leq L_h + N_w - 1$. The number of additions is one less than the number of multiplications. The $*$ operator is convolution.

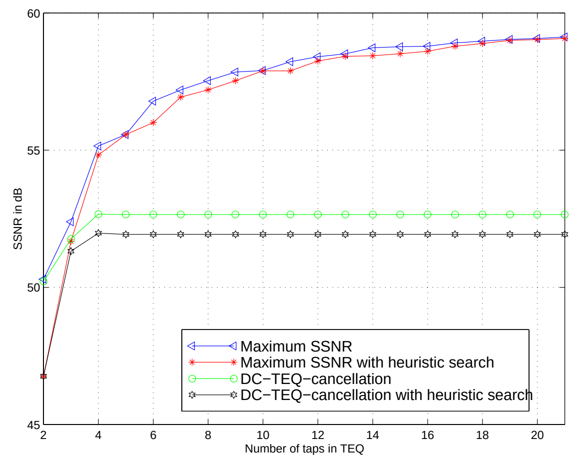
Methods	$\times$	$+$	$\div$
Maximum SSNR	120379	118552	441
DC-TEQ-cancellation	41000	40880	20

Table 3: Computational cost for maximum SSNR method and two proposed methods with $\nu = 32$, $N_w = 21$, and $L_h = 512$ for a fixed value of Δ.

the maximum SSNR and DC-TEQ-cancellation methods are similar. Likewise, the difference in SSNR with and without heuristic search is also similar. Our analysis of the computational cost compared to maximum SSNR method shows that our proposed method is suitable for the real-time implementation.



(a) optimal Δ and heuristic value given by (7)



(b) SSNR results

Figure 3: Determination of Δ for carrier-serving-area DSL 1 channel with $L_h = 512$ and $\nu = 32$.

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