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Fast Time-Domain Equalization for Discrete Multitone Modulation Systems

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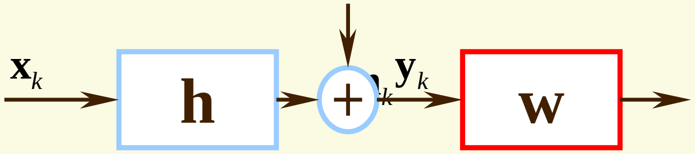
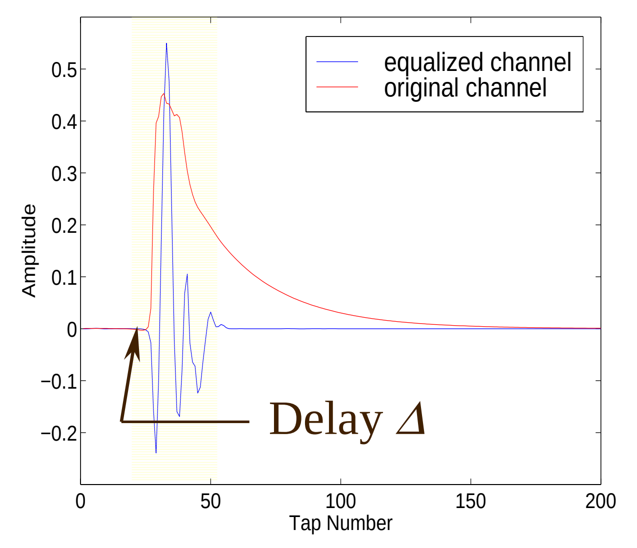
Problem Statement

- ✔ Effective channel impulse response



h: L_h-tap channel impulse response

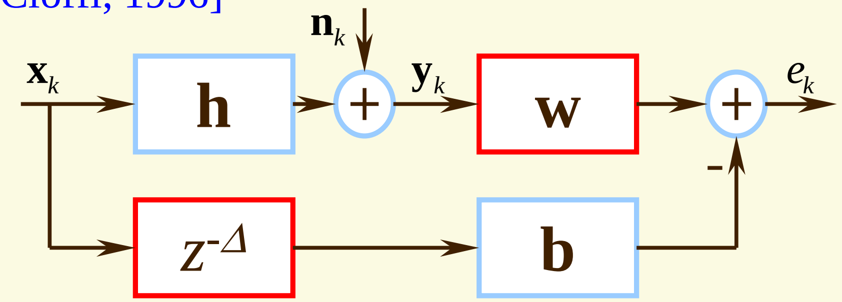
w: N_w-tap time-domain equalizer (TEQ)



- ✔ Problem: High computational cost of optimal TEQ design during ADSL transceiver initialization
- ✔ Goal: Low complexity suboptimal TEQ design

Minimum Mean Squared Error Method

- ✔ Minimize MSE [Falconer & Magee, 1973][Chow & Cioffi, 1992][Al-Dhahir & Cioffi, 1996]



$$\mathbf{w} = \left[w(1) \quad \cdots \quad w(N_w)\right]^T$$

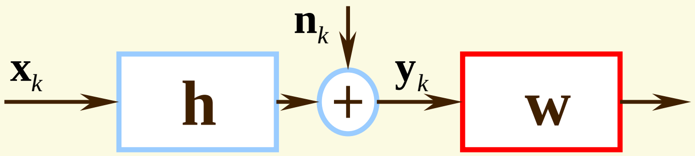
$$\mathbf{b} = \left[b(0) \quad b(1) \quad \cdots \quad b(\nu)\right]^T$$

$$\mathrm{MSE} = E\left\{\left(\mathbf{w}^T\mathbf{y}_k - \mathbf{b}^T\mathbf{x}_{k-\Delta}\right)^2\right\}$$

- ✔ Constraints to avoid trivial solution
  - Unit tap constraint: $b(i) = 1, i \in \{0, 1, \cdots, \nu\}$
  - Unit norm constraint: $\|\mathbf{b}\| = 1$ or $\|\mathbf{w}\| = 1$
- ✔ Computational cost for each Δ
  - Matrix inversion
  - Eigenvalue decomposition

Maximum Shortening SNR Method

- ✔ Minimize energy outside a window of length (ν+*1*) [Melsa, Younce, & Rohrs, 1996]

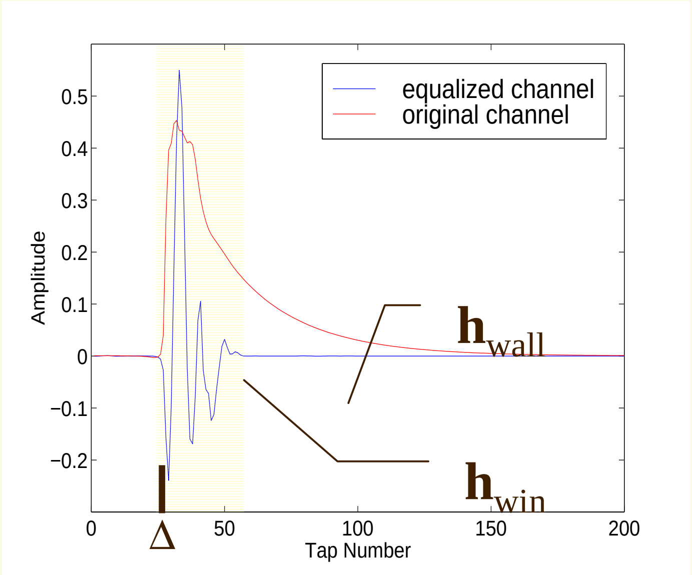


$$\mathbf{h}_{\text{wall}}^T\mathbf{h}_{\text{wall}} = \mathbf{w}^T\mathbf{H}_{\text{wall}}^T\mathbf{H}_{\text{wall}}\mathbf{w} = \mathbf{w}^T\mathbf{A}\mathbf{w}$$

$$\mathbf{h}_{\text{win}}^T\mathbf{h}_{\text{win}} = \mathbf{w}^T\mathbf{H}_{\text{win}}^T\mathbf{H}_{\text{win}}\mathbf{w} = \mathbf{w}^T\mathbf{B}\mathbf{w}$$

$$\text{SSNR} = 10\log_{10}\left(\frac{\mathbf{w}^T\mathbf{B}\mathbf{w}}{\mathbf{w}^T\mathbf{A}\mathbf{w}}\right)$$

- ✔ Constraint: $\mathbf{w}^T\mathbf{B}\mathbf{w} = 1$
- ✔ Computational cost for each *Δ*
  - Matrix inversion
  - Cholesky decomposition
  - Eigenvalue decomposition



Motivation

- ✔ MMSE method
  - Minimizes MSE both inside window and outside window
  - Slow Convergence
- ✔ Maximum SSNR method
  - Requires high computational cost
- ✔ Both methods search for optimal delay *Δ*

$$0 \le \Delta \le L_h + N_w - \nu - 2 \Rightarrow 0 \le \Delta \le 512 + 21 - 32 - 2 = 499$$

Divide-and-Conquer TEQ

- Use Divide-and-Conquer algorithm to minimize $\mathbf{h}_{\text{wall}}^{T}\mathbf{h}_{\text{wall}}$
- Divide a N_w-tap TEQ filter into (N_w-1) two-tap TEQs
- Each two-tap filter is initialized as

$$\mathbf{w}_i = [1, g_i]^T$$

- Calculate each g_i at i th iteration

$$g_i = -\frac{\sum_{k\in S}\tilde{h}_{i-1}(k-1)\tilde{h}_{i-1}(k)}{\sum_{k\in S}\tilde{h}_{i-1}^{2}(k-1)},\ S=\left\{1,2,\cdots,\Delta,\Delta+\nu+2,\cdots,L_{\tilde{h}_{i-1}}\right\}$$

- Convolve all two-tap filters to obtain the N_w-tap TEQ

Heuristic Search of Delay Δ

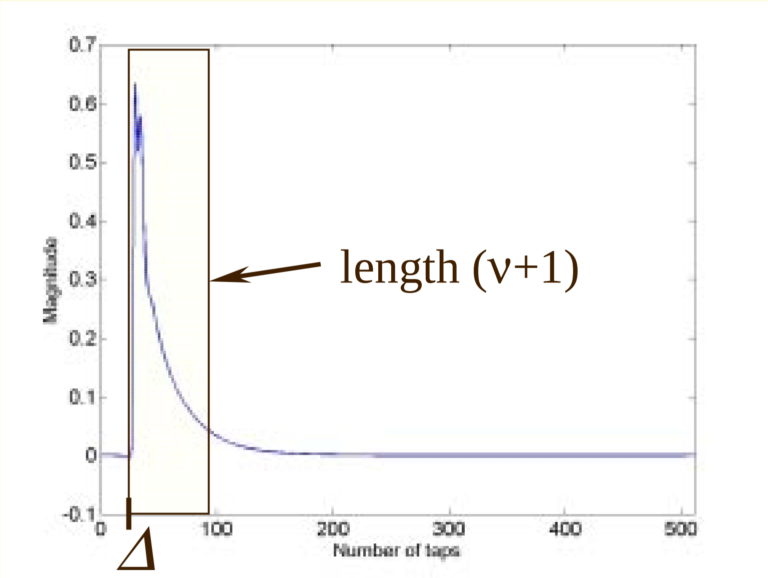
✔ Estimate the optimal delay

$$\Delta_{\text{ratio}} = \arg\max_{\Delta} \frac{\text{energy inside a window of original }\mathbf{h}}{\text{energy outside a window of original }\mathbf{h}}$$

✔ Computational cost

- L_h multiplications
- $(L_h - 2)\Delta_{range}$ additions
- Δ_{range} division

Δ_{range} : the number of Δs to be searched



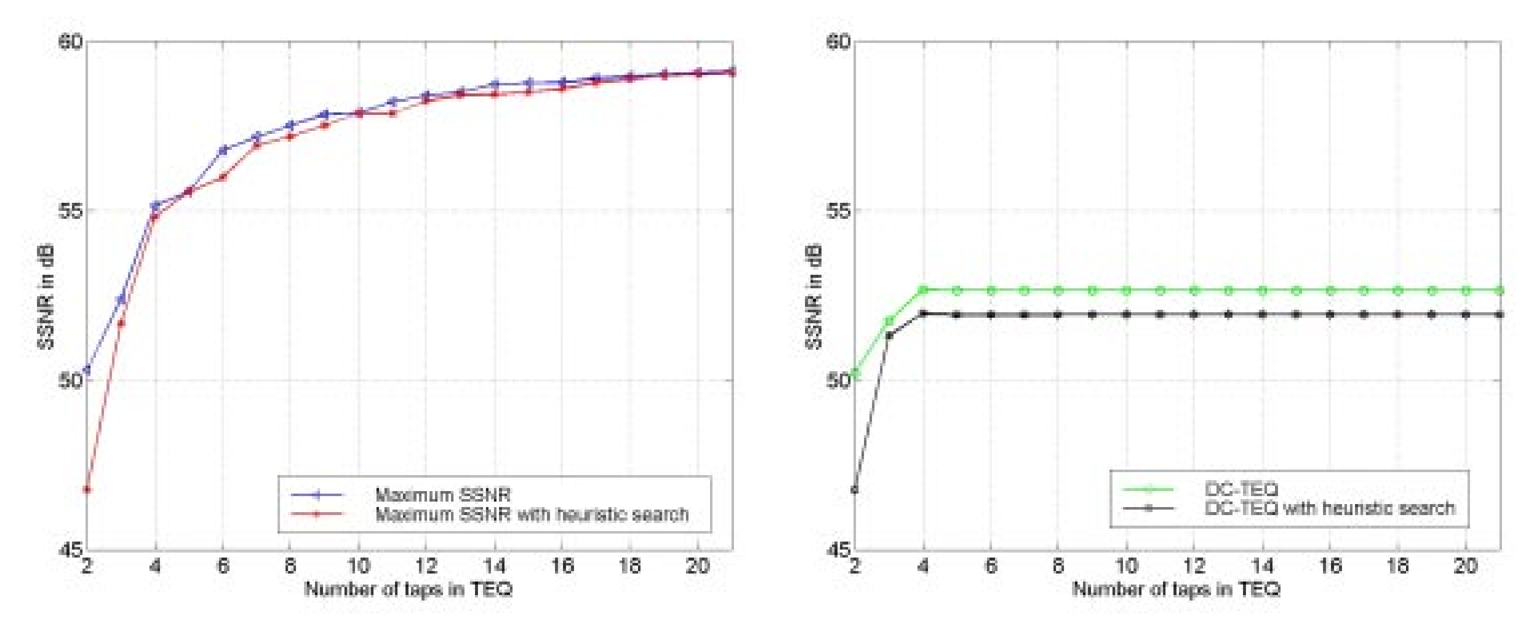
Computer Simulation

- Channel: carrier-serving-area digital subscriber loop 1
- Sampling rate: 2.208 MHz
- Cyclic prefix: $\nu = 32$
- Number of samples per symbol: 512+32
- Equalizer taps: $N_w = 21$

Methods	×	+	÷
Maximum SSNR	120379	118552	441
Divide-and-Conquer TEQ	41000	40880	20

Computations for each value of delay Δ considered

Simulation Results


Maximum SSNR

DC-TEQ

- ✔ Heuristic search: 1 dB loss for four or more TEQ taps
- ✔ Divide-and-conquer TEQ: 4-8 dB loss of SSNR

Conclusion

- ✔ Derive Divide-and-Conquer TEQ design method
  - Requires fewer computations than maximum SSNR method
  - For G.DMT ADSL, $L_h = 512$, $\nu = 32$, and $N_w = 21$,
    - » Reduces multiplications and additions by a factor of 3
    - » Reduces divisions by a factor of 22
  - Comparable performance to the maximum SSNR method
- ✔ Develop heuristic search to find delay
  - Find Δ by maximizing energy ratio of original channel impulse response
  - Reduces computational complexity in return for minimal performance loss
    - » Loss is 4 dB for 2-tap TEQ and 1 dB for 4-tap TEQ