Joint Optimization of Multiple Behavioral and Implementation Properties of Digital IIR Filter Designs

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Introduction

- Problem
  - Simultaneously optimize multiple characteristics of an existing digital IIR lowpass filter design
- Goal
  - Develop an extensible automated framework
- Solution
  - Solve constrained nonlinear optimization problem by using sequential quadratic programming (SQP)
  - Program Mathematica to derive formulas and generate Matlab programs to perform optimization

Modeling

- Free parameters
  - Set of *n* conjugate pole pairs $a_k \exp(\pm j\, b_k)$
  - Set of *m* conjugate zero pairs $c_i \exp(\pm j\, d_i)$
- Properties
  - *Behavioral:* magnitude response, phase response
  - *Implementation:* quality factors
- Compute
  - Cost function as a weighted mean of distance measures
  - Constraints to enforce numerical stability
  - Closed-form symbolic gradients for robustness

Objective Measures

- Magnitude response
  - Scale to be unity at DC
- Unwrapped phase response
  - Constrain zeros to be outside passband
- Quality factor Q
  - For each pole pair $a_k \exp(\pm j\, b_k)$: $0.5 \le Q_k \le \infty$

$$Q_k = \frac{\sqrt{(1 + a_k^2)^2 - 4a_k^2 \cos^2(b_k)}}{2(1 - a_k^2)}$$

  - Effective quality factor is geometric mean of Q_k factors

Distance Measures

- Non-negative differentiable measures
  - A value of zero means the ideal case
  - Differentiability necessary for SQP formulation
- Deviation in magnitude response
  - L_2 norm of deviation from ideal in passband, stopband and transition bands
- Deviation of quality factors
  - Minimum effective quality factor Q_{eff} is 0.5
  - Deviation measured as $\sigma_q = Q_{eff} - 0.5$

Distance Measures

- Deviation from linear phase in passband
  - L_2 norm of deviation from perfect linear phase at optimal slope within $(0, \omega_l)$ where $\omega_l \leq \omega_p$
  - Approximate optimal slope to obtain analytic form
    - Weighted mean of two first-order estimates
    - Accurate up to four Taylor series terms
    - $\alpha = 0.4163$; $\beta = 0.5837$; $r_1 = 0.5385$; $r_2 = 0.9062$

$$\tilde{m} = \alpha \frac{\angle \tilde{H}(e^{j r_1 \omega_l})}{r_1 \omega_l} + \beta \frac{\angle \tilde{H}(e^{j r_2 \omega_l})}{r_2 \omega_l}$$

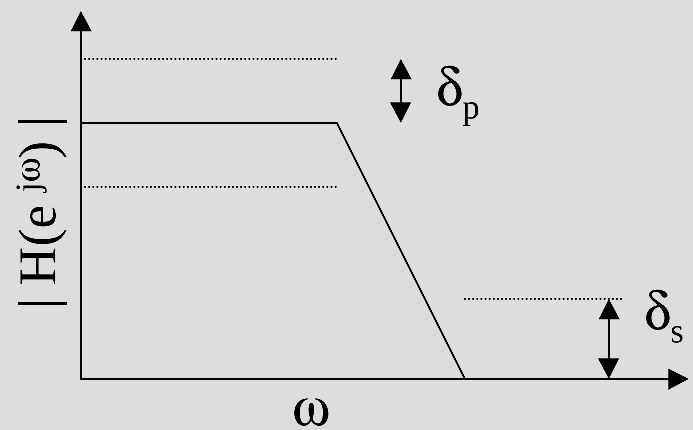
Overall Cost Function

$$\sigma = \frac{W_p}{\omega_p}\sigma_p + \frac{W_t}{\omega_s - \omega_p}\sigma_t + \frac{W_s}{\pi - \omega_s}\sigma_s + \frac{W_{phase}}{\omega_l}\sigma_{phase} + W_q\sigma_q$$

- Weighted sum of distance measures
  - Passband, transition band, stopband, and phase response are normalized by bandwidth
  - Quality factor
- User-defined weights
  - W_p, W_t, W_s, and W_{phase} for passband, transition band, stopband, and phase response, respectively
  - W_q for quality factor

Constraints

- Zero locations outside of the passband
  - Numerical stability of phase response
- Quality factor of each pole pair less than Q_{max}
  - Q_{max} determined by technology
- Pole locations inside unit circle
- Magnitude constraints
  - Passband
    $1 - \delta_p < | H(e^{j\omega}) | < 1 + \delta_p$
  - Stopband
    $| H(e^{j\omega}) | < \delta_s$



Implementation

- Mathematica
  - Compute cost function, constraints, and gradients
  - Generate efficient Matlab code for SQP solution to optimization problem
- Matlab
  - Set all user-definable parameters
  - Initial filter design
    - User-supplied or default computed elliptic filter
    - Preferably overdesigned

Design Example

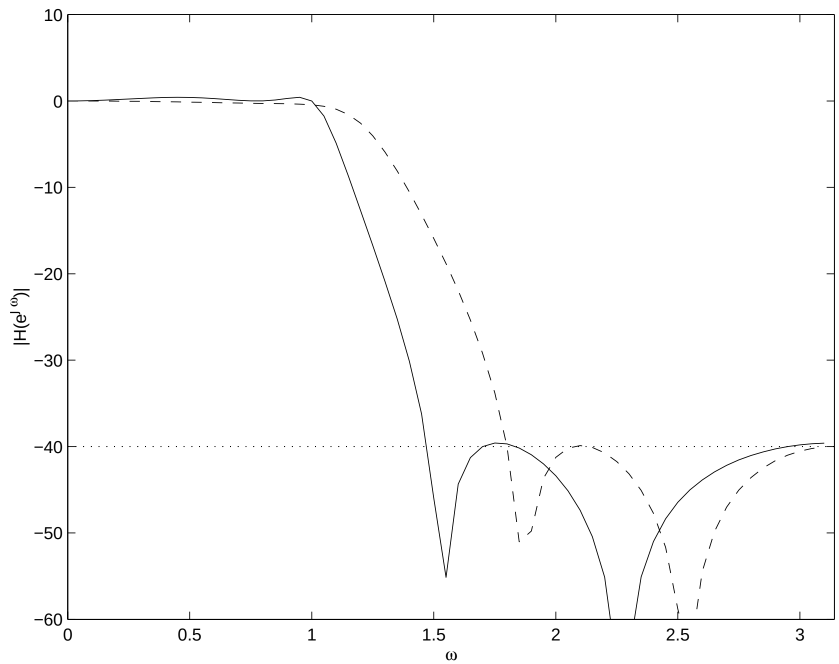
- Optimization of phase response with constraints on quality factors and magnitude response
- Phase optimized over the entire passband
- Magnitude response constraints
  - Passband: (0 - 1) rad, ripple 0.05
  - Stopband: (1.8 - π) rad, ripple 0.01
- Initial filter
  - Fourth-order elliptic filter generated by Matlab
  - Fails quality factor constraints (SQP relaxation)

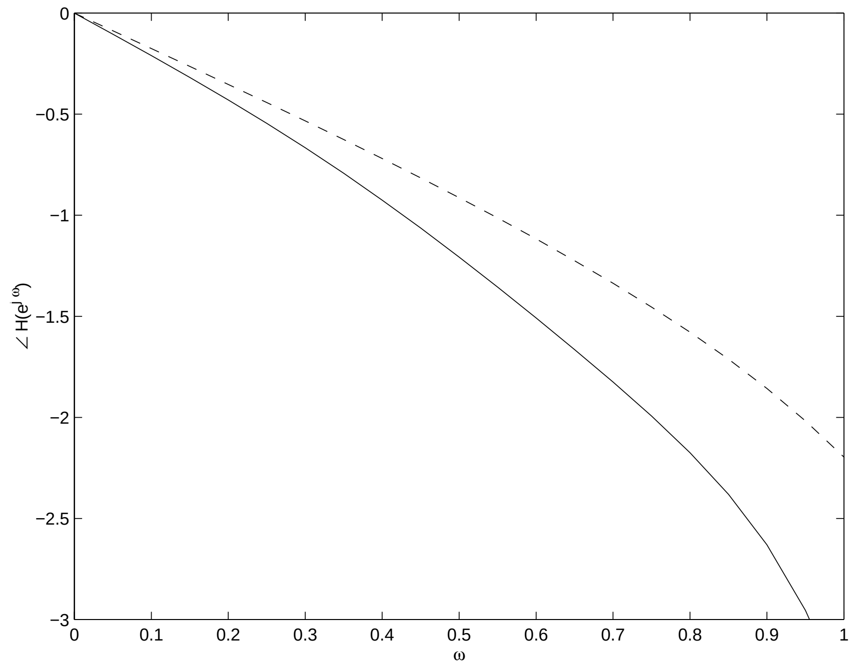
Design Example

- Optimized filter
  - Phase response closer to linear (shown on next slide)
  - Lower quality factors
  - Satisfies magnitude response constraints

	Initial	*Final*
Pole pair 1	$0.5176 \pm j0.3264$	$0.2145 \pm j0.1651$
Pole pair 2	$0.4584 \pm j0.7602$	$0.2982 \pm j0.7306$
Quality factors	0.72, 3.62	0.57, 2.00

Design Example





Magnitude Response

Phase Response

—— Initial filter

-------- Optimized filter

Conclusions

- Extensible framework for automated digital IIR filter design optimization
- Symbolic computation eliminates algebraic errors
- Error-free generation of source code
- Robust due to symbolic computation of gradients
- Easy to change objective functions, measures and constraints
- Software available at
  http://www.ece.utexas.edu/~bevans/projects/syn_filter_software.html