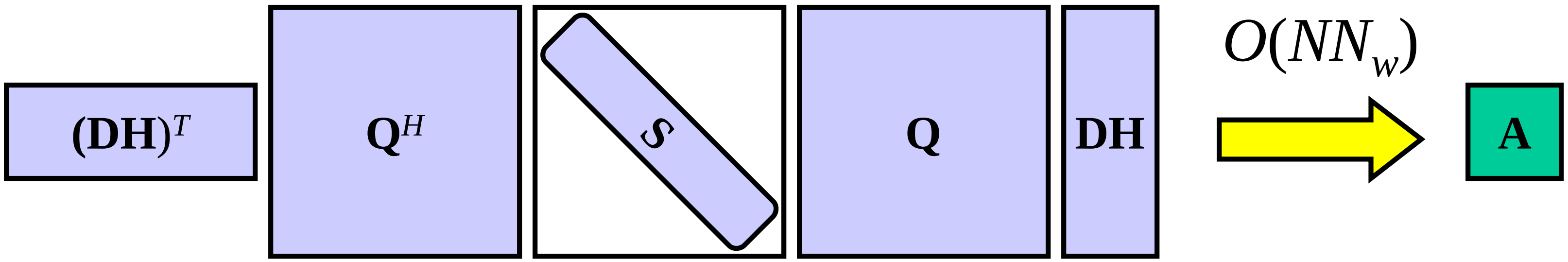
Efficient Matrix Multiplication Methods to Implement a Near-optimum Channel Shortening Method for DMT Transceivers



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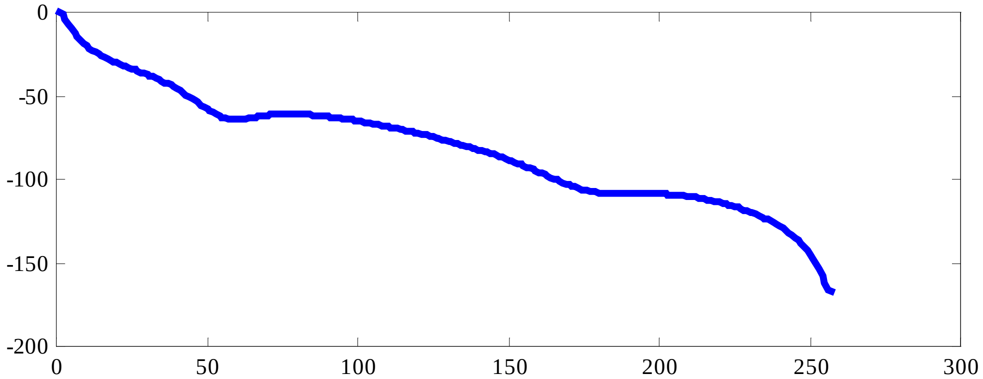
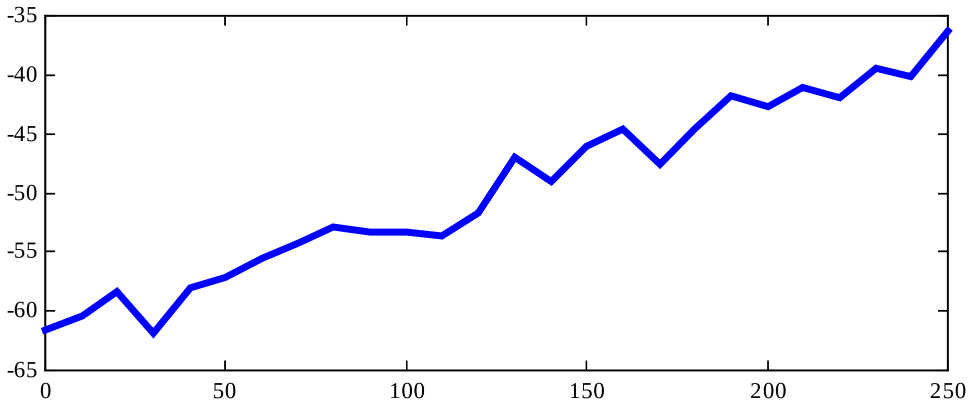
Why DMT?

Typical frequency response of ADSL channels

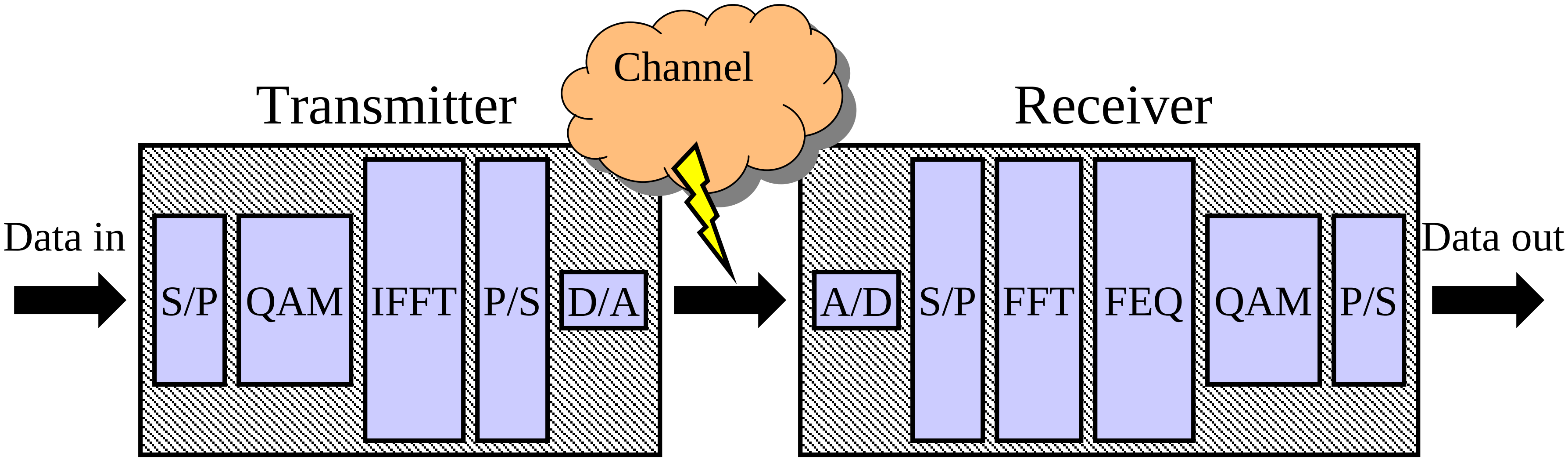


Typical noise spectrum of ADSL channels

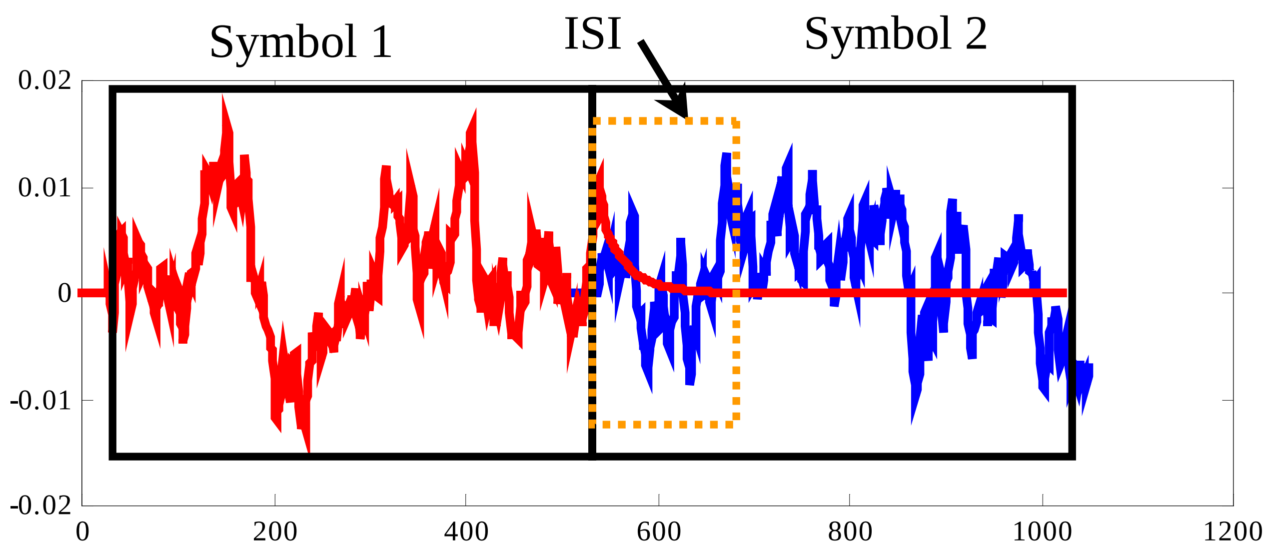


Solution: Partition channel into subchannels using FFT

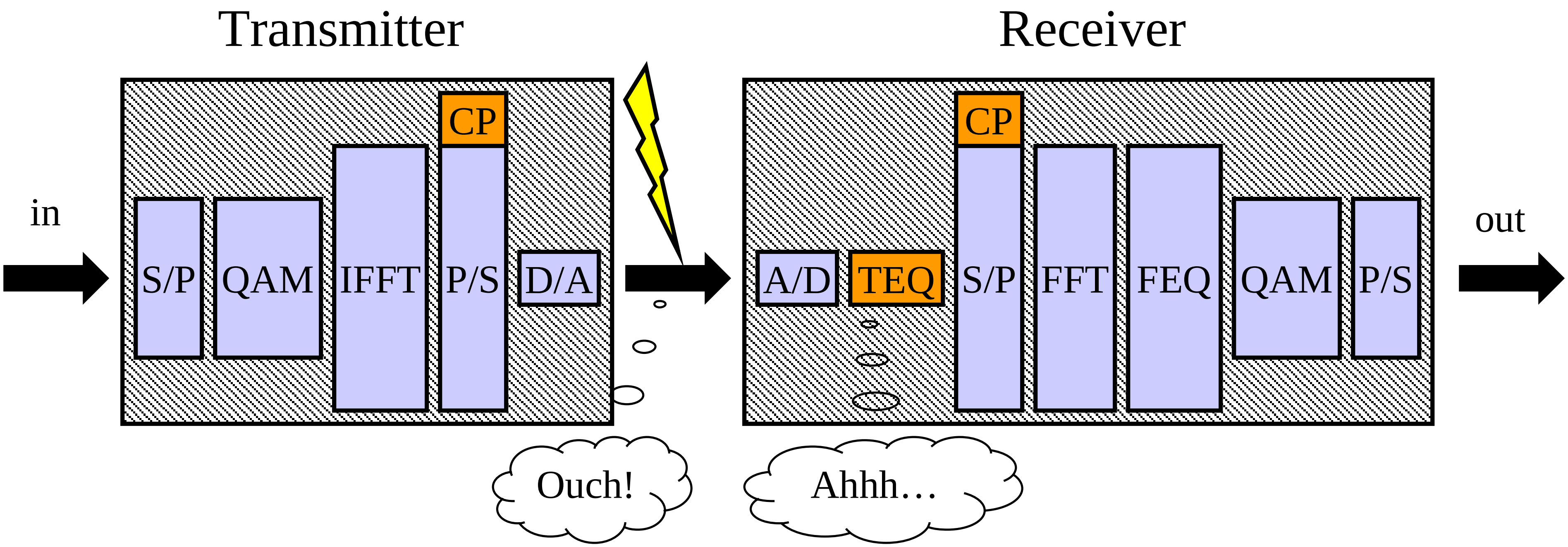
Simplest possible DMT transceiver

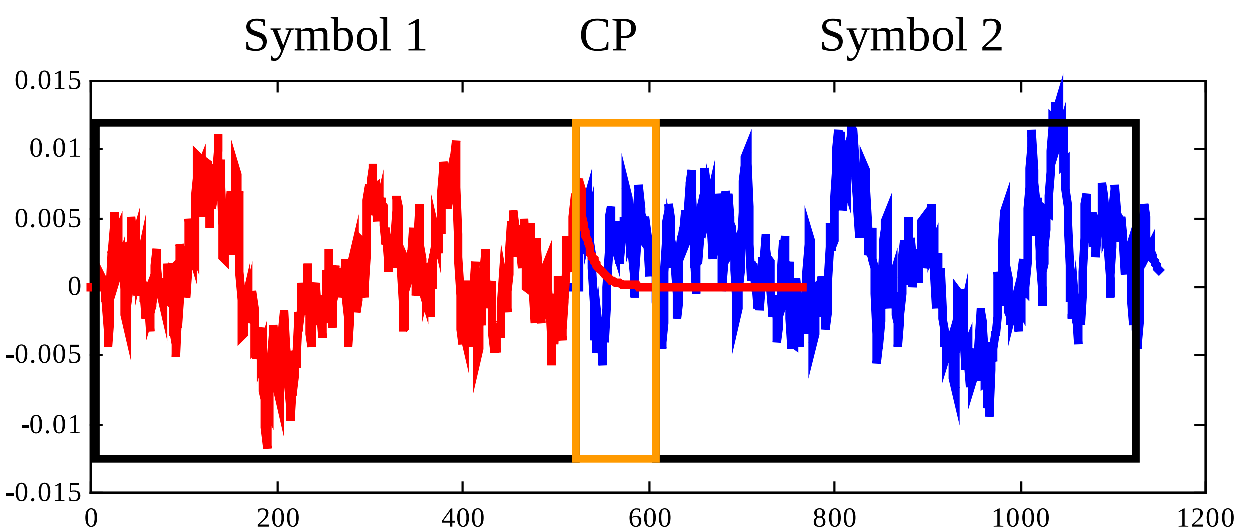


"–Houston, we have a problem…"



Solution to the ISI problem





- CP creates room between symbols
- TEQ shortens channel to fit within CP

Goal: Given a CP of length ?, design the TEQ such that it shortens channel to ? + 1 sample periods.

Objection: But this is generally impossible to do perfectly! ☹

Revised Goal: Design TEQ so that it minimizes ISI in such a way that maximizes channel capacity.

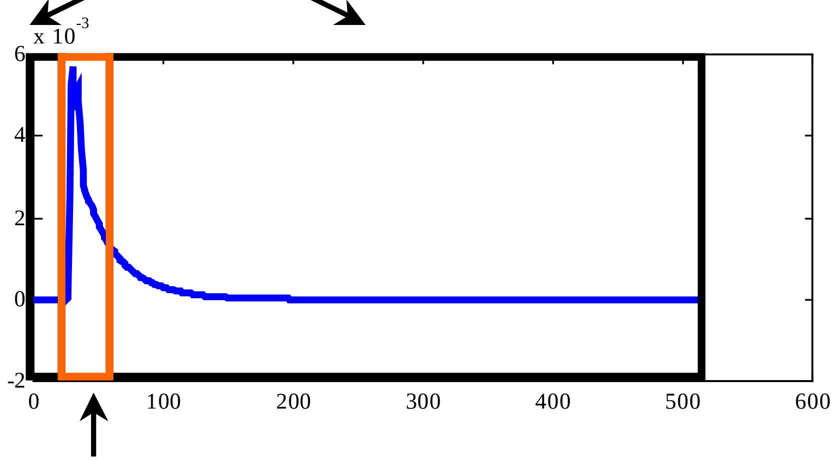
Response: Sounds good, but how do accomplish that? 😐

The min-ISI method [Arslan, Evans, Kiaei, 2000]

Observation: Given a channel impulse response of *h* and an equalizer *w*, there is a part of *h* * *w* that causes ISI and a part that doesn't.

Causes ISI (will extend beyond cyclic prefix)

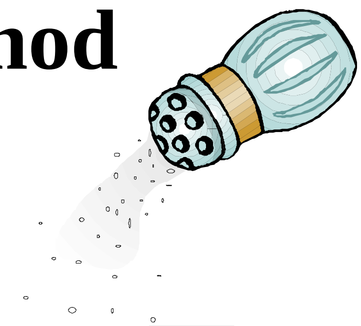
Equalized impulse response, *h* * *w*



Does not cause ISI (will stay within cyclic prefix)

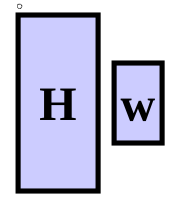
- The length of the window is ? + 1
- Heuristic determination of the optimal window offset, denoted as ? , is given by Lu (2000).

Matrix ingredients of the min-ISI method

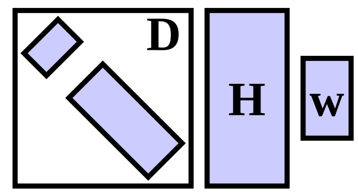


- The equalizer **w**. This is a little vector.

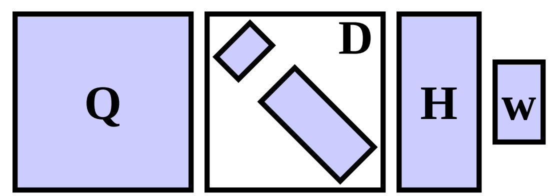
- The convolution matrix **H**, such that $(\mathbf{Hw})_k = (h * w)_k$.



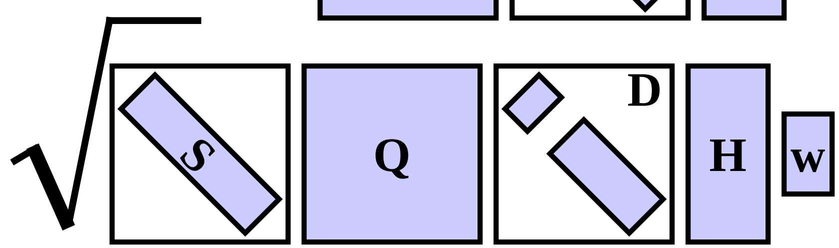
- The windowing matrix **D**. This is a diagonal matrix that isolates the part of $h * w$ causing ISI.



- The FFT matrix **Q**. Takes FFT of **DHw**.



- The weighting matrix diag(**S**).



The Optimization Problem

Goal: Find **w** that minimizes $\left\| \sqrt{\mathrm{diag}(\mathbf{S})}\mathbf{QDHw} \right\|^2$

Translation: Find **w** that minimizes a weighted sum of the ISI power gains in each subchannel.

Something is missing here… a constraint!

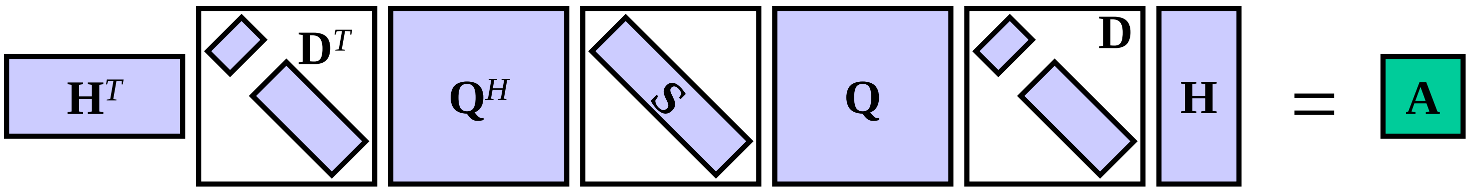
Constrain $\| \mathbf{GHw} \|^2 = 1$, where **G = I – D**

Translation: Prevent **w** from also minimizing the *desired* part of the $h * w$!

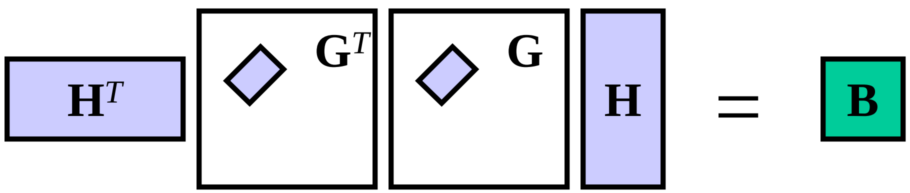
The Matrix Multiplication Problem (finally!)

The optimization problem can be restated as:

Minimize $\mathbf{w}^T\mathbf{A}\mathbf{w}$, where **A** is defined as



Subject to $\mathbf{w}^T\mathbf{B}\mathbf{w} = 1$, where **B** is



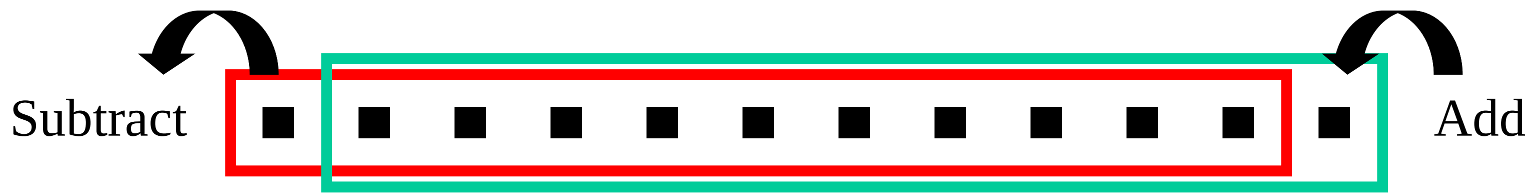
A and **B** are small, so problem can be solved quickly
so long as we can find **A** *and* **B**.

Turns out to be a BIG problem for real-time implementation.

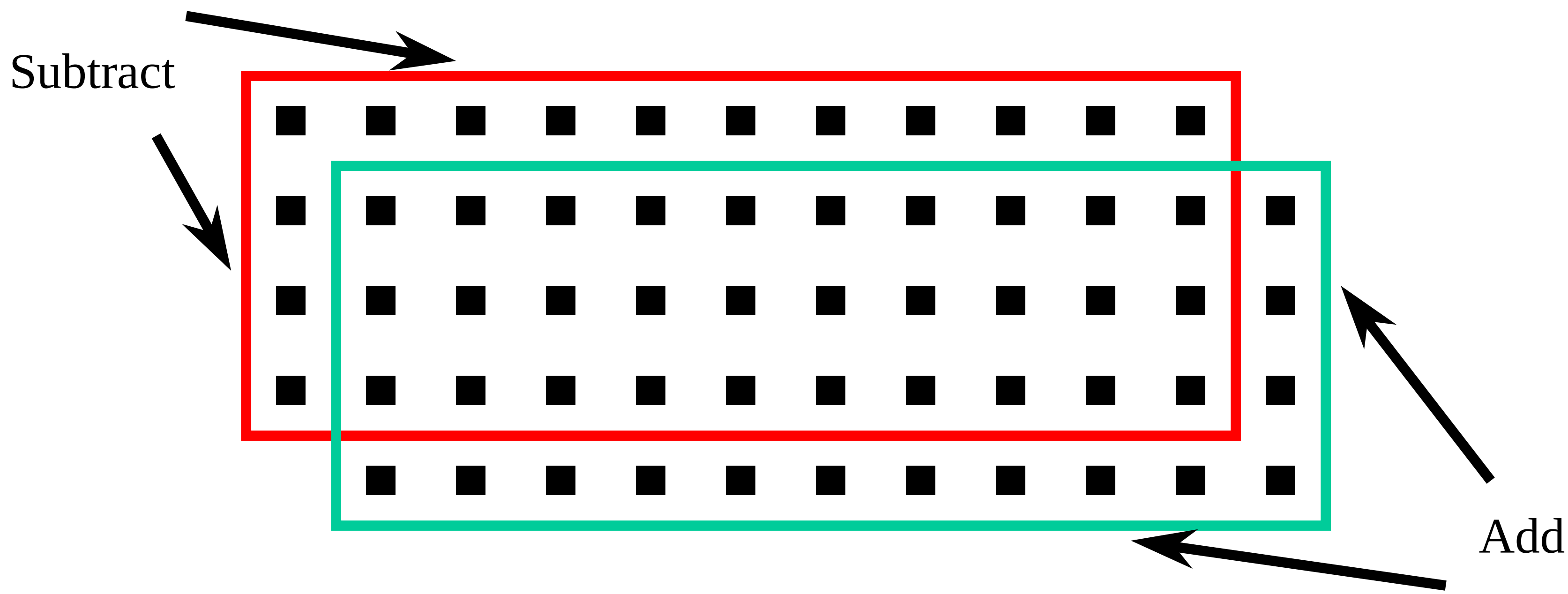
The Solution to the Problem – Sliding Windows

Arises frequently with Toeplitz matrices (i.e. **H**)

Sliding windows for single sums:



Sliding windows for double sums:



Fast Algorithm for Matrix B

Explicit formula:

$$B_{m,n} = \sum_{k=\Delta}^{\Delta+\nu} h_{k-m} h_{k-n}$$

The sliding window:

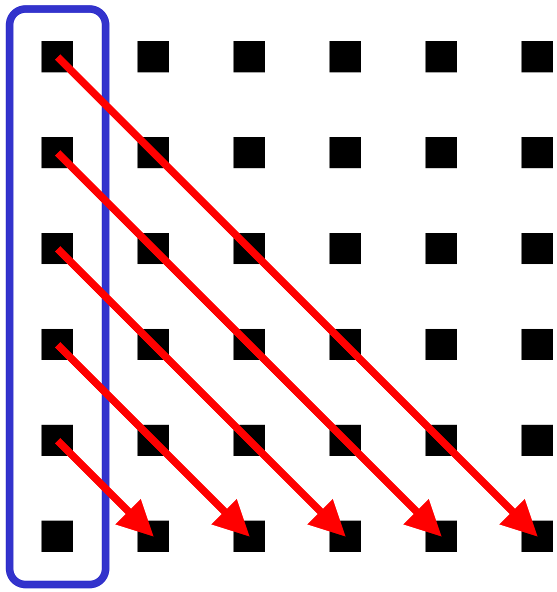
$$B_{m+1,n+1} = \sum_{k=\Delta-1}^{\Delta+\nu-1} h_{k-m} h_{k-n}$$

The recursion:

$$B_{m+1,n+1} = B_{m,n} - h_{\Delta+\nu-m} h_{\Delta+\nu-n} + h_{\Delta-1-m} h_{\Delta-1-n}$$

Very nice!

What is going on…



Fast Algorithm for Matrix A

Explicit formula:

$$A_{m,n} = \sum_{k=0}^{\Delta-1}\sum_{l=0}^{\Delta-1} h_{k-m}h_{l-m}s_{k-l} + \sum_{k=0}^{\Delta-1}\sum_{l=\Delta+\nu+1}^{N-1} h_{k-m}h_{l-m}s_{k-l} + \sum_{k=\Delta+\nu+1}^{N-1}\sum_{l=0}^{\Delta-1} h_{k-m}h_{l-m}s_{k-l} + \sum_{k=\Delta+\nu+1}^{N-1}\sum_{l=\Delta+\nu+1}^{N-1} h_{k-m}h_{l-m}s_{k-l}$$

where $s_{k-l} = \sum_{p=0}^{N-1} S_p e^{j2\pi(k-l)p/N}$

The sliding window:

$$C_{m+1,n+1} = \sum_{k=a-1}^{b-1}\sum_{l=c-1}^{d-1} h_{k-m}h_{l-m}s_{k-l}$$

Recursive formula:

$$\begin{aligned}C_{m+1,n+1} = C_{m,n} &- h_{b-m}h_{d-n}s_{b-d} \\ &- h_{b-m}f(c,d,b,n) \\ &- h_{d-n}f(a,b,d,m) \\ &+ h_{a-1-m}h_{c-1-n}s_{a-c} \\ &+ h_{a-1-m}f(c,d,a-1,n) \\ &+ h_{c-1-n}f(a,b,d-1,m)\end{aligned}$$

where $f(\alpha,\beta,\gamma,\delta) = \sum_{k=\alpha}^{\beta-1} h_{k-\delta}s_{k-\gamma}$

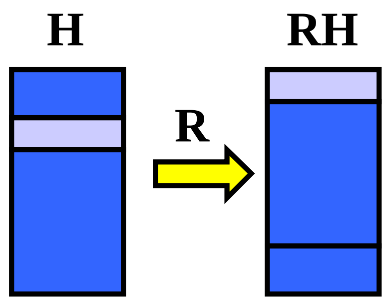
Variants of the algorithm

Row-rotation method

• "Rotates" the rows in the convolution matrix **H** to simplify the explicit formula of **A** to one double sum.

• Assumes last few samples of impulse response close to zero. GOOD assumption.

• Virtually same performance as original method.



No-weighting method

• Assuming equal weighting of subchannels in the optimization problem. Equivalent to maximum SSNR method by Melsa, Younce, and Rohrs (1996).

• Simplifies calculation of **A** to a single sum.

• Almost as good performance as original method.

Results

Method	Channel capacity %	SSNR (dB)	Complexity	MACs
Original min-ISI	99.6	37.8	½ $(N + ?)\, N_w (N_w + 1)$ + $5\, N\, (N_w - 1) + NN_w$	132896
Recursive min-ISI	99.5	37.9	$4(N_w - 1)\,(N + 4N_w - ? - 2)$ + $N_w\,(N + N_w - 1) + N$	44432
Row rotation min-ISI	99.5	37.5	$2(N_w - 1)\,(N + 2N_w - ? - 2)$ + $N_w\,(N + N_w - 1) + N$	25872
Original max SSNR	97.9	58.9	½ $N\, N_w (N_w + 1)$	78836
No-weighting min-ISI	97.8	55.4	$N\, N_w + 5\, N_w\,(N_w - 1)$	10064

N = size of FFT
N_w = size of TEQ
$?$ = size of cyclic prefix

Channel used : CSA loop 1
System margin: 6dB
$N = 512$, $N_w = 17$, $? = 32$