Adaptive Multiscale Estimation for Fusing Remotely Sensed Imagery

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Outline

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  - Kalman filter for data fusion
  - Multiscale Kalman filter
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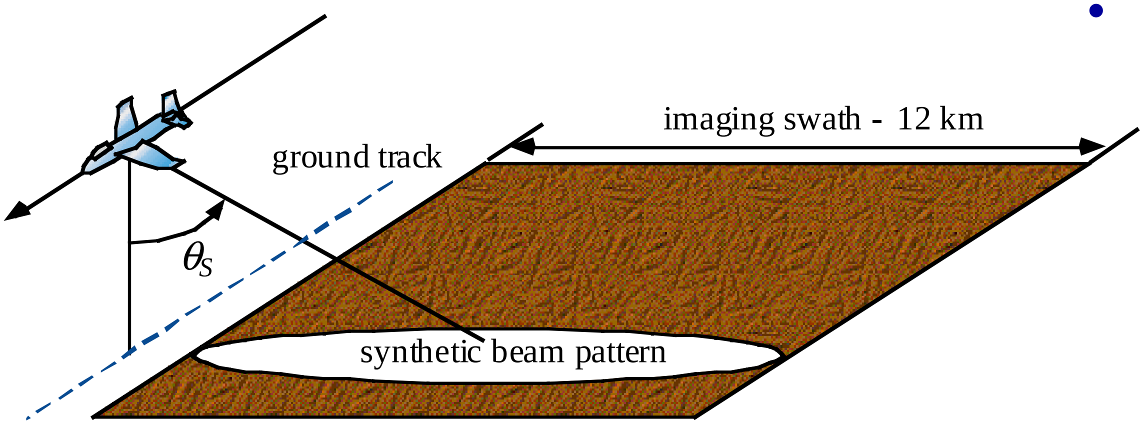
- Conclusions

Research Motivation

- Need exists for mapping surface topography over large areas
  - Covering ≥100 km^2 area requires remote sensing methods
  - Applications require ≤10 m horizontal resolution and ≤1 m vertical accuracy
- Critical applications (for low-relief topography)
  - *Hydrology*: shallow water runoff channels (~0.1 m vertical accuracy)
  - *Seismology*: active faults (~1 m vertical accuracy)
- Best technologies for low-relief topographic mapping
  - Interferometric synthetic aperture radar (INSAR)
    - Covers large area, but poor accuracy (especially when vegetation is present)
  - Laser altimeter (LIDAR))
    - Excellent accuracy, but covers small area
- Need both sensors (data fusion)
  - Exploit advantages of both sensors

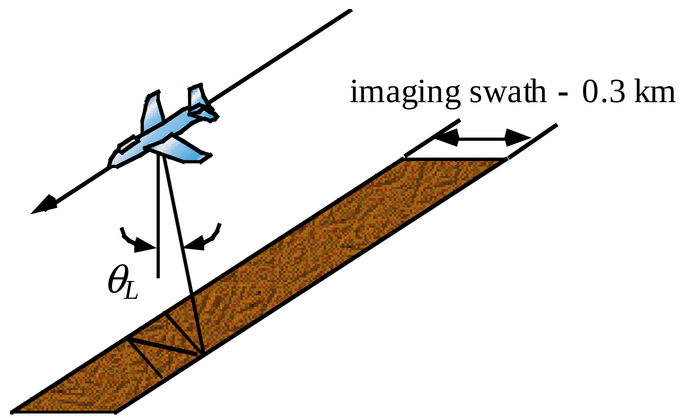
INSAR and LIDAR Imaging



- INSAR *(nominal)*
  - Side-looking
  - Fixed illumination
  - 6 cm wavelength
  - Vertical accuracy ≥ 2 m
  - 10 m pixel spacing
  - NASA/JPL TOPSAR

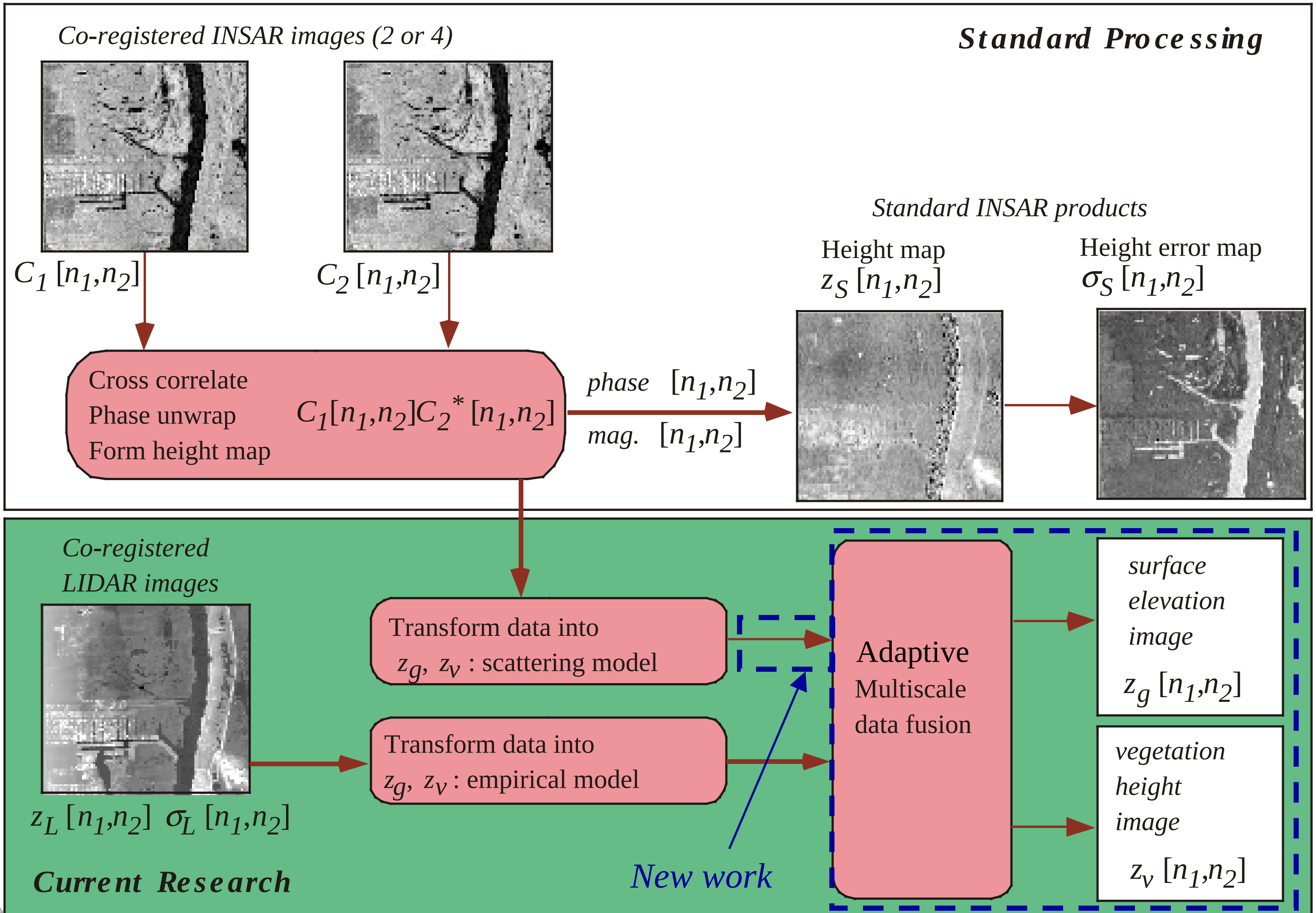
Large coverage area → primary sensor

- LIDAR *(nominal)*
  - Downward-looking
  - Scanning illumination
  - 1 µm wavelength
  - Vertical accuracy ≥ 0.1 m
  - ≤ 5 m pixel spacing
  - Optech, Inc.



complementary sensor

Estimation Framework



Choosing the Approach

- *Smooth* noisy estimates from INSAR inversion
- *Combine* data in a formal way, i.e. account for measurement errors
- Choices:
  - Wiener filter provides MMSE linear smoothing
  - Median filter provides nonlinear smoothing
    - Not suited to MIMO models
    - Not suited to indirect measurements
  - Wavelet denoising provides local smoothing
    - Provides multiresolution analysis, but
    - Not suited to MIMO models
    - Not suited to indirect measurements
  - Kalman filter provides MMSE linear smoothing
    - Allows multiresolution analysis
    - Handles MIMO models
    - Handles indirect measurements
    - Provides error measure automatically

Linear Dynamic Model

- State-space approach
  - Can model any random process having rational spectral density function using state model with finite dimensionality [Brown and Hwang 1997]
  - Can estimate *internal* variables not directly observed
- Use discrete formulation
  - Data from sampled (imaged) continuous process

$$\boxed{\begin{aligned} \boldsymbol{x}_{k+1} &= \Phi_k \boldsymbol{x}_k + \boldsymbol{w}_k \\ \boldsymbol{y}_k &= \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k \end{aligned}}$$

$\boldsymbol{x}_k$ = vector state process at time t_k; $\boldsymbol{x}_k = \boldsymbol{x}(t_k)$

Φ_k = state transition matrix; relates $\boldsymbol{x}_k$ to $\boldsymbol{x}_{k+1}$ in the absence of process noise

$\boldsymbol{w}_k$ = process noise; vector of Gaussian white sequences

$\boldsymbol{H}_k$ = linear mapping matrix between observations $\boldsymbol{y}_k$ and state $\boldsymbol{x}_k$

$\boldsymbol{v}_k$ = measurement error; vector of Gaussian white sequences

Kalman Filter: Algorithm

- Kalman filter is widely used to estimate stochastic signals
  - Requires prior model for Φ, $\boldsymbol{Q}$, $\boldsymbol{H}$, $\boldsymbol{R}$
- $\{\Phi, \boldsymbol{Q}, \boldsymbol{H}, \boldsymbol{R}\}$ assumed known and constant
- Recursive algorithm:

enter priors $(\hat{\boldsymbol{x}}_{1|0}, \boldsymbol{P}_{1|0})$:

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T\left(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}_k^T + \boldsymbol{R}\right)^{-1}$$

enter observations $(\boldsymbol{y}_k)$:

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{y}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k|k-1}\right)$$

$$\boldsymbol{P}_k = \left(I - \boldsymbol{K}_k\boldsymbol{H}_k\right)\boldsymbol{P}_{k|k-1}$$

project ahead:

$$\hat{\boldsymbol{x}}_{k+1|k} = \Phi_k\boldsymbol{x}_{k|k}$$

$$\boldsymbol{P}_{k+1|k} = \Phi_k\boldsymbol{P}_k\Phi_k^T + \boldsymbol{Q}_{\mathrm{k}}$$

repeat:

- $\boldsymbol{K}$ is Kalman gain
- $\boldsymbol{H}$ reduces to indicator function {0, 1}
  - because raw observations are transformed via pre-processing

Kalman Smoothing

- Make a return sweep through the data
  - Obtain estimates conditioned on *all* of the data
  - Variance of the estimate error is reduced
  - ***N*** is total number of samples

$$\hat{\boldsymbol{x}}(k \mid N) = \hat{\boldsymbol{x}}(k \mid k) + \boldsymbol{A}(k)[\hat{\boldsymbol{x}}(k+1 \mid N) - \boldsymbol{P}(k+1 \mid k)]$$

$$\boldsymbol{P}(k \mid N) = \boldsymbol{P}(k \mid k) + \boldsymbol{J}(k)[\boldsymbol{P}(k+1 \mid N) - \boldsymbol{P}(k+1 \mid k)]\boldsymbol{J}^{\boldsymbol{T}}(k)$$

$$\boldsymbol{J}(k) = \boldsymbol{P}(k \mid k)\Phi^{T}(k+1, k)\boldsymbol{P}^{\boldsymbol{-1}}(k+1 \mid k)$$

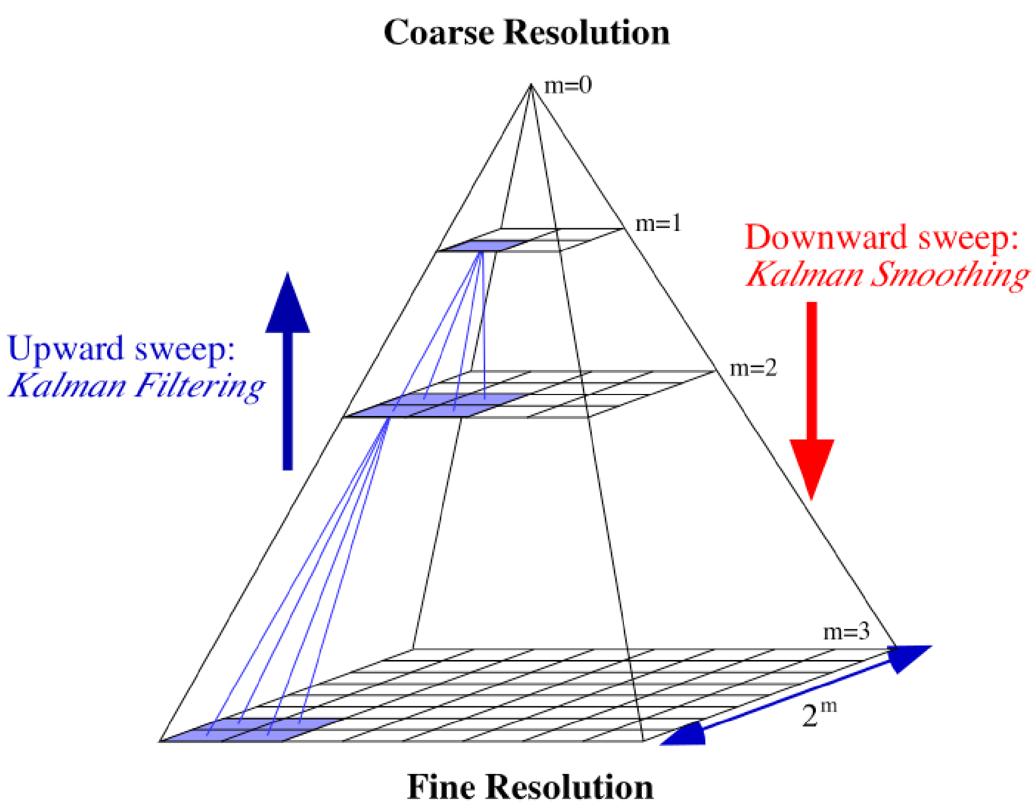
- Also called *fixed-interval smoothing* or *Rauch-Tung-Striebel smoothing*

Multiscale Data Fusion

- Multiscale signal modeling has been heavily studied in recent years

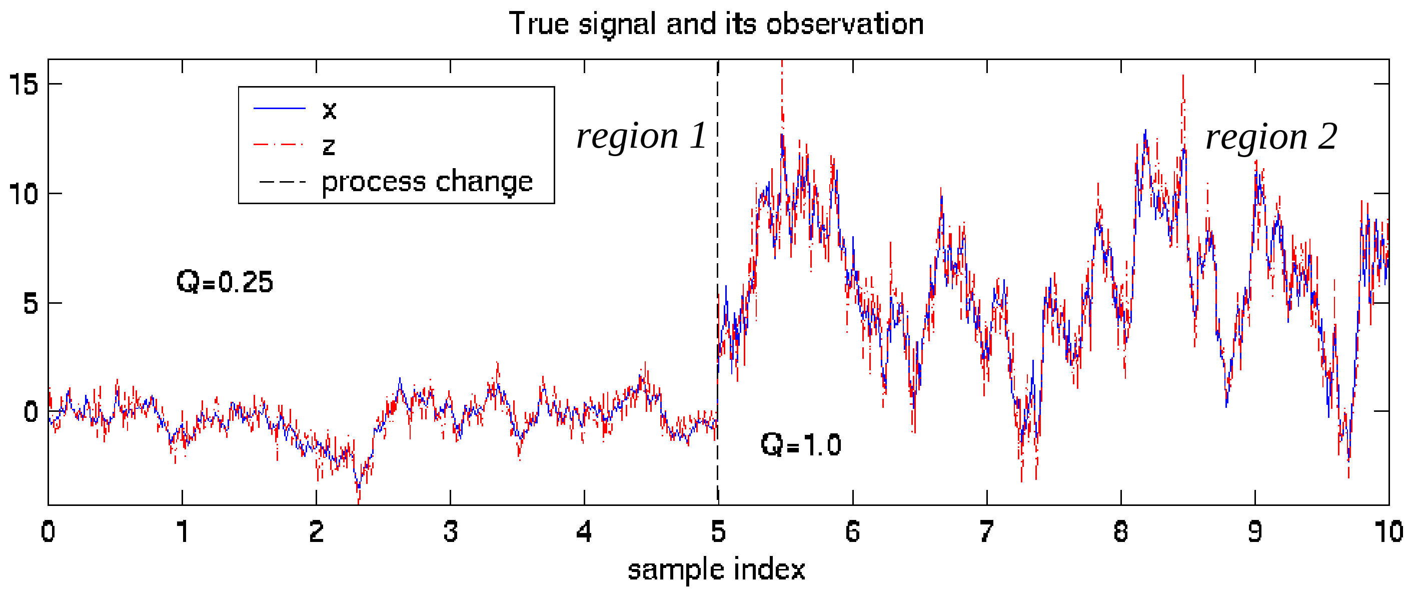
- Motivation
  - Captures multiscale character of natural processes or signals
  - Combines signals or measurements having different resolutions

- Common methods
  - Fine-to-coarse transformations of spatial models
  - Direct modeling on multiscale data structures, e.g. quadtree



Effect of Model Errors

- Example: a piece-wise WSS signal
  - True Q and R different in regions 1 to 2
  - Non-adaptive filter uses values for region 1 throughout
- Effects of non-adaptive estimation in region 2
  - Estimates become suboptimal
  - Estimates may still track data because of (y-Hx) term
  - But true error increases
  - Calculated error uncertainty is incorrect



Use Innovations to Find Model Errors

- Innovations are estimate residuals $\nu_k = y_k - Hx_{k|k-1} = He_k + v_k$
  - Where $e_k = x_k - x_{k|k-1}$ denotes error in estimate
  - They represent "innovative" information provided by observations
- Sequence ν_k is white in the optimal filter

$$\left.\begin{array}{l} E\left[\nu_k \nu_j^T\right] = E\left[\left(He_k + v_k\right)\left(He_j + v_j\right)^T\right] \\ v_k \text{ uncorrelated with } e_j \text{ and } v_j \text{ for } k \neq j \\ \text{for } k \neq j,\ e_k \text{ uncorrelated with } y_j \text{ and } \hat{x}_{j|j-1} \text{ depends only on } y_j \end{array}\right\} E\left[\nu_k \nu_j^T\right] = 0 \ , \ \text{for } k \neq j$$

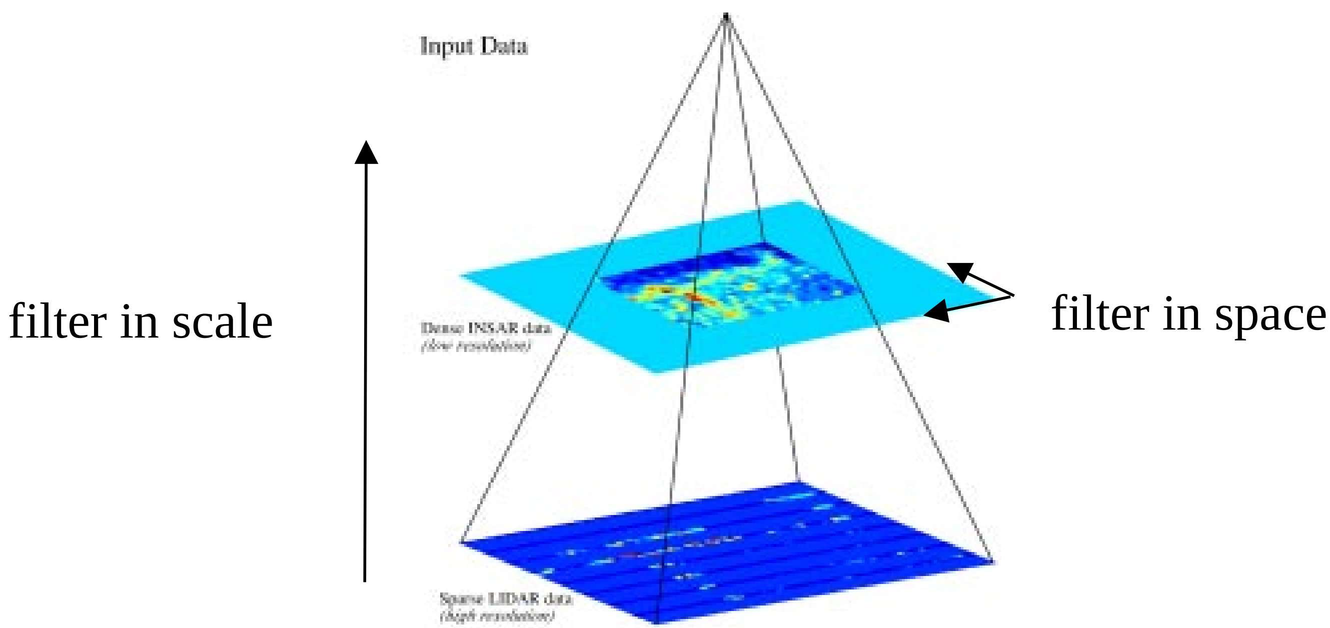
- Sequence ν_k is sum of Gaussian rv's, $\rightarrow$ Gaussian
- Model errors cause violation of uncorrelatedness assumptions
  - Yield correlation in ν_k , $E\{\nu_k \nu_j^T\} \neq 0$ in general
  - Compute ACF and test for nonzero values at nonzero lags
- Relate model parameters and ACF (ν_k) to update Q

$$\mathrm{ACF}\left(\nu_k\right) = E\left[\nu_k \nu_{k-j}^T\right] = HE\left[e_k e_{k-j}^T\right]H^T + HE\left[e_k v_{k-j}^T\right] \text{for } k \neq j$$

$$\boxed{E\left[e_k e_{k-j}^T\right]} \rightarrow f(Q)$$

Adaptive Estimation

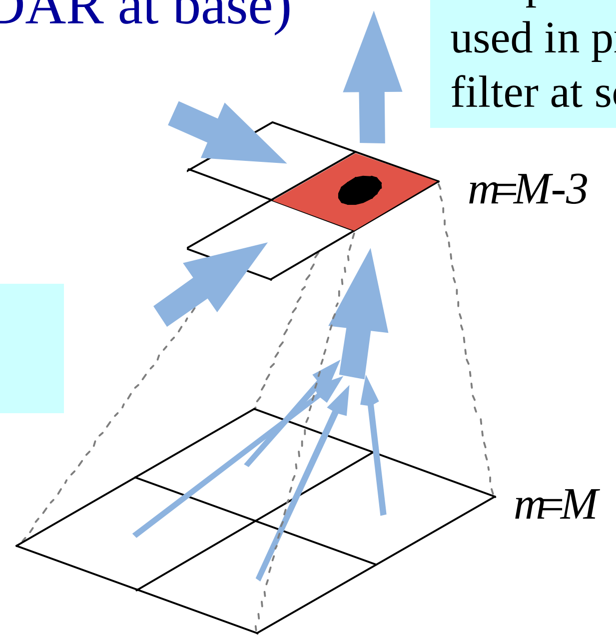
- Most adaptive estimation methods assume Q unknown but constant [Mehra 1972]
- But Q not constant for INSAR images in general
  - Non-stationary terrain, e.g. forest changing to grassland
  - Update Q in a spatial Kalman filter over the INSAR image
- Estimate Q locally with sliding window [Noriega and Pasupathy 1997]
  - Incorporate into multiscale framework
  - Use *innovation-correlation* method in sliding window of N_s samples
  - Solve $O(N_s)$ simultaneous linear equations to estimate Q



Apply to Multiscale Framework

- Use separable linear model to address non-stationary data in spatial dimension [Fornesini and Marchesini 1978]
  - Extend to multiscale framework
- Start up pyramid as before (sparse LIDAR at base)

Adaptive spatial estimates used in priors of scale-wise filter at scale *M-4*

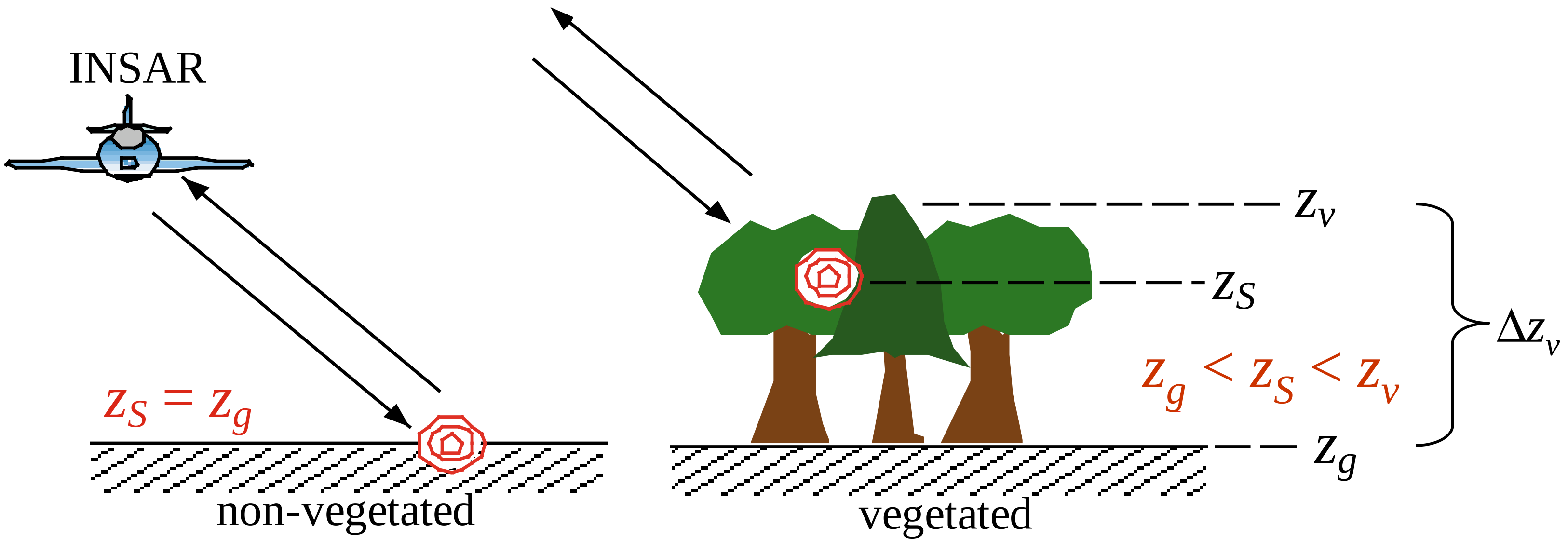


At scale *M-3*, scale-wise estimates used in priors of spatial filter

- Reach dense observations (INSAR) level (*m=M-3*)
  - Use estimates from scale-wise filter at *m=M-3* in spatial filter priors
  - Adaptively filter along rows and columns of INAR image
  - Propagate spatial filter estimates into scale-wise filter priors at level *m=M-4*
- Proceed up and down pyramid with scale-wise filter as before

Measuring Topography with INSAR

- *Problem*: no direct measurement of z_g in presence of vegetation
  - INSAR data provide height of phase scattering center z_S
  - Cannot distinguish surface elevation z_g from vegetation elevation z_v
  - Neglecting noise, $z_S = z_g$ for bare surfaces



- *Proposed solution*:
  - Estimate z_g and Δz_v from INSAR data using electromagnetic scattering model
  - Incorporate additional high-resolution measurements (LIDAR)

Transforming LIDAR Data

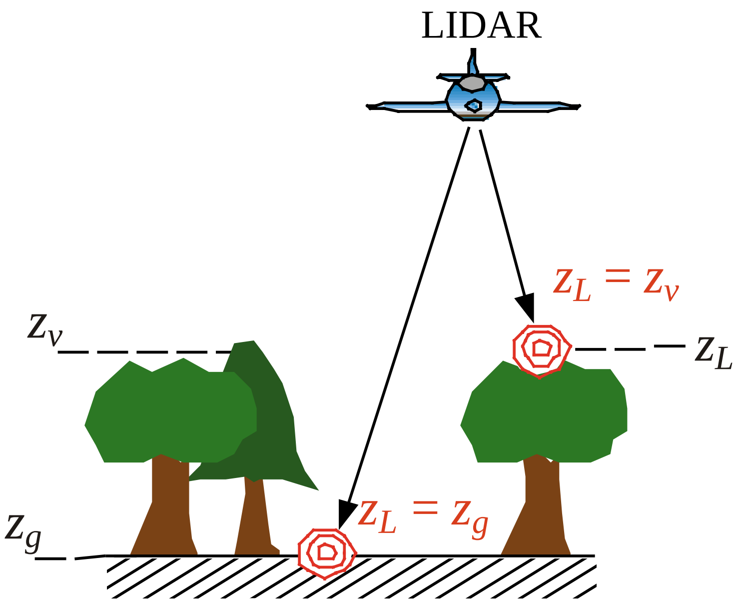
- LIDAR measures z_v directly
  - Optical wavelengths do not penetrate vegetation, except in gaps
- To get z_g at every pixel, threshold vegetated pixels and interpolate
  - standard deviation of heights σ_L indicates presence of vegetation [Neuenschwander, Crawford, Weed, and Gutierrez, 2000]

$$z_v = z_L \quad \forall (n_1, n_2) \in N$$

$$z_g = \begin{cases} z_L \ \forall (n_1, n_2) \in N \mid \sigma_L < \text{threshold} \\ \text{linear interpolation, otherwise} \end{cases}$$

$$(n_1, n_2) = \text{image pixels}$$

$$N = \text{set of all image pixels}$$



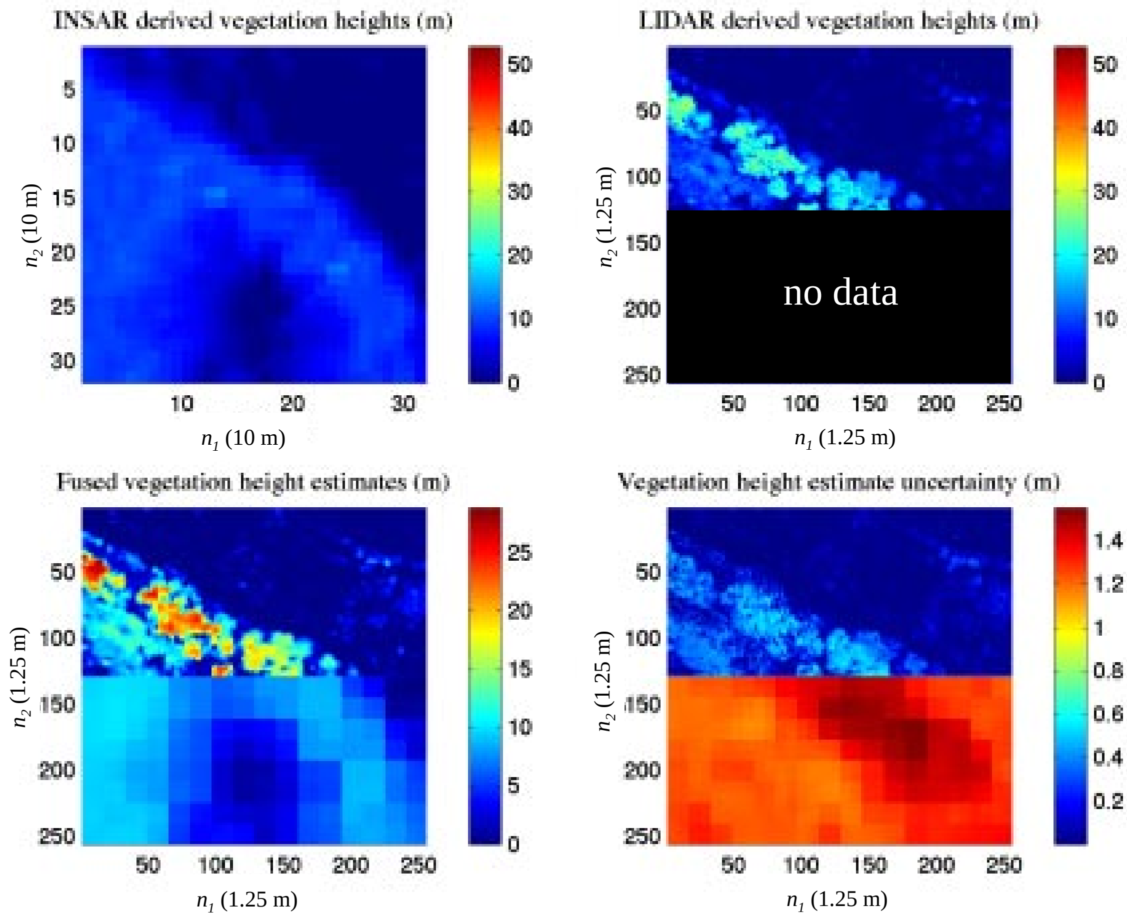
Test Area

- Austin, TX: floodplain area with trees and grassland
- Small area for testing fusion with high resolution LIDAR data
  - LIDAR @ 1.15 m, INSAR @ 10 m

2 km

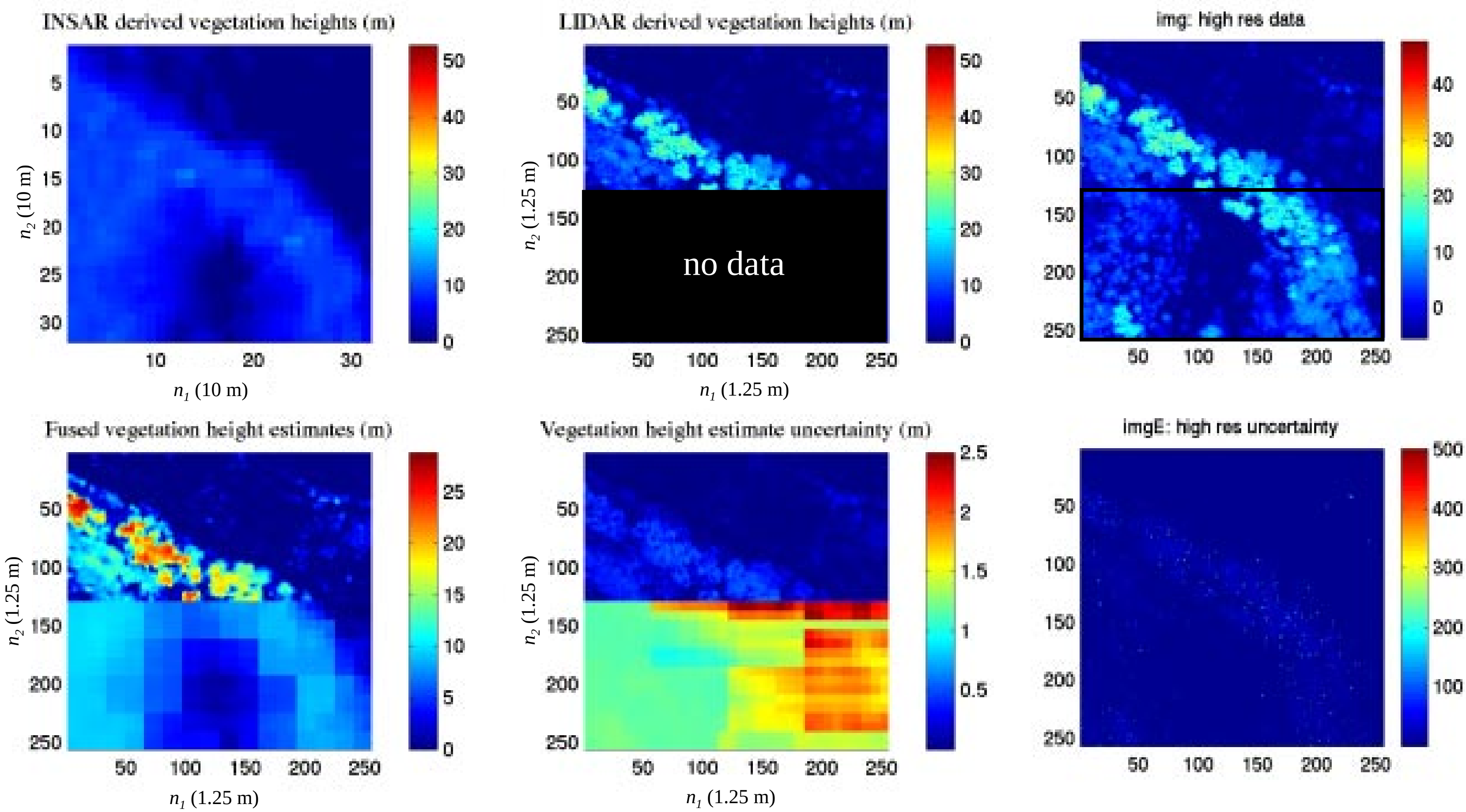
Non-Adaptive Estimates

- Transformed INSAR and LIDAR data into estimates of z_g and Δz_v
  - Physical modeling provides observations to multiscale estimation
- *No-data* areas indicate where LIDAR data were omitted



Adaptive Estimates

- ACF(ν_k) used to indicate non-white innovations
- Q updated in the spatial filter



Estimate Errors

- Reduction in global mean absolute error (MAE) relative to original INSAR
  - After multiscale Kalman smoothing (MKS): 34.9%
  - After adaptive multiscale Kalman smoothing (AMKS): 35.2%

MAE: prior to multiscale Kalman smoothing

vegetation height: $\Delta z_v^{(LIDAR)} - \Delta z_v^{(INSAR)}$	
$H = \{0 \text{ or } 1\}$	3.44 m
$H = \{1\}$	3.18 m
$H = \{0\}$	3.69 m

MAE: after multiscale Kalman smoothing

vegetation height: $\Delta z_v^{(LIDAR)} - \Delta z_v^{(MKS)}$	
$H = \{0 \text{ or } 1\}$	2.24 m
$H = \{1\}$	0.589 m
$H = \{0\}$	3.86 m

MAE: after spatially adaptive multiscale Kalman smoothing

vegetation height: $\Delta z_v^{(LIDAR)} - \Delta z_v^{(AMKS)}$	
$H = \{0 \text{ or } 1\}$	2.23 m
$H = \{1\}$	0.588 m
$H = \{0\}$	3.84 m

Conclusions

- Physical modeling of INSAR plus multiscale estimation yields statistically optimal estimates of z_g and Δz_v (MSE sense)
- Estimates improve when include LIDAR data
- Physical modeling and target-dependent measurement errors allow proper fusion of dissimilar data
- Adaptive framework accommodates non-stationary processes
- Contributions
  – Combined physical modeling with multiscale estimation to accommodate nonlinear measurement-state relations
  – Improved estimates of z_g and Δz_v for remote sensing applications
  – Developed adaptive multi-dimensional Kalman filter in scale and space