Adaptive Multiscale Estimation for Fusing Remotely Sensed Imagery

- Clint Slatton, Melba Crawford, and Brian Evans

Center for Space Research
The University of Texas at Austin

crawford@csr.utexas.edu
http://www.csr.utexas.edu
Outline

• Introduction

• Data Fusion
  – Kalman filter for data fusion
  – Multiscale Kalman filter
  – Adaptive estimation

• Measurements
  – INSAR and LIDAR measurements
  – Transforming into state variables

• Results
  – Selected data sets
  – Parameter estimates

• Conclusions
Research Motivation

• Need exists for mapping surface topography over large areas
  – Covering $\geq 100$ km² area requires remote sensing methods
  – Applications require $\leq 10$ m horizontal resolution and $\leq 1$ m vertical accuracy

• Critical applications (for low-relief topography)
  – Hydrology: shallow water runoff channels ($\sim 0.1$ m vertical accuracy)
  – Seismology: active faults ($\sim 1$ m vertical accuracy)

• Best technologies for low-relief topographic mapping
  – Interferometric synthetic aperture radar (INSAR)
    • Covers large area, but poor accuracy (especially when vegetation is present)
  – Laser altimeter (LIDAR)
    • Excellent accuracy, but covers small area

• Need both sensors (data fusion)
  – Exploit advantages of both sensors
INSAR and LIDAR Imaging

- **INSAR (nominal)**
  - Side-looking
  - Fixed illumination
  - 6 cm wavelength
  - Vertical accuracy \( \geq 2 \) m
  - 10 m pixel spacing
  - NASA/JPL TOPSAR

Large coverage area \( \rightarrow \) primary sensor

- **LIDAR (nominal)**
  - Downward-looking
  - Scanning illumination
  - 1 \( \mu \)m wavelength
  - Vertical accuracy \( \geq 0.1 \) m
  - \( \leq 5 \) m pixel spacing
  - Optech, Inc.

complementary sensor
Estimation Framework

Co-registered INSAR images (2 or 4)

\[ C_1 [n_1,n_2] \]
\[ C_2 [n_1,n_2] \]

Cross correlate
Phase unwrap
Form height map

\[ C_1[n_1,n_2]C_2^* [n_1,n_2] \]

Height map
\[ z_S [n_1,n_2] \]
Height error map
\[ \sigma_S [n_1,n_2] \]

Co-registered LIDAR images

\[ z_L [n_1,n_2] \]
\[ \sigma_L [n_1,n_2] \]

Transform data into
\[ z_g, z_v \): scattering model

Transform data into
\[ z_g, z_v \): empirical model

Adaptive
Multiscale
data fusion

Surface elevation
image
\[ z_g [n_1,n_2] \]
Vegetation
height
image
\[ z_v [n_1,n_2] \]

Current Research

New work
Choosing the Approach

- **Smooth** noisy estimates from INSAR inversion
- **Combine** data in a formal way, i.e. account for measurement errors
- **Choices:**
  - Wiener filter provides MMSE linear smoothing
  - Median filter provides nonlinear smoothing
    - Not suited to MIMO models
    - Not suited to indirect measurements
  - Wavelet denoising provides local smoothing
    - Provides multiresolution analysis, but
    - Not suited to MIMO models
    - Not suited to indirect measurements
  - Kalman filter provides MMSE linear smoothing
    - Allows multiresolution analysis
    - Handles MIMO models
    - Handles indirect measurements
    - Provides error measure automatically
Linear Dynamic Model

- State-space approach
  - Can model any random process having rational spectral density function using state model with finite dimensionality [Brown and Hwang 1997]
  - Can estimate internal variables not directly observed
- Use discrete formulation
  - Data from sampled (imaged) continuous process

\[
\begin{align*}
x_{k+1} &= \Phi_k x_k + w_k \\
y_k &= H_k x_k + v_k
\end{align*}
\]

- \(x_k\) = vector state process at time \(t_k\); \(x_k = x(t_k)\)
- \(\Phi_k\) = state transition matrix; relates \(x_k\) to \(x_{k+1}\) in the absence of process noise
- \(w_k\) = process noise; vector of Gaussian white sequences
- \(H_k\) = linear mapping matrix between observations \(y_k\) and state \(x_k\)
- \(v_k\) = measurement error; vector of Gaussian white sequences
Kalman Filter: Algorithm

- Kalman filter is widely used to estimate stochastic signals
  - Requires prior model for $\Phi, Q, H, R$
- $\{\Phi, Q, H, R\}$ assumed known and constant
- Recursive algorithm:

  enter priors $(\hat{x}_{[1|0]}, P_{[1|0]})$:
  
  $$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R)^{-1}$$
  enter observations ($y_k$):
  
  $$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - H_k \hat{x}_{k|k-1})$$
  
  $$P_k = (I - K_k H_k) P_{k|k-1}$$

  project ahead:
  
  $$\hat{x}_{k+1|k} = \Phi_k x_{k|k}$$
  $$P_{k+1|k} = \Phi_k P_k \Phi_k^T + Q_k$$

  repeat:

- $K$ is Kalman gain
- $H$ reduces to indicator function $\{0, 1\}$
  - because raw observations are transformed via pre-processing
Kalman Smoothing

- Make a return sweep through the data
  - Obtain estimates conditioned on all of the data
  - Variance of the estimate error is reduced
  - $N$ is total number of samples

\[
\hat{x}(k \mid N) = \hat{x}(k \mid k) + A(k)[\hat{x}(k + 1 \mid N) - P(k + 1 \mid k)]
\]

\[
P(k \mid N) = P(k \mid k) + J(k)[P(k + 1 \mid N) - P(k + 1 \mid k)]J^T(k)
\]

\[
J(k) = P(k \mid k)\Phi^T(k + 1, k)P^{-1}(k + 1 \mid k)
\]

- Also called fixed-interval smoothing or Rauch-Tung-Striebel smoothing
Multiscale Data Fusion

- Multiscale signal modeling has been heavily studied in recent years

- Motivation
  - Captures multiscale character of natural processes or signals
  - Combines signals or measurements having different resolutions

- Common methods
  - Fine-to-coarse transformations of spatial models
  - Direct modeling on multiscale data structures, e.g. quadtree

Upward sweep: Kalman Filtering
Downward sweep: Kalman Smoothing
Effect of Model Errors

- Example: a piece-wise WSS signal
  - True $Q$ and $R$ different in regions 1 to 2
  - Non-adaptive filter uses values for region 1 throughout

- Effects of non-adaptive estimation in region 2
  - Estimates become suboptimal
  - Estimates may still track data because of $(y-Hx)$ term
  - But true error increases
  - Calculated error uncertainty is incorrect
Use Innovations to Find Model Errors

• Innovations are estimate residuals $\nu_k = y_k - Hx_{k|k-1} = He_k + \nu_k$
  – Where $e_k = x_k - x_{k|k-1}$ denotes error in estimate
  – They represent “innovative” information provided by observations

• Sequence $\nu_k$ is white in the optimal filter

$$E[\nu_k \nu_j^T] = E \left[ (He_k + \nu_k) (He_j + \nu_j)^T \right]$$

$\nu_k$ uncorrelated with $e_j$ and $\nu_j$ for $k \neq j$

for $k \neq j$, $\nu_k$ uncorrelated with $y_j$ and $\hat{x}_{j|j-1}$ depends only on $y_j$.

$E[\nu_k \nu_j^T] = 0$, for $k \neq j$

• Sequence $\nu_k$ is sum of Gaussian rv’s, $\rightarrow$ Gaussian

• Model errors cause violation of uncorrelatedness assumptions
  – Yield correlation in $\nu_k$, $E\{\nu_k \nu_j^T\} \neq 0$ in general
  – Compute ACF and test for nonzero values at nonzero lags

• Relate model parameters and ACF ($\nu_k$) to update $Q$

$$\text{ACF}(\nu_k) = E[\nu_k \nu_{k-j}^T] = HE[e_k e_{k-j}^T]H^T + HE[e_k \nu_{k-j}^T]$$

for $k \neq j$.

Center for Space Research
The University of Texas at Austin
Adaptive Estimation

• Most adaptive estimation methods assume $Q$ unknown but constant \[\text{[Mehra 1972]}\]

• But $Q$ not constant for INSAR images in general
  – Non-stationary terrain, e.g. forest changing to grassland
  – Update $Q$ in a spatial Kalman filter over the INSAR image

• Estimate $Q$ locally with sliding window \[\text{[Noriega and Pasupathy 1997]}\]
  – Incorporate into multiscale framework
  – Use innovation-correlation method in sliding window of $N_s$ samples
  – Solve $O(N_s)$ simultaneous linear equations to estimate $Q$
Apply to Multiscale Framework

- Use separable linear model to address non-stationary data in spatial dimension [Fornesini and Marchesini 1978]
  - Extend to multiscale framework
- Start up pyramid as before (sparse LIDAR at base)
  - Reach dense observations (INSAR) level ($m=M-3$)
    - Use estimates from scale-wise filter at $m=M-3$ in spatial filter priors
    - Adaptively filter along rows and columns of INAR image
    - Propagate spatial filter estimates into scale-wise filter priors at level $m=M-4$
- Proceed up and down pyramid with scale-wise filter as before
• **Problem**: no direct measurement of \(z_g\) in presence of vegetation
  – INSAR data provide height of phase scattering center \(z_S\)
  – Cannot distinguish surface elevation \(z_g\) from vegetation elevation \(z_v\)
  – Neglecting noise, \(z_S = z_g\) for bare surfaces

• **Proposed solution**:
  – Estimate \(z_g\) and \(\Delta z_v\) from INSAR data using electromagnetic scattering model
  – Incorporate additional high-resolution measurements (LIDAR)
Transforming LIDAR Data

- LIDAR measures $z_v$ directly
  - Optical wavelengths do not penetrate vegetation, except in gaps
- To get $z_g$ at every pixel, threshold vegetated pixels and interpolate
  - standard deviation of heights $\sigma_L$ indicates presence of vegetation [Neuenschwander, Crawford, Weed, and Gutierrez, 2000]

$z_v = z_L \quad \forall (n_1, n_2) \in N$

$z_g = \begin{cases} z_L \quad \forall (n_1, n_2) \in N | \sigma_L < \text{threshold} \\ \text{linear interpolation, otherwise} \end{cases}$

$(n_1, n_2) = \text{image pixels}$

$N = \text{set of all image pixels}$
Test Area

- Austin, TX: floodplain area with trees and grassland
- Small area for testing fusion with high resolution LIDAR data
  - LIDAR @ 1.15 m, INSAR @ 10 m

2 km
Non-Adaptive Estimates

- Transformed INSAR and LIDAR data into estimates of $z_g$ and $\Delta z_v$
  - Physical modeling provides observations to multiscale estimation
- *No-data* areas indicate where LIDAR data were omitted
• ACF($\nu_k$) used to indicate non-white innovations
• $Q$ updated in the spatial filter
### Estimate Errors

- Reduction in global mean absolute error (MAE) relative to original INSAR
  - After multiscale Kalman smoothing (MKS): 34.9%
  - After adaptive multiscale Kalman smoothing (AMKS): 35.2%

**MAE: prior to multiscale Kalman smoothing**

| vegetation height: $\Delta z_v^{(\text{LIDAR})} - \Delta z_v^{(\text{INSAR})}$ |
|---|---|
| $H = \{0 \text{ or } 1\}$ | 3.44 m |
| $H = \{1\}$ | 3.18 m |
| $H = \{0\}$ | 3.69 m |

**MAE: after multiscale Kalman smoothing**

| vegetation height: $\Delta z_v^{(\text{LIDAR})} - \Delta z_v^{(\text{MKS})}$ |
|---|---|
| $H = \{0 \text{ or } 1\}$ | 2.24 m |
| $H = \{1\}$ | 0.589 m |
| $H = \{0\}$ | 3.86 m |

**MAE: after spatially adaptive multiscale Kalman smoothing**

| vegetation height: $\Delta z_v^{(\text{LIDAR})} - \Delta z_v^{(\text{AMKS})}$ |
|---|---|
| $H = \{0 \text{ or } 1\}$ | 2.23 m |
| $H = \{1\}$ | 0.588 m |
| $H = \{0\}$ | 3.84 m |
Conclusions

• Physical modeling of INSAR plus multiscale estimation yields statistically optimal estimates of $z_g$ and $\Delta z_v$ (MSE sense)

• Estimates improve when include LIDAR data

• Physical modeling and target-dependent measurement errors allow proper fusion of dissimilar data

• Adaptive framework accommodates non-stationary processes

• Contributions
  – Combined physical modeling with multiscale estimation to accommodate nonlinear measurement-state relations
  – Improved estimates of $z_g$ and $\Delta z_v$ for remote sensing applications
  – Developed adaptive multi-dimensional Kalman filter in scale and space