Matrix Gain Model For Vector Color
Error Diffusion

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Outline

• Digital halftoning
  – Modeling grayscale error diffusion halftoning
  – Modeling color error diffusion halftoning

• Matrix gain model for color error diffusion
  – Validating the signal shaping predicted by the model
  – Validating the noise shaping predicted by the model

• Conclusions
Grayscale Error Diffusion

- Shape quantization noise into high frequencies
- Two-dimensional sigma-delta modulation
- Design of error filter is key to high quality
Modeling Grayscale Error Diffusion

- Sharpening is caused by a correlated error image [Knox, 1992]

Floyd-Steinberg

Jarvis

Error images

Halftones
Modeling Grayscale Error Diffusion

- **Apply sigma-delta modulation analysis to two dimensions**
  - Linear gain model for quantizer in 1-D [Ardalan and Paulos, 1988]
  - Linear gain model for grayscale image [Kite, Evans, Bovik, 2000]
  - Signal transfer function (STF) and noise transfer function (NTF)
    - $1 - H(z)$ is highpass so $H(z)$ is lowpass

$$STF = \frac{B_s(z)}{X(z)} = \frac{K_s}{1 + (K_s - 1)H(z)}$$

signal path:

- $u(m) \rightarrow Q(.) \rightarrow b(m)$
- $NTF = \frac{B_n(z)}{N(z)} = 1 - H(z)$

noise path:

- $u_n(m) \rightarrow K_n \rightarrow K_n u_n(m) + n(m)$
- $K_n = 1$
Vector Color Error Diffusion

- Error filter has matrix-valued coefficients
- Algorithm for adapting matrix coefficients

[Akarun, Yardimci, Cetin 1997]

\[ t(m) = \sum_{k \in \phi} h(k) e(m - k) \]

\[ \text{difference} \quad u(m) \quad \text{threshold} \quad b(m) \]

\[ \text{shape error} \quad \text{compute error} \]
The Matrix Gain Model

- Replace scalar gain with a matrix

\[ \tilde{\mathbf{K}}_s = \arg \min_{\tilde{\mathbf{A}}} E \left\{ \| \mathbf{b}(m) - \tilde{\mathbf{A}} \mathbf{u}(m) \|^2 \right\} = \tilde{\mathbf{C}}_{bu} \tilde{\mathbf{C}}_{uu}^{-1} \]

\[ \tilde{\mathbf{K}}_n = \tilde{\mathbf{I}} \]

- Noise is uncorrelated with signal component of quantizer input
- Convolution becomes matrix–vector multiplication in frequency domain

\[ \mathbf{B}_n(z) = \left( \tilde{\mathbf{I}} - \tilde{\mathbf{H}}(z) \right) \mathbf{N}(z) \]

Noise component of output

\[ \mathbf{B}_s(z) = \tilde{\mathbf{K}} \left( \tilde{\mathbf{I}} + \tilde{\mathbf{H}}(z) \left( \tilde{\mathbf{K}} - \tilde{\mathbf{I}} \right) \right)^{-1} \mathbf{X}(z) \]

Signal component of output

\[ \mathbf{u}(m) \text{ quantizer input} \]

\[ \mathbf{b}(m) \text{ quantizer output} \]
How to Construct an Undistorted Halftone

- Pre-filter with inverse of signal transfer function to obtain undistorted halftone
  \[ \tilde{G}(z) = \left(1 + \tilde{H}(z)(\tilde{K} - \tilde{I})\right)\tilde{K}^{-1} \]

- Pre-filtering is equivalent to the following when \( \tilde{L} = \tilde{K}^{-1} - \tilde{I} \)

Modified error diffusion
Validation #1 by Constructing Undistorted Halftone

- Generate linearly undistorted halftone
- Subtract original image from halftone
- Since halftone should be “undistorted”, the residual should be uncorrelated with the original

Correlation matrix of residual image (undistorted halftone minus input image) with the input image

\[
\tilde{C}_{rx} = \begin{bmatrix}
0.0052 & 0.0009 & 0.0040 \\
0.0054 & 0.0023 & 0.0020 \\
0.0058 & 0.0011 & 0.0027
\end{bmatrix}
\]
Validation #2 by Knox’s Conjecture

\[ E_s(z) = 0 \]
\[ E_n(z) = N(z) \]

Correlation matrix for an error image and input image for an error diffused halftone

\[ \tilde{C}_{ex} = \begin{bmatrix} 0.3204 & 0.2989 & 0.0999 \\ 0.2787 & 0.3295 & 0.1605 \\ 0.2063 & 0.2952 & 0.1836 \end{bmatrix} \]

Correlation matrix for an error image and input image for an undistorted halftone

\[ \tilde{C}_{ex} = \begin{bmatrix} 0.0455 & 0.0235 & 0.0122 \\ 0.0493 & 0.0144 & 0.0164 \\ 0.0428 & 0.0142 & 0.0150 \end{bmatrix} \]
Validation #3 by Distorting Original Image

- Validation by constructing a linearly distorted original
  - Pass original image through error diffusion with matrix gain substituted for quantizer
  - Subtract resulting color image from color halftone
  - Residual should be shaped uncorrelated noise

Correlation matrix of residual image (halftone minus distorted input image) with the input image

\[
\mathbf{C}_{rx} = \begin{bmatrix}
0.0067 & 0.0007 & 0.0051 \\
0.0065 & 0.0039 & 0.0049 \\
0.0082 & 0.0040 & 0.0062 \\
\end{bmatrix}
\]
Validation #4 by Noise Shaping

• Noise process is error image for an undistorted halftone
• Use model noise transfer function to compute noise spectrum
• Subtract actual halftone from modeled halftone and compute actual noise spectrum
Conclusions

• Modeling of color error diffusion in the frequency domain using a coupled matrix gain and noise injection approach
• Linearizes error diffusion
• Predicts linear distortion and noise shaping effects
• Signal frequency distortion may be “cancelled”
• Filters may be designed for optimum noise shaping