Sensitivity Analysis of a Spatially-Adaptive Estimator for Data Fusion

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Introduction

- *Problem*: No general capability for mapping and updating remote sensing data acquired from multiple sensors
  - Historical data often acquired from different sensors
  - Data sets may have different extents and resolutions
  - Different imaging geometry and wavelengths yield sensor-dependent errors
- *Solution*: Develop multiscale data fusion framework to combine image data
  - Use Kalman-based method to provide robust performance and limit computational complexity
- *Application*: Mapping topography
  - Fuse Digital Elevation Models (DEMs) acquired from different sensors and at different times
  - Obtain improved DEM with sparse high-resolution data combined with dense low-resolution data
  - Obtain error map
  - Incorporate spatially-adaptive capability

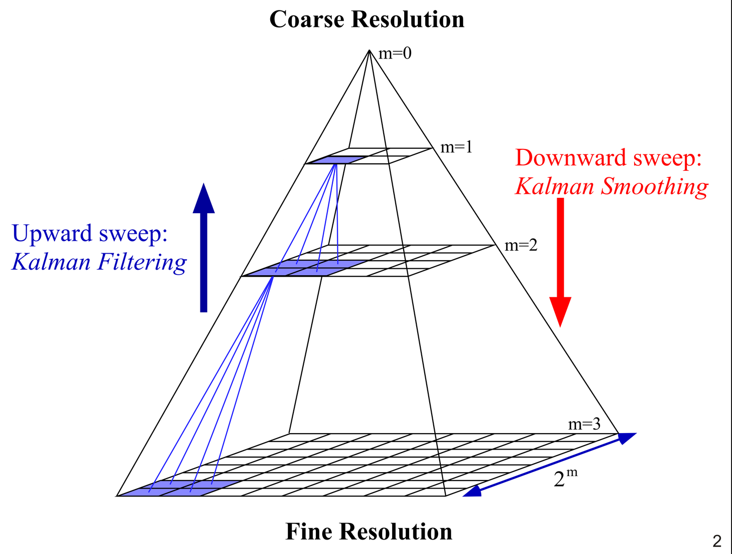
Data Fusion Framework

- Multiscale Kalman smoother (MKS) operates on a quad-tree
  - Begins with fine-to-coarse sweep up the tree (Kalman filtering with merge step) [1]
  - Followed by coarse-to-fine sweep down the tree (Kalman smoothing)
  - Accommodates sparse and irregularly spaced measurements
  - Computes minimum mean squared error estimates of state variables
  - Allows explicit separation of state variables and observations

<u>*Coarse-to-Fine Dynamic Model*</u>

$x(s)=\Phi(s)x(Bs)+\Gamma(m)w(s)$ state equation

$y(s)=H(s)x(s)+v(s)$ measurement equation



m	= scale	$y(s)$	= sensor measurement
s	= node index on multiresolution tree	$v(s)$	= measurement noise process ~$N(0,R(s))$
Bs	= backshift from s (coarse to fine)	$\Phi(s)$	= coarse-to-fine state transition
$x(s)$	= state variable	$\Gamma(s)$	= stochastic detail model
$w(s)$	= white noise process ~$N(0,1)$	$H(s)$	= observation-state mapping

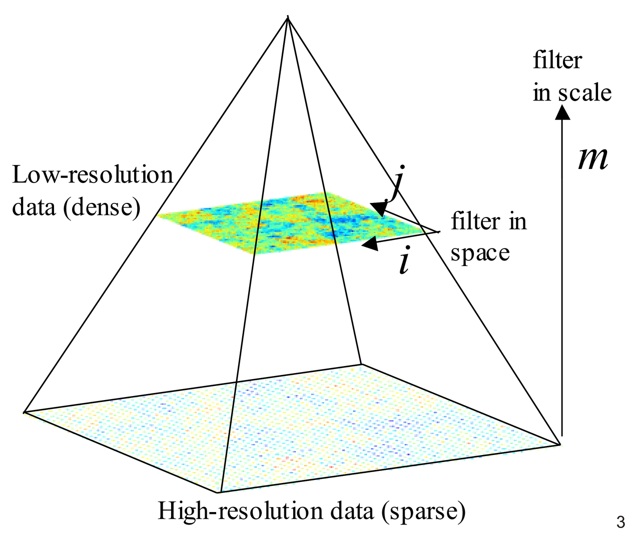
Spatially-Adaptive MKS

- Kalman process noise variance Q is derived from the downward model [1]

$$P_s(s) = \mathrm{E}[x(s)x(Bs)] = \Phi^2 P_s(Bs) + \Gamma^2(s)$$

$$Q(s) = P_s(Bs)[1 - \Phi^2 P_s(Bs)/P_s(s)]$$

- MKS has no mechanism to make Γ, P_s, or Q vary spatially
  - No variation with (i, j), only with scale m
- Benefits of adaptive MKS (AMKS) fusion [2]
  - Compensates for modeling errors in Q
  - Spatially-varying Q represents actual topography more accurately
  - Updated Q images provide insight about topographic features



Estimating Q In Spatial Dimensions

- Innovations represent prediction error $\upsilon_k = y_k - Hx_{k|k-1} = He_k + v_k$
  - Where $e_k = x_k - x_{k|k-1}$ denotes error in estimate
- The sequence υ_k is Gaussian, white sequence for optimal filter

$$\left.\begin{array}{l} E\left[\upsilon_k \upsilon_j^T\right] = E\left[(He_k + v_k)(He_j + v_j)^T\right] \\ v_k \text{ uncorrelated with } e_j \text{ and } v_j \text{ for } k \neq j \\ \text{for } k \neq j,\ e_k \text{ uncorrelated with } y_j \text{ and } \hat{x}_{j|j-1} \text{ depends only on } y_j \end{array}\right\} E\left[\upsilon_k \upsilon_j^T\right] = 0 \quad \text{for } k \neq j$$

- Model errors cause assumptions of uncorrelated noise to be violated
  - Yields correlation in υ_k, $\mathrm{E}[\upsilon_k \upsilon_j^T] \neq 0$ in general
- Detect correlation in υ_k using autocorrelation function (ACF) [3] [4]
  - Non-white ACF(υ_k) implies model errors
  - Relate Kalman parameters $\{\Phi, H, Q, R\}$ to ACF(υ_k) and update Q using innovation-correlation method

$$\mathrm{ACF}(\upsilon_k) = E\left[\upsilon_k \upsilon_{k-j}^T\right] = H\ \underbrace{E\left[e_k e_{k-j}^T\right]}_{f(Q)} H^T + H\ E\left[e_k v_{k-j}^T\right] \quad \text{for } k \neq j$$

Sensitivity to Process Noise Variance

- Spatial Kalman filter used in the data fusion is scalar-valued and reaches steady state quickly
  - Use solution to the scalar time-invariant discrete-time Ricatti equation to examine sensitivity
  - The Ricatti difference equation describes the evolution of the discrete-time a priori error variance P_k^-
  - The difference equation can be transformed into a system of two simultaneous linear difference equations [5]
  - The general solution indicates the Kalman filter rate of convergence grows with Q and Q/R

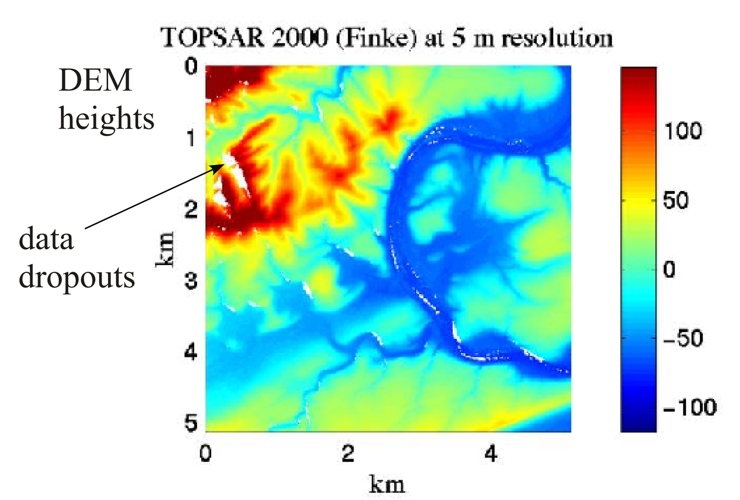
$$P_{k+1} = \Phi_k^2 \left\{ P_k - \frac{(P_k^- H_k)^2}{P_k H_k^2 + R_k} \right\} + Q_k \longrightarrow \begin{bmatrix} A_k \\ B_k \end{bmatrix} = \Psi^k \begin{bmatrix} P_0 \\ 1 \end{bmatrix} \longrightarrow \Psi = \begin{bmatrix} \Phi + \frac{QH^2}{\Phi R} & \frac{Q}{\Phi} \\ \frac{H^2}{\Phi R} & \frac{1}{\Phi} \end{bmatrix}$$

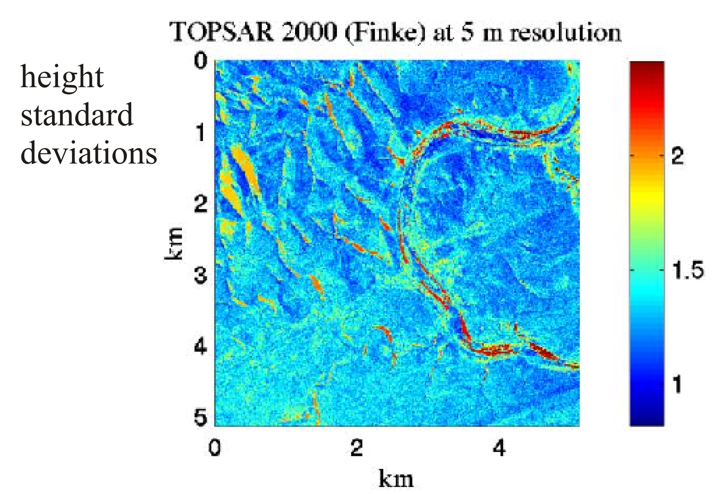
- Numerical results were computed for different values of $\Delta Q(Q/R)$
  - Relative improvement of the adaptive estimator degraded for large $\Delta Q(Q/R)$ values

Q	$\Delta Q(Q/R)$	Δ% MSE
10	0.10	52.4
1	0.01	38.5
10	1	53.7
1	0.1	44.7
10	10	16.6
1	1	30.5
10	100	2.2
1	10	18.8

Results

- Fused multiple DEMs from different radars with standard deviations σ_h >2.5 m





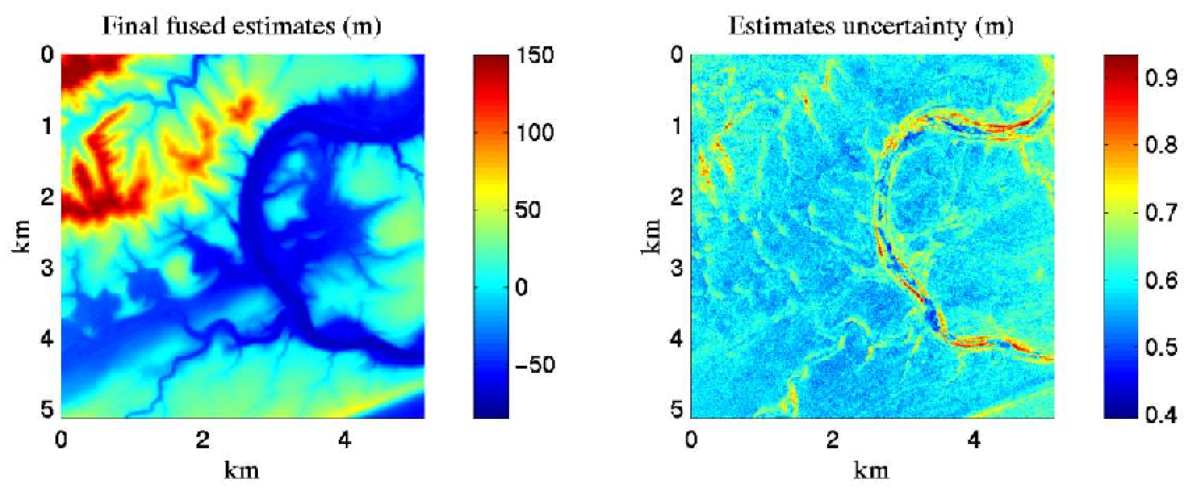
- Mean height standard deviations for each observation σ_h and fusion results sqrt(P^S)

Observation	Mean σ_h (m)
ERS	18
TOPSAR-1	1.9
TOPSAR-2	1.9
TOPSAR-3	1.6

Fused results	Mean sqrt(P^s) (m)
ERS + TOPSAR-1	1.1
prior + TOPSAR-2	0.76
prior + TOPSAR-3	0.60

Results

- Obtain improved DEM and lower uncertainty sqrt(P^S) without spatial blurring



Complexity Analysis

- MKS
  - Non-iterative, $O(N)$ operations, where N is number of leaf nodes
- AMKS
  - Non-iterative, solve for Q over contiguous pixel segements where non-white innovations detected
  - Implement at scale m=M-1, additional operations over MKS $O(N/4)$
- Adaptive algorithm represents approximately 25% additional complexity and achieves up to 15% reduction in MSE

Conclusions

- Multiscale estimation represents an improvement over splicing high-resolution data by accounting for realizations of the state process at multiple scales
- AMKS data fusion provides smaller MSE than standard MKS data fusion
  - The amount of improvement depends on the values of Q, ΔQ, and R
  - Improvements in the MKS implementation are generally less than in 1-D Kalman filter
  - Updated images of Q provide information about where the MKS algorithm is suboptimal
- Alternate approaches to spatially-adaptive MKS will be investigated, such as multiple-model filter banks

References

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[3] R. K. Mehra, "Approaches to Adaptive Filtering," *IEEE Trans. Automatic Control*, pp. 693-698, Oct. 1972.

[4] G. Noriega and S. Pasupathy, "Adaptive Estimation of Noise Covariance Matrices in Real-Time Preprocessing of Geophysical Data," *IEEE Trans. Geosci. Remote Sensing*, vol. 35, pp. 1146-1159, Sep. 1997.

[5] M. S. Grewal and A. P. Andrews, *Kalman Filtering: Theory and Practice*, Englewood Cliffs, NJ: Prentice-Hall, 1993.

Acknowledgments

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