Optimal Power Allocation in Multiuser OFDM Systems

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Why Multiuser OFDM?

- Orthogonal Frequency Division Multiplexing (OFDM) efficiently provides high data rates without intersymbol interference.
- For 4G cellular and broadband wireless, many users.
- Time division or carrier sensing will have large overhead, poor utilization of diversity.

What is OFDM?

- Discrete-time baseband system model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- Cyclic prefix $\Rightarrow$ $\mathbf{H}$ is a circular matrix, thus $\mathbf{H} = \mathbf{Q}^*\mathbf{D}\mathbf{Q}$.
- Prefiltering with IFFT matrix $\mathbf{Q}^*$ and postfiltering with the FFT matrix $\mathbf{Q}$ yields

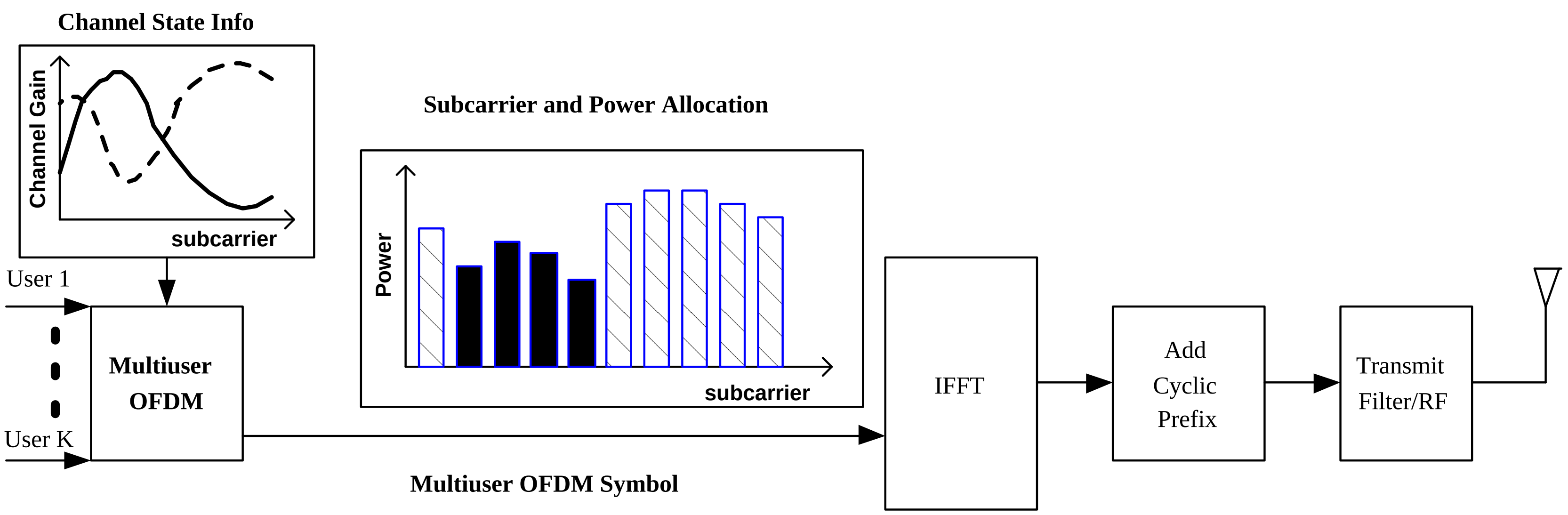
$$\mathbf{Y} = \mathbf{D}\mathbf{X} + \mathbf{N}$$

$$\mathbf{D} = \text{diag}\{\lambda_1, \lambda_2, ..., \lambda_N\}$$

- Basic Idea of Multiuser OFDM: Each user has unique $\{\lambda_n\}$, just use the good ones!

How to partition resources?

- Intuitively, it is obvious that good subcarriers for each user should be used, poor subcarriers should be left for other users.



OK, but really, how to partition resources?

- Fixed resource allocation is clearly suboptimal, but is simple.
  - TDMA: one user occupies all subchannels at each time slot
  - FDMA: each user occupies some subchannels all the time
- Adaptively allocating resources based on channel conditions is obviously better, but...
  - Channel state information at the transmitter is required. (We'll assume we have perfect CSI in this talk).
  - How to optimally assign subcarriers and power?
  - Optimal in what sense?

Rate & Margin Adaptation

- Rate adaptation: maximize capacity with a total power constraint
  - Maximize sum capacity [Jang & Lee, 2003]
  - Maximize minimum user's capacity [Rhee & Cioffi, 2000]
- Margin adaptation: minimize total transmit power given users' data rate and bit error rate requirements [Wong, Cheng, Letaief, & Murch, 1999]

Commentary

- Maximize sum capacity
  - *Good:* Overall capacity is maximized
  - *Bad:* No fairness considered. Some users may not get chance to transmit
- Maximize minimum user's capacity
  - *Good:* Each user gets approximately same capacity $\Rightarrow$ maximum fairness
  - *Bad:* Overall capacity is not maximized, inflexible allocation of rates.
- To balance sum capacity and fairness, we propose proportional fairness.
  - *In theory*, fill in the gap between two extreme cases: maximum sum capacity & maximum minimum user's capacity
  - *In practice*, allow different service privileges $\Rightarrow$ different pricing

Formal problem definition

$$\max_{p_{k,n},\mathbf{1}_{k,n}} \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{\mathbf{1}_{k,n}}{N} \log_2\left(1+\frac{p_{k,n}h_{k,n}^2}{N_0\frac{B}{N}}\right)$$

$$\begin{aligned}
\text{subject to} \quad & \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n} \le P_{total} \\
& p_{k,n} \ge 0 \text{ for all } k,n \\
& \mathbf{1}_{k,n} = \{0,1\} \text{ for all } k,n \\
& \sum_{k=1}^{K} \mathbf{1}_{k,n} = 1 \text{ for all } n \\
& R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K
\end{aligned}$$

$$R_k = \sum_{n=1}^{N} \frac{\mathbf{1}_{k,n}}{N} \log_2\left(1+\frac{p_{k,n}h_{k,n}^2}{N_0\frac{B}{N}}\right)$$

Optimization

- There are three resources that should be optimized
  1. Subcarriers
  2. Power amongst users
  3. Power amongst subcarriers, for a given user
- These quantities must be jointly allocated for global optimality.
  - KN binary integer variables $\rho_{k,n}$
  - KN continuous variables $p_{k,n}$
  - Non-convex feasible set
  - K^N subchannel allocations. $K = 4$ users, $N = 64$ subcarriers $\Rightarrow$ $K^N = 3.4 \times 10^{38}$
- Clearly, separate subchannel and power allocation is required.

Subchannel Allocation

- Modified method of [Rhee & Cioffi, 2000], assume equal power distribution to all subchannels

  1. Initialization (Enforce zero initial conditions)
     set $R_k = 0$, $\Omega_k = \emptyset$ for $k = 1, 2, ..., K$ and $A = \{1, 2, ..., N\}$

  2. For $k = 1$ to K (Allocate best subcarrier for each user)
     (a) find n satisfying $| H_{k,n} | \geq | H_{k,j} |$ for all $j \in A$
     (b) let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update R_k

  3. While $A \neq \emptyset$ (Then iteratively give lowest rate user first choice)
     (a) find k satisfying $R_k/\gamma_k \leq R_i/\gamma_i$ for all i, $1 \leq i \leq K$
     (b) for the found k, find n satisfying $| H_{k,n} | \geq | H_{k,j} |$ for all $j \in A$
     (c) for the found k and n, let $\Omega_k = \Omega_k \cup \{n\}$, $A = A - \{n\}$ and update R_k

Power Allocation for a Single User

- For user k, given total power $P_{k,tot}$, use water-filling to maximize capacity
- Optimal power distribution for user k

$$p_{k,n} = p_{k,1} - \frac{H_{k,1} - H_{k,n}}{H_{k,n} H_{k,1}}$$

$$P_{k,tot} = \sum_{n=1}^{N_k} p_{k,n} = N_k p_{k,1} - \sum_{n=2}^{N_k} \frac{H_{k,1} - H_{k,n}}{H_{k,n} H_{k,1}}$$

  - $H_{k,n}$ is the channel-to-noise ratio for user k at subchannel n
  - N_k is the number of subchannels used by user k
  - $H_{k,1} \leq H_{k,2} \leq ... \leq H_{k,N_k}$
- How to find $P_{k,tot}$?

Power Allocation among Users

- Use the proportional fairness and total power constraints

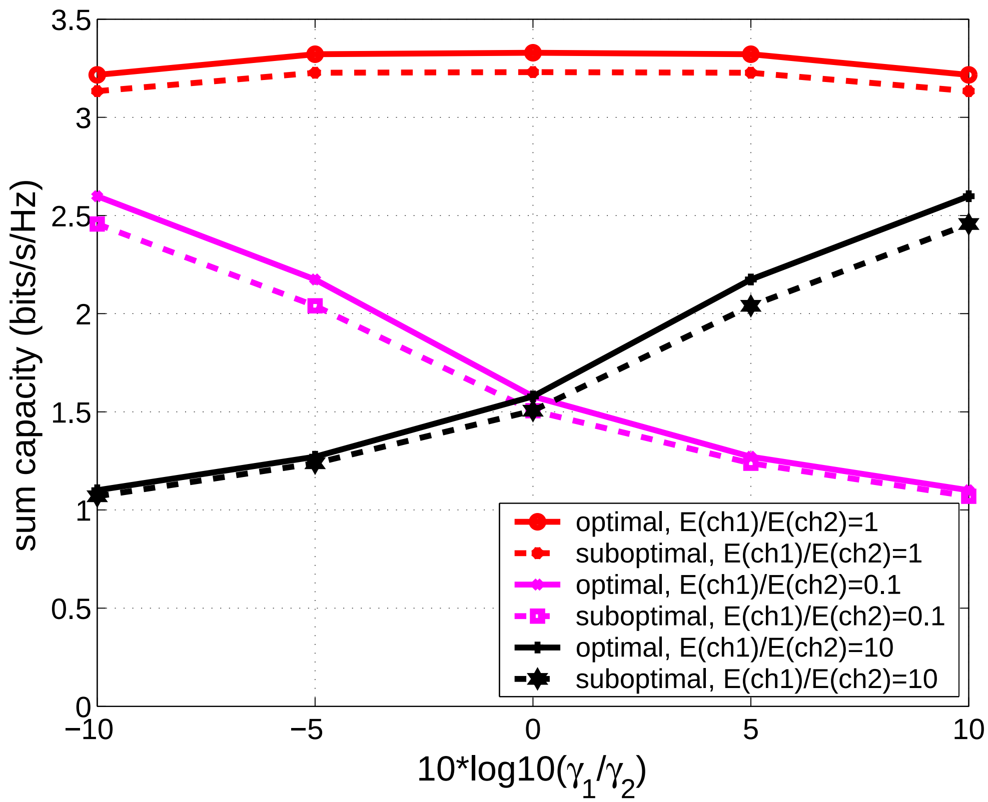
$$\frac{1}{\gamma_1} \cdot \frac{N_1}{N} \left(\log_2 \left(1 + H_{1,1} \frac{P_{1,tot} - V_1}{N_1} \right) + \log_2 W_1 \right)$$
$$= \frac{1}{\gamma_k} \cdot \frac{N_k}{N} \left(\log_2 \left(1 + H_{k,1} \frac{P_{k,tot} - V_k}{N_k} \right) + \log_2 W_k \right)$$

$$\sum_{k=1}^{K} P_{k,tot} = P_{total}$$

$$V_k = \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,n} H_{k,1}} \qquad W_k = \left(\prod_{n=2}^{N_k} \frac{H_{k,n}}{H_{k,1}} \right)^{\frac{1}{N_k}}$$

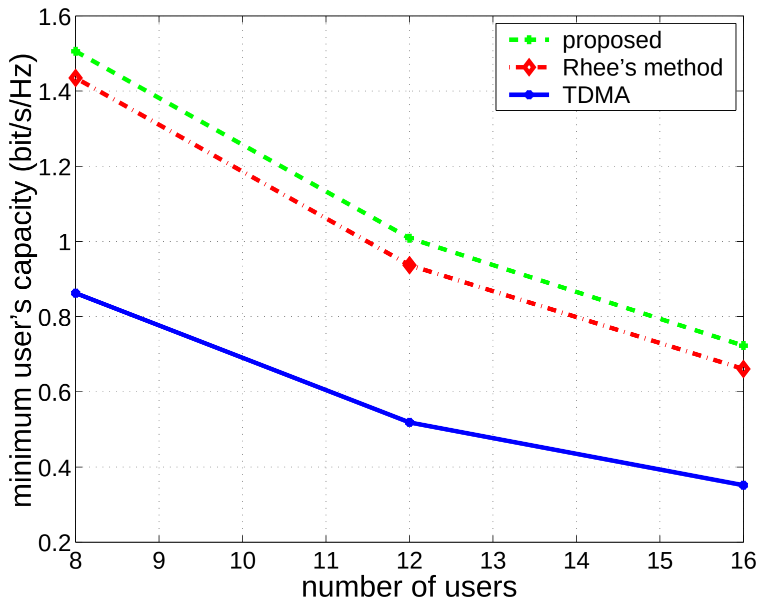
- Solving the set of nonlinear equations iteratively: $\mathcal{O}(K)$/iteration
- Two special cases
  - Linear case: $N_1 : N_2 : ... : N_K = \gamma_1 : \gamma_2 : ... : \gamma_K$, closed form solution
  - High channel-to-noise ratio: $V_k = 0$ and $H_{k,1} P_{k,tot} / N_k \gg 1$

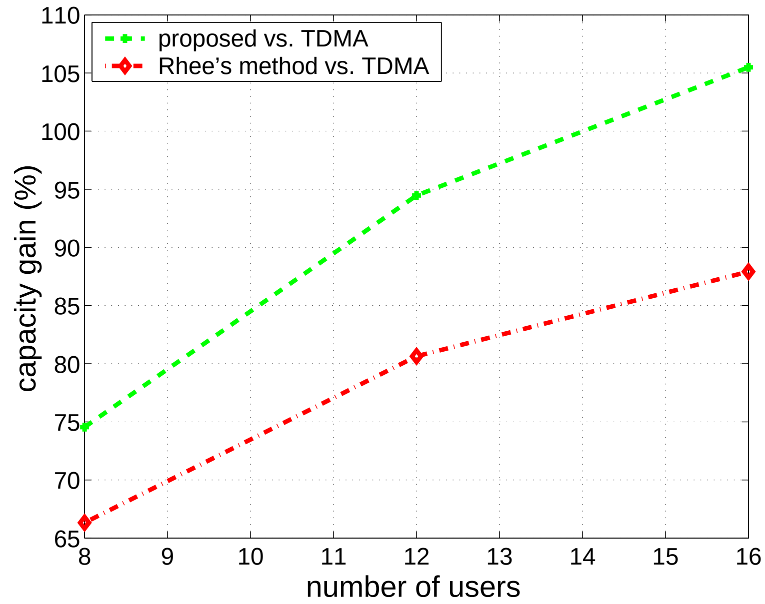
Comparison with Optimal



- Suboptimal gets very close to optimal performance.
- $K = 2$ users, $N = 10$ subcarriers, $N_0 = -100$ dBm/Hz. $P_{tot} = -30$ dBm/Hz.

Comparison with Prior Work





- Even for equal rates, proportional fairness with optimal power allocation improves upon max-min scheme.
- $N_0 = -110$ dBm/Hz, $P_{tot} = -30$ dBm/Hz, $N = 64$. The maximum path loss difference is 40 dB, and $\gamma_1 : \gamma_2 : ... : \gamma_K = 1 : 1 : ... : 1$.

Conclusion

- Two main contributions:
  1. Introduced a proportional fairness rate adaptive optimization for multiuser OFDM
  2. Provided an optimal power allocation for an arbitrary subchannel allocation
- With a separate (suboptimal) subchannel and power allocation, the complexity (in K) is reduced from exponential to linear.
  1. Loss in data rate is only a few percent relative to global optimum.
  2. Gain in data rate is around a factor of 2 relative to fixed allocations.