Comparison of Space-Time Water-Filling with Spatial Water-Filling for MIMO Fading Channels

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Background

- MIMO systems increase the spectral efficiency by exploiting the degree of freedom in the spatial dimension.
- Transmit power adaptation is possible when channel state information is known at the transmit side.
  - Temporal water-filling for SISO channels [Goldsmith & Varaiya, 1997]
  - Spatial water-filling for MIMO channels [Telatar, 1999]
  - Space-time water-filling for MIMO channels [Biglieri, Caire, & Taricco, 2001], [Jayaweera & Poor, 2004]

Motivation

- Two-dimensional space-time water-filling over one-dimensional spatial water-filling: how much gain in capacity?
- A comprehensive comparison of space-time water-filling vs. spatial water-filling
  - Capacity
  - Outage probability
  - Computational complexity
- Performance comparison in a composite channel model
  - Rayleigh fast fading
  - Log-normal shadowing

Problem formulation

- Discrete-time baseband model of a narrowband MIMO system

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v}$$

- Composite MIMO fading channel model

$$\mathbf{H} = \sqrt{s}\mathbf{H}_w$$

  - $\mathbf{H}_w$: i.i.d. Rayleigh fast fading
  - s: log-normal shadowing

- Spatial water-filling

$$\max_{\mathbf{Q}} \log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger \right| \quad \text{s. t.} \quad \mathrm{tr}(\mathbf{Q}) \leq P; \quad \text{where} \quad \mathbf{Q} = E[\mathbf{x}\mathbf{x}^\dagger]$$

- Space-time water-filling

$$\max_{\mathbf{Q}} E\left[\log \left| \mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{Q}\mathbf{H}^\dagger \right| \right] \quad \text{s. t.} \quad E\left[\mathrm{tr}(\mathbf{Q})\right] \leq \overline{P}$$

Channel capacity with space-time water-filling

- Converting two-dimensional space-time water-filling to one-dimensional temporal water-filling

$$
\begin{aligned}
E\left[\log\left|\mathbf{I}+\frac{1}{\sigma^2}\mathbf{HQH}^\dagger\right|\right] &= E\left[\sum_{k=1}^{M}\log\left(1+\frac{p(\lambda_k)\lambda_k}{\sigma^2}\right)\right] \\
&= M\ E\left[\log\left(1+\frac{p(\lambda)\lambda}{\sigma^2}\right)\right]
\end{aligned}
$$

  where λ denotes an eigenvalue of $\mathbf{H}^\dagger\mathbf{H}$; $p(\lambda)$ is power adaption for λ.

- Equivalent to temporal water-filling for a SISO channel with effective channel gain distribution $f(\lambda)$ [Goldsmith & Varaiya, 1997]
  - Optimal power adaptation $p(\lambda) = \left(\Gamma_0 - \frac{\sigma^2}{\lambda}\right)^+$
  - What is the eigenvalue distribution $f(\lambda)$?

Eigenvalue distribution of the composite channel

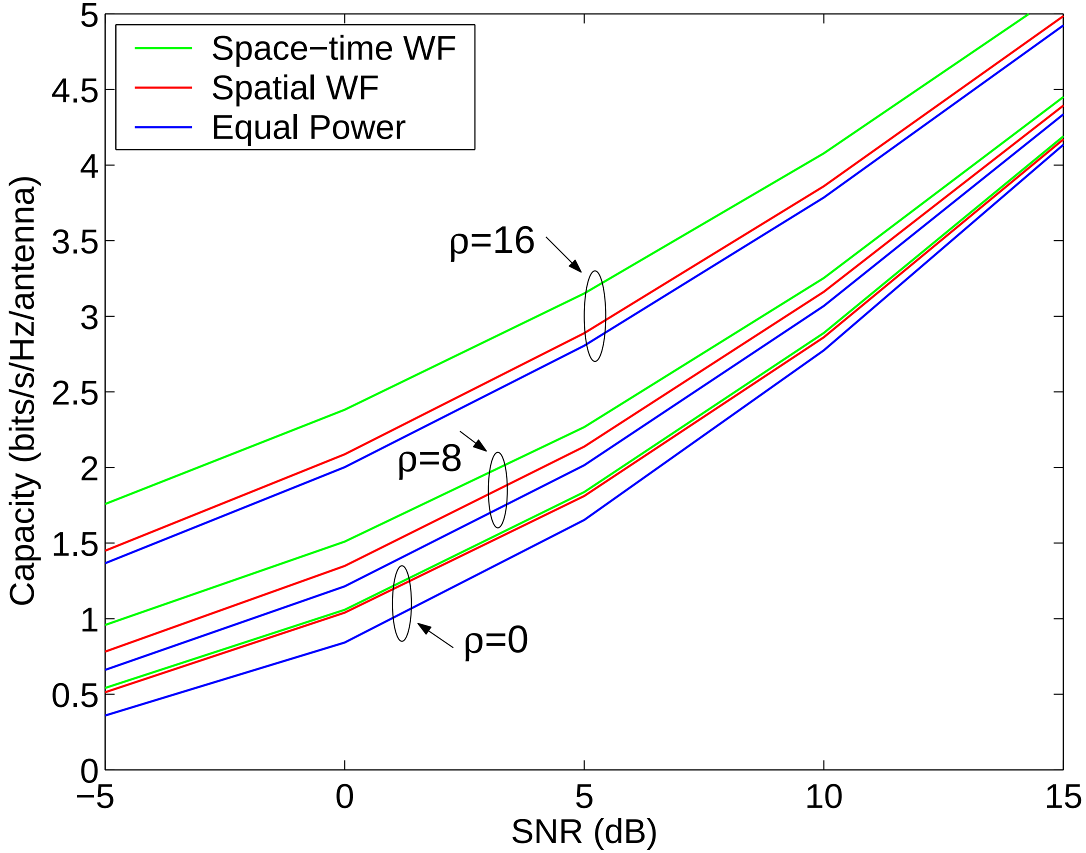
- Recall $\mathbf{H} = \sqrt{s}\mathbf{H}_w$, hence $\lambda = s\, t$, where λ is an eigenvalue of $\mathbf{H}^\dagger\mathbf{H}$ and t is an eigenvalue of $\mathbf{H}_w^\dagger\mathbf{H}_w$
- Composite channel eigenvalue distribution $f(\lambda)$
  - Eigenvalue distribution of $\mathbf{H}_w^\dagger\mathbf{H}_w$, $g(t)$ [Telatar, 1999]
  - Log-normal distribution of s, with log-normal variance ρ^2

$$f(\lambda) = \frac{10}{\rho \log 10\sqrt{2\pi}} \int_0^\infty g\left(\frac{\lambda}{s}\right) \frac{1}{s^2} e^{-\frac{(10\log_{10} s)^2}{2\rho^2}} ds$$

- Channel capacity with space-time water-filling

$$E\left[\log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{HQH}^\dagger\right|\right] = M\int_{\frac{\sigma^2}{\Gamma_0}}^\infty \log\left(\frac{\Gamma_0\lambda}{\sigma^2}\right) f(\lambda)d\lambda$$

Simulation results: capacity



- A 2×2 MIMO system, with log-normal variance ρ^2
- Maximal power mismatch: 0.02% for $\rho = 0$; 0.05% for $\rho = 8$; 1.8% for $\rho = 16$

Outage probability

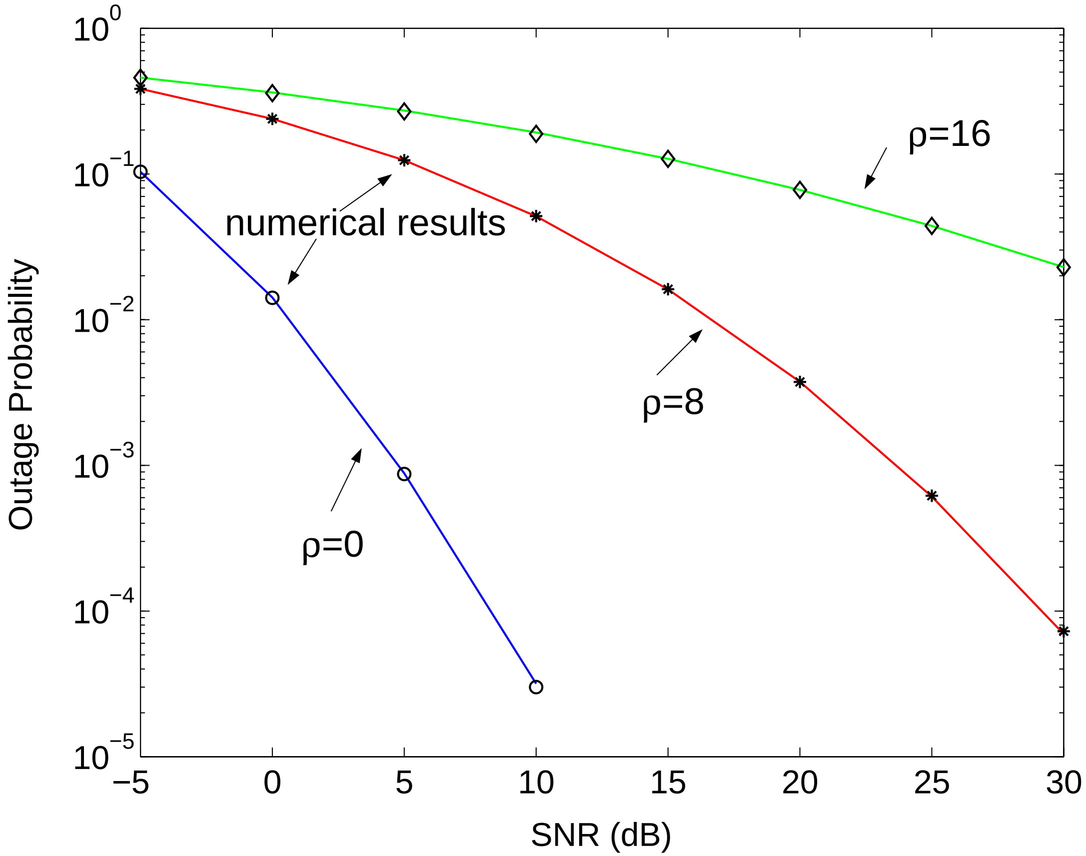
- Data transmission is blocked if effective channel gain is less than a threshold
- Definition of outage probability [Jayaweera & Poor, 2004]

$$P_{out}(\sigma^2, M) = P\{\lambda_1 \leq \frac{\sigma^2}{\Gamma_0}\}$$

- Outage probability of space-time water-filling, where $g_{max}(t_1)$ and $f_{max}(\lambda_1)$ are the maximal eigenvalue distribution of $\mathbf{H}_w^\dagger \mathbf{H}_w$ and $\mathbf{H}^\dagger \mathbf{H}$ respectively

$$P_{out} = \frac{10}{\rho \log 10 \sqrt{2\pi}} \int_0^{\sigma^2/\Gamma_0} \underbrace{\int_0^{\infty} g_{max}\left(\frac{\lambda_1}{s}\right) \frac{1}{s^2} e^{-\frac{(10\log_{10} s)^2}{2\rho^2}} ds}_{f_{max}(\lambda_1)} \; d\lambda_1$$

Simulation results: outage probability



- A 2×2 MIMO system, with log-normal variance ρ^2
- Shadowing effect dominates the outage probability.

Approximated Capacity and Outage Probability Analysis

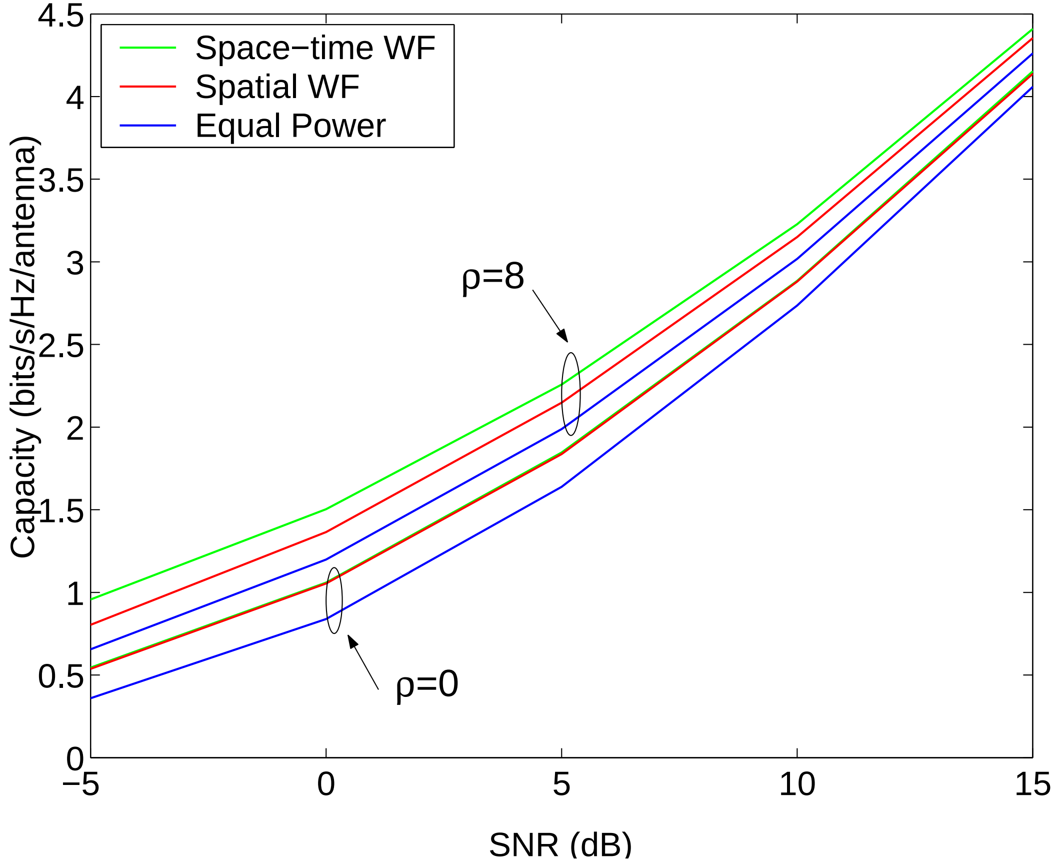
- Computationally complex to calculate the eigenvalue distributions exactly
- Limiting eigenvalue distribution for a $M \times M$ MIMO system [Telatar, 1999]

$$g(t) \approx \frac{1}{2\pi}\sqrt{\frac{4}{tM} - \frac{1}{M^2}} \qquad t \in (0, 4M)$$

- *Nakagami-m* approximation for maximal singular value of $\mathbf{H}_w$ [Wong, 2004]

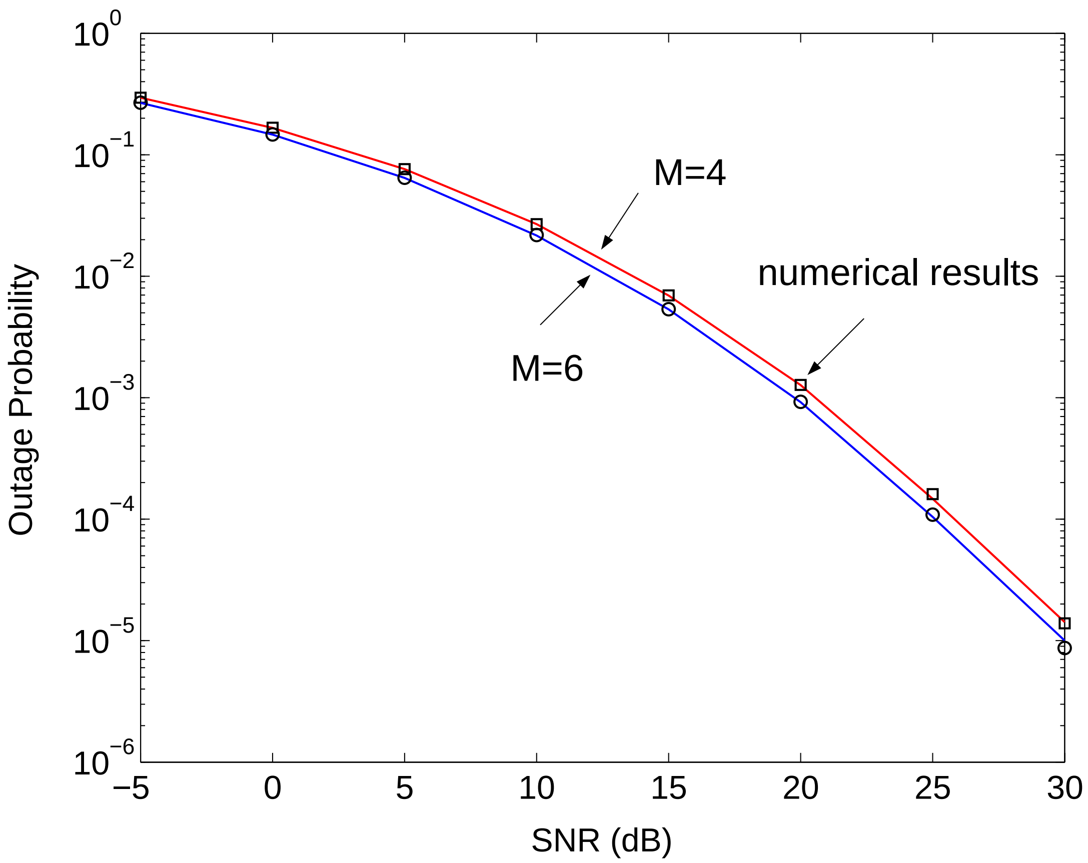
$$g_{max}(t_1) \approx \frac{m^m}{\Gamma(m)\Omega^m} t_1^{m-1} e^{-\frac{mt_1}{\Omega}}$$

Simulation results: approximated capacity



- A 4×4 MIMO system, with log-normal variance ρ^2
- Maximal power mismatch: 2.5% for $\rho = 0$; 1.8% for $\rho = 8$

Simulation results: approximated outage probability



- 4×4 and 6×6 MIMO systems; $\rho = 8$
- The number of antennas has little impact on outage probability.

Space-time vs. spatial water-filling

	space-time water-filling	spatial water-filling
spectral efficiency	optimal	suboptimal
complexity	low	high
eigenvalue distribution	required	not required
transmission mode	burst	continuous

Conclusion

- Comprehensive comparison of space-time water-filling over spatial water-filling in a composite channel model
  - Space-time water-filling gains little over spatial water-filling unless shadowing is severe.
  - Outage probability is dominated by shadowing.
- Exact method to evaluate the ergodic channel capacity and outage probability of space-time water-filling
- Approximated eigenvalue distributions to reduce the complexity