Low Complexity User Selection Algorithms for Multiuser MIMO Systems with Block Diagonalization

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***Abstract*— Block diagonalization (BD) is a precoding technique that eliminates inter-user interference in downlink multiuser multiple-input multiple-output (MIMO) systems. With the assumptions that all users have the same number of receive antennas and utilize all receive antennas when scheduled for transmission, the number of simultaneously supportable users with BD is limited by the ratio of the number of basestation transmit antennas to the number of user receive antennas. In a downlink MIMO system with a large number of users, the basestation may select a subset of users to serve in order to maximize the total throughput. The brute-force search for the optimal user set, however, is computationally prohibitive. We propose two low-complexity suboptimal user selection algorithms for multiuser MIMO systems with BD. Both algorithms aim to select a subset of users such that the total throughput is nearly maximized. The first user selection algorithm greedily maximizes the total throughput, whereas the criterion of the second algorithm is based on the channel energy. We show that both algorithms have linear complexity in the total number of users and achieve around 95% of the total throughput of the complete search method in simulations.**

I. Introduction

Multiple-input multiple-output (MIMO) systems have drawn a lot of attention in the past decade. A pioneering paper on point-to-point MIMO channel capacity by Telatar [1] showed that the MIMO channel capacity scales linearly with the minimum number of transmit and receive antennas in Rayleigh fading channels. For Gaussian broadcast multiuser MIMO channels, it was conjectured in [2] and recently proven in [3] that Dirty Paper Coding (DPC) [4] can achieve the capacity region. The sum capacity in a multiuser broadcast channel is defined to be the maximum aggregation of all users' data rates. Although DPC can achieve the sum capacity [2], deploying DPC in real-time systems is impractical because of the complicated encoding and decoding schemes [5]. An alternative and more practical precoding technique for downlink broadcast MIMO channels is Block Diagonalization (BD) [6]–[10]. With BD, each user's signal is multiplied by a precoding matrix before transmission. Every user's precoding matrix is restricted to be in the null space of all other users' channels. Hence if the channel matrices of all users are perfectly known at the transmitter, each user perceives an interference-free channel. On the other hand, BD is inferior in terms of sum capacity relative to DPC, since the users' transmit signal covariance matrices are generally not optimal.

Z. Shen is supported by Texas Instruments. R. Chen is supported by SBC Laboratories. R. Heath was supported by the National Science Foundation under grant CCF-514194 and the Office of Naval Research under grant number N00014-05-1-0169. All authors are supported by an equipment donation from Intel.

Due to the rank condition imposed by the fact that each user's precoding matrix must lie in the null space of all other users' channels, the number of users that can be simultaneously supported with BD is limited by the number of transmit antennas, the number of receive antennas, and the richness of the channels [6]. In this paper, we consider the problem of choosing a subset of users that maximize the total throughput for a multiuser system with a large number of users. We assume that every user utilizes all its receive antennas. A brute-force complete search over all possible user sets guarantees that the total throughput is maximized. The complexity, however, is prohibitive if the number of users in the system is large. For example, if $\hat{K}$ is the maximum number of users that can be simultaneously supported by BD and K is the total number of users, then the complete search for the optimal user set has combinatoric complexity because every i ($1 \leq i \leq \hat{K}$) out of K users must searched.

A user selection algorithm for downlink multiuser MISO systems has been proposed in [11], where the users are equipped with one receive antenna and zero-forcing beamforming is performed at the transmitter, which is equivalent to BD. The algorithm in [11] constructs a set of semi-orthogonal users whose total throughput is close the sum capacity achieved by DPC. Analogous to the user selection problem is the antenna selection problem where the transmitter and receiver select a subset of antennas to transmit and receive signals. A low-complexity antenna selection algorithm is proposed in [12] that achieves almost the same outage capacity as the optimal selection method. Antenna selection has also been considered in downlink multiuser MIMO systems with BD [13], where it has been shown that a significant reduction in symbol error rate can be achieved even with one extra transmit antenna. Space division multiple access (SDMA) with scheduling for multimedia services has been studied in [14]. It was shown in [14] that the system throughput-delay characteristics can be improved by scheduling the users with nearly orthogonal spatial signatures at each time slot. Several other scheduling as well as admission control algorithms for downlink SDMA systems can also be found in [15].

In this paper, we propose two suboptimal user selection algorithms for BD with the aim of maximizing the total throughput while keeping the complexity low. Both algorithms iteratively select users until the maximum number of simultaneously supportable users are reached. The first user selection algorithm greedily maximizes the total throughput. In each user selection step, the algorithm selects a user who provides the maximum total throughput with those already selected users. While the first algorithm requires frequent singular value decomposition (SVD) of the channel matrices, the second proposed algorithm selects the users based on the channel energy, thus reducing the computational complexity. We show that the proposed algorithms achieve around 95% of the total throughput of the optimal user set, and the complexity of the proposed algorithms is linear in the total number of users.

II. System Model and Background

In this section, we introduce the system model and briefly describe the block diagonalization method for multiuser MIMO systems presented in [6] [7]. In a downlink multiuser MIMO system with K users, we denote the number of transmit antennas at the base station as n_t and the number of receive antennas for the jth user as $n_{r,j}$. The transmitted symbol of user j is denoted as a N_j-dimensional vector $\mathbf{x}_j$, which is multiplied by a $n_t \times N_j$ precoding matrix $\mathbf{T}_j$ and sent to the basestation antenna array.

The received signal $\mathbf{y}_j$ for user j can be represented as

$$\mathbf{y}_j = \mathbf{H}_j\mathbf{T}_j\mathbf{x}_j + \sum_{k=1,k\neq j}^{K} \mathbf{H}_j\mathbf{T}_k\mathbf{x}_k + \mathbf{v}_j \tag{1}$$

where the second item in the right-hand-side (RHS) of (1) is the interference seen by user j from other users' signals and $\mathbf{v}_j$ denotes the Additive Gaussian White Noise (AWGN) vector for user j with variance $E[\mathbf{v}_j\mathbf{v}_j^*] = \sigma^2\mathbf{I}$. Matrix $\mathbf{H}_j \in \mathbb{C}^{n_{r,j}\times n_t}$ denotes the channel transfer matrix from the basestation to the jth user, with each entry following an i.i.d. complex Gaussian distribution $\mathcal{CN}(0,1)$ [1], which is a valid channel model if the transmit and receive antennas are in rich-scattering environments and the antenna spacing is larger than the coherence distance. Other non-physical and physical MIMO channel models can be found in [16]. For analytical simplicity, we assume that $\text{rank}(\mathbf{H}_j) = \min(n_{r,j}, n_t)$ for all users. It is also assumed that the channels $\mathbf{H}_j$ experienced by different users are independent. The key idea of block diagonalization is to precode each user's data $\mathbf{x}_j$ with the precoding matrix $\mathbf{T}_j$, such that

$$\begin{aligned} &\mathbf{T}_j \in \mathbb{U}(n_t, N_j) \\ &\mathbf{H}_i\mathbf{T}_j = 0 \quad \text{for all } i \neq j \ \text{ and } 1 \leq i,j \leq K, \end{aligned} \tag{2}$$

where $\mathbb{U}(n,k)$ represents the class of $n \times k$ unitary matrices, i.e. the collection of vectors $(\mathbf{u}_1,\ldots,\mathbf{u}_k)$ where $\mathbf{u}_i \in \mathbb{C}^n$ for all i, and the k-tuple $(\mathbf{u}_1,\ldots,\mathbf{u}_k)$ is orthonormal.

Hence with precoding matrices $\mathbf{T}_j$, the received signal for user j can be simplified to

$$\begin{aligned} \mathbf{y}_j &= \mathbf{H}_j\mathbf{T}_j\mathbf{x}_j + \sum_{k=1,k\neq j}^{K} \mathbf{H}_j\mathbf{T}_k\mathbf{x}_k + \mathbf{v}_j \\ &= \mathbf{H}_j\mathbf{T}_j\mathbf{x}_j + \mathbf{v}_j. \end{aligned} \tag{3}$$

Let $\widetilde{\mathbf{H}}_j = [\mathbf{H}_1^T \ \cdots \ \mathbf{H}_{j-1}^T \ \mathbf{H}_{j+1}^T \ \cdots \ \mathbf{H}_K^T]^T$. In order to satisfy the constraint in (2), $\mathbf{T}_j$ shall be in the null space of $\widetilde{\mathbf{H}}_j$. Let $\widetilde{N}_j$ denote the rank of $\widetilde{\mathbf{H}}_j$. Let the singular value decomposition of $\widetilde{\mathbf{H}}_j$ be $\widetilde{\mathbf{H}}_j = \widetilde{\mathbf{U}}_j\widetilde{\mathbf{\Lambda}}_j[\widetilde{\mathbf{V}}_j^1 \ \widetilde{\mathbf{V}}_j^0]^*$, where $\widetilde{\mathbf{V}}_j^1$ contains the first $\widetilde{N}_j$ right singular vectors and $\widetilde{\mathbf{V}}_j^0$ contains the last $(n_t - \widetilde{N}_j)$ right singular vectors of $\widetilde{\mathbf{H}}_j$. The columns in $\widetilde{\mathbf{V}}_j^0$ form a basis set in the null space of $\widetilde{\mathbf{H}}_j$, and hence the columns in $\mathbf{T}_j$ are linear combinations of those in $\widetilde{\mathbf{V}}_j^0$.

In the rest of the paper, we assume that every user has and uses the same number of receive antennas, i.e. $n_{r,j} = n_r$ for $j = 1,2,\cdots,K$ for simplicity. With the assumption that each element in $\mathbf{H}_j$ is i.i.d. complex Gaussian, the rank condition [6] indicates that the maximum number of simultaneous users is $\left\lceil \frac{n_t}{n_r} \right\rceil$, where $\lceil\cdot\rceil$ is the ceiling operation.

III. Low Complexity User Selection Algorithms

In this section, we first define the sum capacity (i.e. the maximum total throughput) of BD. Two suboptimal user selection algorithms are then proposed to reduce the complexity of finding the optimal user set.

Consider a set of channels $\{\mathbf{H}_j\}_{j=1}^K$ for a multiuser MIMO system. Let $\mathcal{K} = \{1,2,\cdots,K\}$ denote the set of all users, and $\mathcal{A}_i$ be a subset of $\mathcal{K}$, where the cardinality of $\mathcal{A}_i$ is less than or equal to the maximum number of simultaneous users $\hat{K}$. Let $\overline{\mathbf{H}}_j = \mathbf{H}_j\mathbf{T}_j$ denote the effective channel after precoding for user $j \in \mathcal{A}_i$. The total throughput achieved with BD applied to the user set $\mathcal{A}_i$ with total power P can be expressed as

$$C_{BD|\mathcal{A}_i}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2) = \max_{\{\mathbf{Q}_j:\, \mathbf{Q}_j \geq 0,\, \sum_{j\in\mathcal{A}_i} \text{Tr}(\mathbf{Q}_j) \leq P\}} \sum_{j\in\mathcal{A}_i} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\overline{\mathbf{H}}_j\mathbf{Q}_j\overline{\mathbf{H}}_j^*\right| \tag{4}$$

where $\mathbf{Q}_j = E[\mathbf{x}_j\mathbf{x}_j^*]$ is user j's input covariance matrix of size $N_j \times N_j$ and $\mathbf{H}_{\mathcal{A}_i}$ denotes the set of channels for those users in $\mathcal{A}_i$. Notice that the solution to the RHS of (4) can be obtained by the water-filling algorithm over the eigenvalues of $\{\overline{\mathbf{H}}_j\overline{\mathbf{H}}_j^*\}_{j\in\mathcal{A}_i}$ with total power constraint P as in [6].

Let $\mathcal{A}$ be the set containing all possible $\mathcal{A}_i$, i.e. $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \cdots\}$, then the sum capacity (maximum total throughput) with BD can be defined as

$$C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) = \max_{\mathcal{A}_i\in\mathcal{A}} C_{BD|\mathcal{A}_i}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2). \tag{5}$$

TABLE I

CAPACITY-BASED SUBOPTIMAL USER SELECTION ALGORITHM

1) Initially, let $\Omega = \{1, 2, \cdots, K\}$ and $\Upsilon = \emptyset$. Let $s_1 = \arg\max_{k\in\Omega} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^*\right|$ where $\mathrm{Tr}(\mathbf{Q}_k) \le P$ and $\mathbf{Q}_k$ is semi-positive definite. Let $\Omega = \Omega - \{s_1\}$ and $\Upsilon = \Upsilon + \{s_1\}$. Let $C_{temp} = \max_{k\in\Omega} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}_k\mathbf{Q}_k\mathbf{H}_k^*\right|$.
2) for $i = 2 : \hat{K}$
   a) For every $k \in \Omega$,
      i) Let $\overline{\Upsilon}_k = \Upsilon + \{k\}$.
      ii) Find the precoding matrix $\mathbf{T}_j$ for each $j \in \overline{\Upsilon}_k$, and obtain the effective channel $\overline{\mathbf{H}}_j = \mathbf{H}_j\mathbf{T}_j$ for each $j \in \overline{\Upsilon}_k$.
      iii) Perform a singular value decomposition (SVD) on $\overline{\mathbf{H}}_j$, and obtain the M singular values $\{\lambda_{j,m}\}_{i=1}^{M}$.
      iv) Water-fill over $\lambda_{j,m}^2$ for $j \in \overline{\Upsilon}_k$ and $1 \le m \le M$. Find the total throughput to the user set $\overline{\Upsilon}_k$, denoted as C_k.
   b) Let $s_i = \arg\max_{k\in\Omega} C_k$.
   c) If $\max_{k\in\Omega} C_k < C_{temp}$
      Algorithm terminated. The selected user set is Υ.
      else
      Let $\Omega = \Omega - \{s_i\}$ and $\Upsilon = \Upsilon + \{s_i\}$. And let $C_{temp} = \max_{k\in\Omega} C_k$.

Denote $\hat{K} = \left\lceil \frac{n_t}{n_r} \right\rceil$ as the maximum number of simultaneous users, and the Cardinality of $\mathcal{A}$ is $|\mathcal{A}| = \sum_{i=1}^{\hat{K}} {}_K C_i$, where ${}_n C_m$ denotes the combination of n choosing m. Hence, it is clear that a brute-force exhaustive search over $\mathcal{A}$ is computationally prohibitive if $K \gg \hat{K}$.

A. Capacity-Based Suboptimal User Selection Algorithm

The exhaustive search method needs to consider roughly $\mathcal{O}(K^{\hat{K}})$ possible user sets. In this section, we present a suboptimal algorithm whose complexity is $\mathcal{O}(\hat{K}K)$.

Let s_i denote the user index selected in the ith iteration, i.e. $s_i \in \{1, 2, \cdots, K\}$ and $1 \le i \le \hat{K}$. Let Ω denote the set of unselected users and Υ denote the set of selected users. The capacity-based user selection algorithm is described in Table I. In words, the algorithm first selects the single user with the highest capacity. Then, from the remaining unselected users, it finds the user that provides the highest total throughput together with those selected users. The algorithm terminates when $\hat{K}$ users are selected or the total throughput drops if more users are selected (the total throughput may decrease with an additional user because the size of the null space for every user reduces in order to meet the zero inter-user interference requirement). Clearly, the proposed algorithm needs to search over no more than $\hat{K}K$ user sets, which greatly reduces the complexity compared to the exhaustive search method. Since the user selection criterion is based on the sum capacity, we name the above algorithm the capacity-based suboptimal user selection algorithm.

B. Frobenius Norm-Based Suboptimal User Selection Algorithm

Although the capacity-based suboptimal user selection algorithm greatly reduces the size of the search set, the algorithm still may not be cost-effective for real-time implementation because singular value decomposition, which is computationally intensive, is required for each user in each iteration to find the total throughput. In this section, we propose another suboptimal user selection algorithm which is based on channel Frobenius norm. The motivation is that the capacity is closely related to eigenvalues of the effective channel after precoding. Although the channel Frobenius norm cannot characterize the capacity completely, it is related to the capacity because the Frobenius norm indicates the overall energy of the channel, i.e. the sum of the eigenvalues of $\mathbf{HH}^*$ equals $||\mathbf{H}||_F^2$.

TABLE II

FROBENIUS NORM-BASED SUBOPTIMAL USER SELECTION ALGORITHM

1) Initially, let $\Omega = \{1, 2, \cdots, K\}$ and $\Upsilon = \emptyset$. Let $s_1 = \arg\max_{k\in\Omega} ||\mathbf{H}_k||_F^2$. Let $\mathbf{V} = \mathbf{V}_{s_1}$. Let $\Omega = \Omega - \{s_1\}$ and $\Upsilon = \Upsilon + \{s_1\}$.
2) for $i = 2 : \hat{K}$
   a) For each $k \in \Omega$, let $\widetilde{\mathbf{H}}_k = \mathbf{H}_k - \mathbf{H}_k\mathbf{V}^*\mathbf{V}$. Then $\widetilde{\mathbf{H}}_k$ is in the null space of $\mathbf{V}$.
      for $j = 1 : i-1$
      i) Let
$$\hat{\mathbf{H}}_{s_j,k} = [\mathbf{H}_{s_1}^T \ \cdots \ \mathbf{H}_{s_{j-1}}^T \ \mathbf{H}_{s_{j+1}}^T \ \cdots \ \mathbf{H}_{s_{i-1}}^T \ \mathbf{H}_k^T]^T.$$
      ii) Let $\mathbf{W}_{s_j,k}$ be the row basis for $\hat{\mathbf{H}}_{s_j,k}$ after Gram-Schmidt orthogonalization.
   b) For each $s \in \Upsilon$, let $\widetilde{\mathbf{H}}_s = \mathbf{H}_s - \mathbf{H}_s\mathbf{W}_{s,k}^*\mathbf{W}_{s,k}$. Then $\widetilde{\mathbf{H}}_s$ is in the null space of $\hat{\mathbf{H}}_{s,k}$. Let
$$s_i = \arg\max_{k\in\Omega}\left(\sum_{s\in\Upsilon} ||\widetilde{\mathbf{H}}_s||_F^2 + ||\widetilde{\mathbf{H}}_k||_F^2\right).$$
   c) Let $\Omega = \Omega - \{s_i\}$ and $\Upsilon = \Upsilon + \{s_i\}$. Apply the Gram-Schmidt orthogonalization procedure to $\widetilde{\mathbf{H}}_{s_i}$ and get $\widetilde{\mathbf{V}}_{s_i}$. Let $\mathbf{V} = [\mathbf{V}^T \ \widetilde{\mathbf{V}}_{s_i}^T]^T$.
3) Apply the capacity-based suboptimal user selection algorithm to the set Υ, and get the final selected user set and the total throughput.

Let s_i denotes the user index selected in the ith iteration, i.e. $s_i \in \{1, 2, \cdots, K\}$ and $1 \le i \le \hat{K}$. Let Ω denote the set of unselected users and Υ denote the set of selected users. Let $\mathbf{V}_k$ be the basis for the row vector space of $\mathbf{H}_k$ after applying the Gram-Schmidt orthogonalization procedure to the rows of $\mathbf{H}_k$. The Frobenius norm-based user selection algorithm is described in Table II. The idea of the norm-based user selection algorithm is to select the set of users such that the sum of the effective channel energy of those selected users is as large as possible. Notice that steps 1 and 2 in the norm-based algorithm are independent with SNR, i.e. P. Once the $\hat{K}$ users are selected, step 3 makes the final user selection (possibly a subset of the $\hat{K}$ users chosen by steps 1 and 2) with the capacity-based algorithm, where the SNR is taken into consideration. Clearly, the norm-based algorithm requires fewer SVD operations than the capacity-based algorithm.

IV. SIMULATION RESULTS

In this section, we compare the performance of the following algorithms:

- optimal user selection by complete search (optimal),

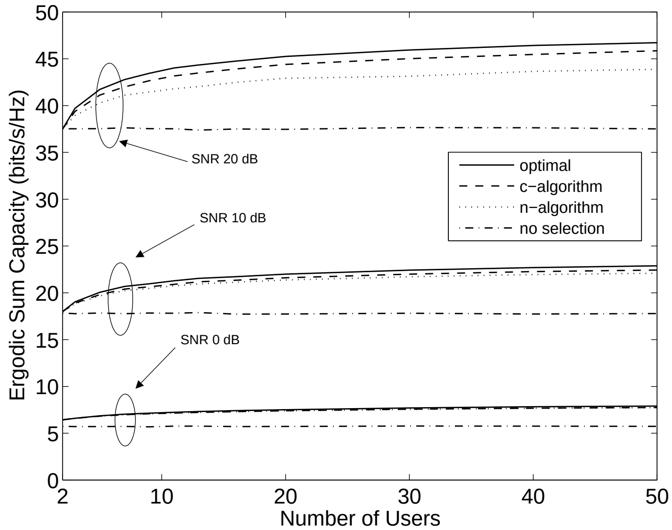

Fig. 1. Ergodic sum capacity vs. the number of users, where $n_t = 8$, $n_r = 4$, and $\hat{K} = 2$.

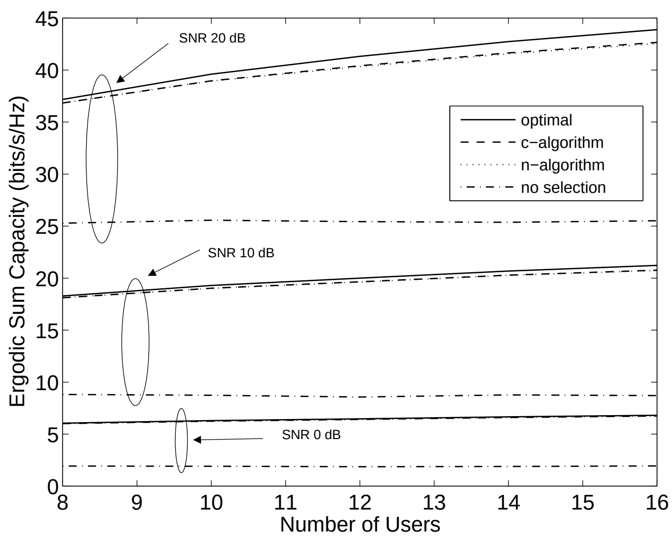

Fig. 2. Ergodic sum capacity vs. the number of users, where $n_t = 8$, $n_r = 1$, and $\hat{K} = 8$.

- capacity-based user selection algorithm (c-algorithm),
- Frobenius norm-based user selection algorithm (n-algorithm),
- round-robin algorithm for $\hat{K}$ simultaneous users (no selection).

Figs. 1-3 show the ergodic sum capacity (averaged over 1000 channel realizations) vs. the number of users for $(n_t = 8, n_r = 4)$, $(n_t = 8, n_r = 1)$, and $(n_t = 12, n_r = 4)$ MIMO systems, where $\hat{K} = 2$, $\hat{K} = 8$, and $\hat{K} = 3$ respectively. It is shown that the capacity-based and the norm-based user selection algorithms achieve around 95% of the total throughput of the complete search method. The capacity-based algorithm performs slightly better than the norm-based algorithm because its user selection criterion is directly based on the sum capacity. For low SNRs, e.g. $SNR = 0$ dB, the proposed algorithms achieve almost the same sum capacity as the exhaustive search method. This is true because beamforming to the user with the highest capacity, which is the first step in the capacity-based user selection algorithm, is asymptotically optimal for sum capacity of BD in the low SNR regime. For high SNRs, although the proposed algorithms may not always find the optimal user set due to their reduced search range, they can still achieve a significant part of the ergodic sum capacity of the exhaustive search method because both algorithms greedily try to maximize the total throughput.

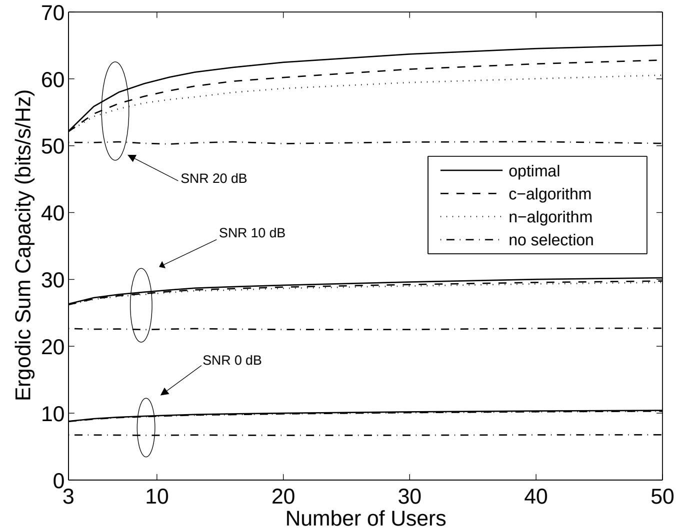

Fig. 3. Ergodic sum capacity vs. the number of users, where $n_t = 12$, $n_r = 4$, and $\hat{K} = 3$.

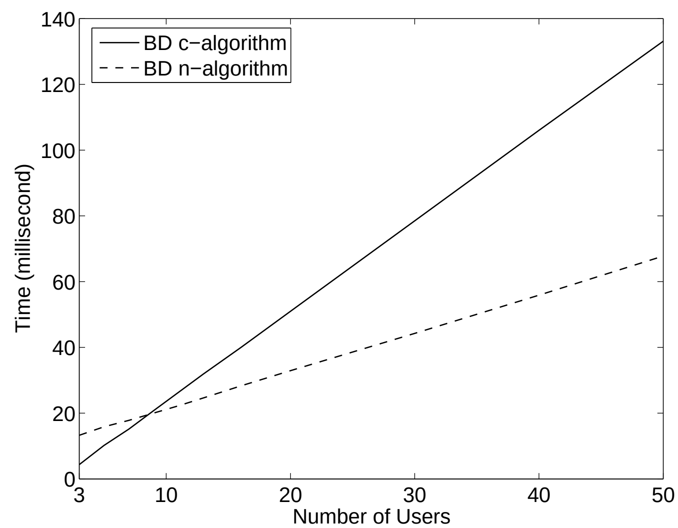

Fig. 4. Average run time for the capacity-based and norm-based user selection algorithms, where $n_t = 12$, $n_r = 4$, $\hat{K} = 3$, and $SNR = 10$ dB.

Fig. 4 shows the average CPU (Pentium M 1.6 GHz PC) run-time of the two proposed algorithms vs. the number of users in the systems. For the curves in Fig. 4, the numbers of transmit and receive antennas are 12 and 4, respectively.

Hence, the maximum number of simultaneously supportable users by BD is 3. Fig. 4 shows that both of the proposed algorithms have linear complexity (in terms of CPU run-time) in the number of total users in the system, because no more than $K\hat{K}$ user sets need to be searched over. The capacity-based algorithm has higher complexity than the norm-based algorithm, since SVD is more frequently performed to obtain the MIMO channel eigenvalues. The ergodic sum capacity of the capacity-based algorithm, however, is higher than the norm-based algorithm as shown in Figs. 1-3. Further, since only tens to hundreds of milliseconds are required by the proposed algorithms to select 3 out of $30-50$ users, the proposed algorithms are suitable for real-time implementations for systems with slow to medium Doppler frequency spread.

The computational complexity of the two proposed algorithms is mainly from the following operations:

- matrix Frobenius norm calculation,
- Gram-Schmidt orthogonalization,
- water-filling algorithm for optimal power allocation,
- singular value decomposition.

A detailed complexity analysis of the proposed algorithms can be found in [18].

V. Conclusion

Two suboptimal user selection algorithms for multiuser MIMO systems with block diagonalization are proposed in this paper. The goal is to select a subset of users to maximize the total throughput while keeping the complexity low. The brute-force complete search method yields the optimal user set with the sum capacity achievement. However, the complexity of the complete search algorithm is roughly $\mathcal{O}\{K^{\hat{K}}\}$, where K is the total number of users and $\hat{K}$ is the maximum number of simultaneous users. Simulations show that the proposed capacity-based and norm-based user selection algorithms achieve about 95% of the sum capacity whereas their complexity is $\mathcal{O}\{K\}$. Although the proposed user selection algorithms are greedy in nature, they can be easily extended to incorporate fairness, e.g. the rate proportional fairness in [19].

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