Joint Interference Cancellation and Channel Shortening in Multi-User MIMO Systems

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Abstract

In this paper, a two-stage receiver structure for interference cancellation in multi-user spatially-multiplexed multiple-antenna systems is presented. A space-time equalizer is used in the first stage for joint coantenna/cochannel interference suppression and shortening of the effective channel for each transmit stream of the desired user. The channel shortening, combined with independent detection, helps reduce the complexity of the second stage Viterbi equalizer, which is used for separate intersymbol interference equalization for each of the streams. Three objective functions are proposed for determining the coefficients of the space-time equalizer using a direct training data-based approach, which does not require estimation of the interferer's channel. Simulation results show good symbol error performance as compared to existing algorithms with asynchronous MIMO interferers.

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I. Introduction

Multiple-input multiple-output (MIMO) wireless systems promise high capacity, diversity, and array gains [1]–[3]. MIMO communication, however, introduces new challenges in receiver design in multi-user systems. For multi-user spatially multiplexed systems, the MIMO receiver needs to suppress coantenna interference (CAI) from different transmit streams, inter-symbol interference (ISI) arising from frequency-selective channels, and cochannel interference (CCI) due to transmission by various users on the same frequency channel. Different spatial-temporal algorithms have been suggested to mitigate interference in [4]–[12]. These range from the high-complexity, optimal multi-user maximum likelihood sequence estimation (MLSE) [6] to the simpler multi-user minimum mean squared error (MMSE) linear receiver, and include a number of decision-feedback equalization (DFE) approaches with performance and complexity between those of the MLSE and MMSE algorithms. Other low complexity detection algorithms are based on prefiltering to shorten the effective channel memory (see, e.g., [13]–[19]). MIMO equalizers for channel shortening in single-user scenarios have been investigated recently in [20] and [21].

In this paper, we propose a low-complexity two-stage receiver structure for interference suppression in MU-MIMO systems in frequency-selective channels. The framework is suitable for mitigating CAI/CCI, including interference from *asynchronous* users, and equalizing ISI. The first stage uses a space-time filter for CAI/CCI cancellation *and* shortening of the equalized effective channels, without equalizing the ISI. The channel shortening, combined with independent detection of the transmit streams, enables the use of reduced complexity Viterbi detectors in the ensuing stage for ISI equalization of each transmit stream. Instead of using an estimate of the channel to design the equalizer in the first stage, a direct training data-based approach is used to jointly optimize the equalizer coefficients and the shortened effective channels, based on a number of proposed objective functions.

The main contributions of our paper include a formulation that eliminates the need for the interferers' training data or channel knowledge and proposed algorithms that are robust to asynchronicity of the cochannel interferers, which arises due to differences in the propagation delays of different users. Many of the previously proposed low complexity approaches [8]–[12], [20], [21] require the interferers' channel information, which inhibits scalability of the MIMO system with the number of transmit and/or receive antennas. In addition, they need to estimate the desired user's channel for the equalizer design and this channel estimation may cause error propagation in the design. The proposed framework is similar to that in [7], but extends the MU-SIMO work to MU-MIMO scenarios and addresses the problem of asynchronicity of the interferers. Moreover, the complexity of the proposed receiver is reduced due to channel shortening. Assuming that the receiver is interested in the detection of one desired user, the complexity of the optimal MLSE receiver is on the order of $M^{M_t(P+1)}$, where M is the constellation size, M_t is the number of transmit antennas of the desired user, and $P+1$ is the channel impulse response length. The complexity of the approach in [7] is linear in M_t and is on the order of $M_t M^{L+1}$, where $L+1$ is the length of the effective channel response after the first-stage of equalization. Note that the effective channel length $L+1$ is longer than the original channel length $P+1$ due to equalization. In certain scenarios, as we shall see in Section VII, $L+1$ may be larger than $M_t(P+1)$, in which case, the complexity of the sub-optimal two-stage approach would exceed that of the optimal MLSE receiver. The proposed receiver has an overall complexity on the order of $M_t M^{\nu+1}$, where $\nu+1$ is the length of the shortened equalized channel. Note that $\nu+1$ is necessarily less than $L+1$, the original length of the equalized channel, and thus the complexity of the proposed receiver is less than that of [7]. Also, by choosing $\nu+1$ carefully, we can ensure that the complexity of our approach is always significantly less than that of the MLSE receiver.

At this point, it is worth noting that the proposed algorithms are for uncoded single carrier systems and that a significant amount of parallel work has been done for coded multicarrier systems (see, e.g., [22], [23]). Our methods are compared with an algorithm proposed in [22] with asynchronous interferers. Through simulation results, we show that our methods have a lower symbol error rate when the difference in propagation delay is large.

The paper is organized as follows. The data and channel models are developed in Section II. They are extended to include the proposed receiver in Section III. The problem is formulated in Section IV and solutions are presented in Section V. This is followed by comparison of the algorithms in terms of computational complexity in Section VI and symbol error performance in Section VII. The key results are summarized and the paper is concluded in Section VIII.

II. MIMO SYSTEM MODEL

In this section, a discrete-time complex baseband model is developed for a MIMO multiple access channel with one desired user and I interferers. Assume that the receiver has M_r receive antennas, the desired user has M_t transmit antennas, and the ith interferer has $M_{t,i}$ transmit antennas. The channel between each transmit antenna of the desired user and each receive antenna includes the pulse-shaping filter, the propagation channel, and matched receive filter, and is assumed to be slow fading and frequency-selective with $P+1$ taps (or a *memory* of P samples). Further, the channel is assumed to be quasi-static, i.e. constant over one frame of symbols, but varying between different frames. The system model is first developed for the desired user and then extended to include the I interferers.

With spatial multiplexing [3], let $x_{m_t}(k)$ be the data symbol transmitted from the m_tth antenna (of the desired user) at the kth instant, $h_{m_r,m_t}(p)$ denote the pth tap of the channel between the m_tth transmit antenna and the m_rth receive antenna, and $n_{m_r}(k)$ be the additive white Gaussian noise (AWGN) at the m_rth receive antenna. Assuming perfect synchronization and sampling, the sample of the received signal at the m_rth receive antenna at time instant k is[1]

$$r_{m_r}(k) = \sum_{m_t=1}^{M_t} \sum_{p=0}^{P} h_{m_r,m_t}(p) x_{m_t}(k-p) + n_{m_r}(k). \tag{1}$$

The symbol duration and the symbol power (or energy), i.e $\mathcal{E}[x_{m_t}(k)x^*_{m_t}(k)]$ are assumed to be unity for simplicity. The noise is uncorrelated with the input symbols and independently and identically distributed (i.i.d.) as $\mathcal{CN}(0, \sigma_n^2)$.

To derive a training based equalization algorithm, a stacked vector notation is developed to account for an equalizer memory of G. Let $\mathbf{x}(k) = [\mathbf{x}_1^T(k)\ \mathbf{x}_2^T(k)\ \cdots\ \mathbf{x}_{M_t}^T(k)]^T$ be the kth instant transmit vector with $\mathbf{x}_{m_t}(k) = [x_{m_t}(k)\ x_{m_t}(k-1)\ \cdots\ x_{m_t}(k-P-G)]^T$ being formed by stacking the $P+G$ previously transmitted data symbols from the m_tth transmit antenna below $x_{m_t}(k)$ for $m_t = 1, \ldots, M_t$. Let the samples received at the m_rth antenna over a block of $G+1$ symbol periods be stacked together to form $\mathbf{r}_{m_r}(k) = [r_{m_r}(k)\ r_{m_r}(k-1)\ \cdots\ r_{m_r}(k-G)]^T$. Further, to enable simultaneous processing of a frame of S symbols over a quasi-static channel, $S-1$ additional snapshots of the input vector $\mathbf{x}(k)$ are included to form the input matrix, $\mathbf{X} = [\mathbf{x}(k)\ \mathbf{x}(k+1)\ \cdots\ \mathbf{x}(k+S-1)]$. The corresponding output matrix can be expressed as $\mathbf{R} = [\mathbf{r}(k)\ \mathbf{r}(k+1)\ \cdots\ \mathbf{r}(k+S-1)]$, where $\mathbf{r}(k)$ is formed by stacking the receive vectors for

[1]In this paper, $(\cdot)^T$ denotes the transpose of a vector or matrix, $(\cdot)^H$ denotes the conjugate transpose of a vector or matrix, $\|(\cdot)\|_2$ is the vector 2-norm of a vector, $\mathcal{E}[\cdot]$ is the expectation operator, $\mathcal{CN}(0, a)$ is the complex Gaussian distribution with i.i.d. real and imaginary parts, each distributed according to the normal distribution with zero mean and variance $a/2$.

the M_r receive antennas to obtain $\mathbf{r}(k) = [\mathbf{r}_1^T(k)\ \mathbf{r}_2^T(k)\ \cdots\ \mathbf{r}_{M_r}^T(k)]^T$. The noise matrix $\mathbf{N}$ is constructed in the same way as $\mathbf{R}$. The channel matrix $\mathbf{H}$ is a block Toeplitz matrix, given by

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1\ \mathbf{0}\ \cdots\ \mathbf{0}\ \mathbf{H}_2\ \mathbf{0}\ \cdots\ \mathbf{0}\ \cdots\ \mathbf{H}_{M_t}\ \mathbf{0}\ \cdots\ \mathbf{0} \\ \mathbf{0}\ \mathbf{H}_1\ \mathbf{0}\ \cdots\ \mathbf{0}\ \mathbf{H}_2\ \mathbf{0}\ \cdots\ \mathbf{0}\ \cdots\ \mathbf{H}_{M_t}\ \mathbf{0}\ \cdots \\ \vdots \qquad \ddots \qquad \vdots \\ \mathbf{0}\ \cdots\ \mathbf{0}\ \mathbf{H}_1\ \mathbf{0}\ \cdots\ \mathbf{0}\ \mathbf{H}_2\ \mathbf{0}\ \cdots\ \mathbf{0}\ \cdots\ \mathbf{H}_{M_t} \end{bmatrix}$$

where $\mathbf{0}$ is a column vector of zeros of length M_r, and for $m_t = 1, \ldots, M_t$, the matrix $\mathbf{H}_{m_t} = [\mathbf{h}_{1,m_t}\ \mathbf{h}_{2,m_t}\ \cdots\ \mathbf{h}_{M_r,m_t}]^T$, with $\mathbf{h}_{m_r,m_t}$ being given by $[h_{m_r,m_t}(0)\ h_{m_r,m_t}(1)\ \cdots\ h_{m_r,m_t}(P)]^T$. Thus, the system equation with one user can be written as

$$\mathbf{R} = \mathbf{H}\mathbf{X} + \mathbf{N}. \tag{2}$$

The channel and transmit streams of the I interferers can be stacked similarly using the preceding steps to create $\mathbf{H}^{(i)}$ and $\mathbf{X}^{(i)}$ respectively, for $i = 1, \ldots, I$. The interfering signals are added to the desired user's signal at the receiver to give the complete input-output relation,

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \sum_{i=1}^{I} \mathbf{H}^{(i)}\mathbf{X}^{(i)} + \mathbf{N}. \tag{3}$$

III. Space-time Equalization Framework

In this section, the system model in Section II is extended to enable the design of the two-stage receiver structure shown in Fig. 1. The receiver is used for independent detection of the desired user's transmit streams, wherein each transmit stream is detected independently of other streams by treating them as interference. In the first stage, a space-time equalizer, $\mathbf{W}$, performs CAI/CCI cancellation *and* shortens the effective channels for the different spatially multiplexed transmit streams. In the second stage, a single channel Viterbi equalizer is used for the detection of each transmit stream independently of other streams by utilizing the shortened effective channel and equalized output for the specific transmit stream. The space-time equalizer is designed directly using a training-based approach as opposed to conventional methods that estimate the channel first.

Let $\mathbf{X}$ in (3) be the input matrix corresponding to the training sequences transmitted on the M_t transmit antennas of the desired source. Let $\mathbf{Y}$ in (3) be the associated output of this training data obtained at the receive antenna array. The length of the training sequence is chosen so as to ensure that $\mathbf{X}_{m_t}\mathbf{X}_{m_t}^H$ has full rank, where $\mathbf{X}_{m_t} = [\mathbf{x}_{m_t}(k)\ \mathbf{x}_{m_t}(k+1)\ \cdots\ \mathbf{x}_{m_t}(k+S-1)]$ is the sub-matrix corresponding to the vth transmit stream. This is the common persistence of excitation assumption. A necessary condition is that $S \geq (P+G+1)$. This is apparent from the dimensions of $\mathbf{X}$ and $\mathbf{Y}$ (see Table I for a summary of the dimensions of the important matrices in this paper). The reason for requiring $\mathbf{X}_v\mathbf{X}_v^H$ to be full rank will become clear in Section V. The receiver is assumed to be ignorant of the training on the I cochannel interferers' channels.

The channel output is equalized with a space-time equalizer of $G+1$ taps, denoted by $\mathbf{W} = [\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_{M_t}]$. Each $\mathbf{w}_u$, $u = 1, \ldots, M_t$, is a column vector of size $M_r(G+1)$. The system equation after equalization can be written as

$$\mathbf{W}^H\mathbf{Y} = \widetilde{\mathbf{Z}}^H\mathbf{X} + \sum_{i=1}^{I} \mathbf{W}^H\mathbf{H}^{(i)}\mathbf{X}^{(i)} + \mathbf{W}^H\mathbf{N} \tag{4}$$

where $\widetilde{\mathbf{Z}}$ is the effective channel matrix for the desired source after equalization. Let the column vector $\widetilde{\mathbf{z}}_{v,u}$ represent the effective channel vector for the sequence transmitted from the vth antenna when $\mathbf{w}_u$ is used to equalize the uth transmit sequence. If the uth transmit sequence is being detected (by equalizing with $\mathbf{w}_u$), the effective channel, $\widetilde{\mathbf{z}}_{u,u}$, is of interest. The other channels, $\widetilde{\mathbf{z}}_{v,u}$ for $v \neq u$, represent the effective channels for the co-antenna interferers, and we refer to them as the *cross-channels*.

To allow for channel shortening for reducing the complexity of the Viterbi equalizer, $(\widetilde{\mathbf{z}}_{u,u})^H$ is forced to $[\mathbf{0}_{1\times\delta}\ \mathbf{z}_{u,u}^H\ \mathbf{0}_{1\times(P+G-\nu-\delta)}]$, where $\mathbf{z}_{u,u}$ is the shortened channel of length $\nu+1$, for the uth transmit stream and δ is a parameter in the range $0 \leq \delta \leq P+G-\nu$ that represents the delay of the shortened channel. Note that if the channel $\mathbf{H}$ is known at the receiver, one would only have to design $\mathbf{W}$ and the shortened effective channels would fall out from the convolution of the equalizer with the channel, i.e., from $\mathbf{W}^H\mathbf{H}$. In our case, a joint optimization is performed to obtain both $\mathbf{W}$ *and* $\widetilde{\mathbf{Z}}$ by minimizing certain objective functions subject to different constraints as shown in the following section.

IV. Problem Formulation

As the detection strategy used in this paper is independent detection, the optimization problem can be formulated separately for each transmit stream, with constraints placed on $\mathbf{w}_u$ and $\widetilde{\mathbf{z}}_{u,u}$, instead of joint constraints on $\mathbf{W}$ and $\widetilde{\mathbf{Z}}$. The basis for the formulation of the cost functions presented is minimization of the energy in the cross-channels and maximization of the energy in the desired effective channel.

A. Optimization Problem I (SAINR-$\mathbf{z}$)

The first optimization problem is to maximize a signal-to-interference-plus-noise ratio (SINR)-like quantity and can be written as

$$\max_{\mathbf{w}_u,\widetilde{\mathbf{z}}_{u,u}} \frac{||\widetilde{\mathbf{z}}_{u,u}^H \mathbf{X}_u||_2^2}{||\mathbf{w}_u^H \mathbf{Y} - \widetilde{\mathbf{z}}_{u,u}^H \mathbf{X}_u||_2^2}, \text{ subject to } ||\widetilde{\mathbf{z}}_{u,u}||_2^2 = 1. \tag{5}$$

The constraint on $||\widetilde{\mathbf{z}}_{u,u}||_2^2$ is to prevent a zero vector solution. Recall that $\mathbf{X}_u$ is the sub-matrix associated with the uth transmit stream. As each $\widetilde{\mathbf{z}}_{u,u}$ has the shortened channel $\mathbf{z}_{u,u}$ embedded in it, with δ zeros preceding it, $\widetilde{\mathbf{z}}_{u,u}^H \mathbf{X}_u$ is equivalent to $\mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}$ where, $\mathbf{X}_u^{(\delta)} = [\mathbf{X}_u^{(\delta)}(k)\ \mathbf{X}_u^{(\delta)}(k+1)\ \cdots\ \mathbf{X}_u^{(\delta)}(k+S-1)]$ is a sub-matrix corresponding to the $\delta+1, \ldots, \delta+\nu+1$ rows of the training matrix of the uth transmit sequence. The optimization can thus be rewritten as

$$\max_{\mathbf{w}_u,\mathbf{z}_{u,u}} \frac{||\mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}||_2^2}{||\mathbf{w}_u^H \mathbf{Y} - \mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}||_2^2}, \text{ subject to } ||\mathbf{z}_{u,u}||_2^2 = 1. \tag{6}$$

One can observe that the denominator absorbs the CAI, CCI, residual interference due to shortening and AWGN in the difference term between the equalized output and the shortened channel output. This enables joint CAI/CCI cancellation and channel shortening. Note that $\mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}$ is basically the uth transmit stream after equalization, and hence, the numerator is the desired signal after equalization. Henceforth, this SINR-like quantity will be called the signal-to-all-interference-plus-noise ratio (SAINR) as the denominator term includes various sources of interference besides noise.

The objective function maximized by the authors in [7] was the post-equalization SINR, subject to a similar constraint on the effective channel. As mentioned in Section I, they specifically considered SIMO systems. The overall complexity of their Viterbi equalizer was on the order of $M_t M^{L+1}$, where $L+1$ is the equalized channel length, while that of the Viterbi equalizer with channel shortening is on the order of $M_t M^{\nu+1}$.

B. Optimization Problem II (SAINR-$\mathbf{w}$)

The other variable $\mathbf{w}_u$ in the optimization problem can also be constrained to prevent a degenerate solution. For this case, the cost function is modified slightly and the optimization problem is stated as

$$\max_{\mathbf{w}_u, \mathbf{z}_{u,u}} \frac{\|\mathbf{w}_u^H \mathbf{Y}\|_2^2}{\|\mathbf{w}_u^H \mathbf{Y} - \mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}\|_2^2}, \text{ subject to } ||\mathbf{w}_u||_2^2 = 1. \tag{7}$$

The constraint on $||\mathbf{w}_u||_2^2$ is sufficient for avoiding a trivial zero vector solution for $\mathbf{w}_u$. The cost function is similar to that in (6), with the difference being in the numerator. With perfect cancellation, $\mathbf{w}_u^H \mathbf{Y} \approx \mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}$. Hence the numerator of the objective function in (7) corresponds to the desired signal after equalization and the cost function is again the post equalization SAINR.

C. Optimization Problem III (LS-$\mathbf{z}$)

In the final optimization problem, we minimize only the all-interference term in the denominator of (6), instead of maximizing the SAINR. The problem is now a least-squares problem with a quadratic constraint and we can expect a reduction in computational complexity if we use one of the many gradient descent algorithms available for solving such constrained least-squares problems, e.g. recursive least squares (RLS) [24]. This is the motivation for the following formulation

$$\min_{\mathbf{w}_u, \mathbf{z}_{u,u}} \|\mathbf{w}_u^H \mathbf{Y} - \mathbf{z}_{u,u}^H \mathbf{X}_u^{(\delta)}\|_2^2, \text{ subject to } ||\mathbf{z}_{u,u}||_2^2 = 1, \tag{8}$$

where the all-interference-plus-noise term is minimized subject to a constraint on the norm of the effective channel. This optimization problem is demonstrated to be practically equivalent to that in (6) for long enough training sequences in Section VII-A. This suggests that this formulation enables the use of low-complexity algorithms for achieving the same system performance as obtained by solving (6).

V. SOLUTIONS

All the three optimization problems have similar forms and the technique of separation of variables is used to solve them. This is stated in Proposition 1. We drop the index u from $\mathbf{w}_u$, $\mathbf{z}_{u,u}$ and $\mathbf{X}_u^{(\delta)}$ for ease of reading.

Proposition 1 (Separation of variables). *[25] Define*

$$\mathcal{F}(\mathbf{w},\mathbf{z}) = \frac{\|\mathbf{w}^H\mathbf{Y}\|_2^2}{\|\mathbf{w}^H\mathbf{Y} - \mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2}, \tag{9}$$

$$f(\mathbf{w}) = \mathbf{z} = \arg\max_{\mathbf{z}} \mathcal{F}(\mathbf{w},\mathbf{z}), \tag{10}$$

$$\mathcal{M}(\mathbf{w}) = \max_{\mathbf{z}} \mathcal{F}(\mathbf{w},\mathbf{z}) = \frac{\|\mathbf{w}^H\mathbf{Y}\|_2^2}{\|\mathbf{w}^H\mathbf{Y} - f(\mathbf{w})^H\mathbf{X}^{(\delta)}\|_2^2}. \tag{11}$$

1) *If $\widehat{\mathbf{w}} = \arg\max_{\mathbf{w}} \mathcal{M}(\mathbf{w})$, and $\widehat{\mathbf{z}} = f(\widehat{\mathbf{w}})$, then $(\widehat{\mathbf{w}},\widehat{\mathbf{z}})$ is the global maximizer for $\mathcal{F}(\mathbf{w},\mathbf{z})$, and $\mathcal{F}(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \mathcal{M}(\widehat{\mathbf{w}})$.*
2) *If $(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \arg\max_{\mathbf{w},\mathbf{z}} \mathcal{F}(\mathbf{w},\mathbf{z})$, then $\widehat{\mathbf{w}}$ is the global maximizer for $\mathcal{M}(\mathbf{w})$, and $\mathcal{F}(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \mathcal{M}(\widehat{\mathbf{w}})$.*

It follows from Proposition 1 that if we define

$$\mathcal{F}(\mathbf{w},\mathbf{z}) = \|\mathbf{w}^H\mathbf{Y} - \mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2, \tag{12}$$

$$f(\mathbf{z}) = \mathbf{w} = \arg\min_{\mathbf{w}} \mathcal{F}(\mathbf{w},\mathbf{z}), \tag{13}$$

$$\mathcal{M}(\mathbf{z}) = \min_{\mathbf{w}} \mathcal{F}(\mathbf{w},\mathbf{z}) = \|f(\mathbf{z})^H\mathbf{Y} - \mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2. \tag{14}$$

then,

1) If $\widehat{\mathbf{z}} = \arg\min_{\mathbf{z}} \mathcal{M}(\mathbf{z})$, and $\widehat{\mathbf{w}} = f(\widehat{\mathbf{z}})$, then $(\widehat{\mathbf{w}},\widehat{\mathbf{z}})$ is the global minimizer for $\mathcal{F}(\mathbf{w},\mathbf{z})$, and $\mathcal{F}(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \mathcal{M}(\widehat{\mathbf{z}})$.
2) If $(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \arg\min_{\mathbf{w},\mathbf{z}} \mathcal{F}(\mathbf{w},\mathbf{z})$, then $\widehat{\mathbf{z}}$ is the global minimizer for $\mathcal{M}(\mathbf{z})$, and $\mathcal{F}(\widehat{\mathbf{w}},\widehat{\mathbf{z}}) = \mathcal{M}(\widehat{\mathbf{z}})$.

Note that the variables $\mathbf{w}$ and $\mathbf{z}$ are not independent of each other. However, the above results enable us to solve our joint optimization problems by solving for one variable first, conditioned on the other. This is followed by substituting the solution back into the cost function and solving for the other variable. The approach is similar to that followed in [7].

A. Solution for Optimization Problem I (SAINR-z)

We first need to find $f(\mathbf{z}) = \mathbf{w} = \arg\max_{\mathbf{w}} \mathcal{F}(\mathbf{w},\mathbf{z})$. This is the same as solving for $f(\mathbf{z}) = \arg\min_{\mathbf{w}} \|\mathbf{w}^H\mathbf{Y} - \mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2$. Differentiating this modified cost function w.r.t. $\mathbf{w}$ and setting the result to zero, we obtain

$$\mathbf{w}^H = \mathbf{z}^H\mathbf{X}^{(\delta)}\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}. \tag{15}$$

Substituting in the optimization problem in (6), we have

$$\mathbf{z}_{opt} = \arg\max_{\mathbf{z}} \frac{\|\mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2}{\|\mathbf{z}^H\mathbf{X}^{(\delta)}\mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y} - \mathbf{z}^H\mathbf{X}^{(\delta)}\|_2^2}, \text{ subject to } ||\mathbf{z}||_2^2 = 1, \tag{16}$$

$$= \arg\max_{\mathbf{z}} \frac{\mathbf{z}^H\mathbf{X}^{(\delta)}\mathbf{X}^{(\delta)H}\mathbf{z}}{\mathbf{z}^H\mathbf{X}^{(\delta)}\mathbf{P}_{\mathbf{Y}}\mathbf{X}^{(\delta)H}\mathbf{z}}, \text{ subject to } ||\mathbf{z}||_2^2 = 1 \tag{17}$$

where $\mathbf{P}_{\mathbf{Y}} = \mathbf{I} - \mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y}$ is a projection matrix. This is a generalized eigenvalue problem and the solution, $\mathbf{z}_{opt}$, is given by the generalized eigenvector corresponding to the largest generalized eigenvalue[2] of $\mathbf{X}^{(\delta)}\mathbf{X}^{(\delta)H}$ and $\mathbf{X}^{(\delta)}\mathbf{P}_{\mathbf{Y}}\mathbf{X}^{(\delta)H}$. The optimum equalizer, $\mathbf{w}_{opt}$, can be obtained by substituting $\mathbf{z}_{opt}$ in (15).

The optimum value of the δ parameter of the shortened channel needs to be found as well. This is done by evaluating (6) for all possible values of δ and choosing the δ that results in the maximum value. Note that the optimal δ can be found using faster techniques [15], [26]. The same algorithm is repeated for all transmit sequences. Each transmit sequence is detected by a single-channel Viterbi equalizer in the second stage of the receiver, which uses the shortened effective channel $\mathbf{z}$ and equalized output $\mathbf{w}$ for each transmit stream to suppress the ISI.

B. Solution for Optimization Problem II (SAINR-$\mathbf{w}$)

Using the same steps as in Section V-A, we have

$$\mathbf{z}^H = \mathbf{w}^H\mathbf{Y}\mathbf{X}^{(\delta)H}(\mathbf{X}^{(\delta)}\mathbf{X}^{(\delta)H})^{-1}, \tag{18}$$

and,

$$\mathbf{w}_{opt} = \arg\max_{\mathbf{w}} \frac{\mathbf{w}^H\mathbf{Y}\mathbf{Y}^H\mathbf{w}}{\mathbf{w}^H\mathbf{Y}\mathbf{P}_{\mathbf{X}}\mathbf{Y}^H\mathbf{w}}, \text{ subject to } ||\mathbf{w}||_2^2 = 1 \tag{19}$$

where $\mathbf{P}_{\mathbf{X}} = \mathbf{I} - \mathbf{X}^{(\delta)H}(\mathbf{X}^{(\delta)}\mathbf{X}^{(\delta)H})^{-1}\mathbf{X}^{(\delta)}$ is a projection matrix. The solution, $\mathbf{w}_{opt}$, is given by the generalized eigenvector corresponding to the largest generalized eigenvalue of $\mathbf{Y}\mathbf{Y}^H$ and $\mathbf{Y}\mathbf{P}_{\mathbf{X}}\mathbf{Y}^H$. The optimum effective channel, $\mathbf{z}_{opt}$, is obtained by substituting $\mathbf{w}_{opt}$ in (18). As before, this algorithm along with δ-optimization is repeated for all transmit streams and the detection is completed by single-channel Viterbi equalizers in the following stage.

[2] A low-complexity division-free algorithm for finding the generalized eigenvector corresponding to the largest eigenvalue has been developed in [26], [27].

C. Solution for Optimization Problem III (LS-$\mathbf{z}$)

As mentioned before, (8) is a constrained least-squares optimization problem. We solve for $f(\mathbf{z}) = \arg\min_{\mathbf{w}} \mathcal{F}(\mathbf{w}, \mathbf{z}) = \arg\min_{\mathbf{w}} \|\mathbf{w}^H \mathbf{Y} - \mathbf{z}^H \mathbf{X}^{(\delta)}\|_2^2$. Differentiating w.r.t. $\mathbf{w}$ and setting the result to zero, we obtain the same expression as in (15)

$$\mathbf{w}^H = \mathbf{z}^H \mathbf{X}^{(\delta)} \mathbf{Y}^H (\mathbf{Y}\mathbf{Y}^H)^{-1}. \tag{20}$$

Now (8) can be rewritten as

$$\mathbf{z}_{opt} = \arg\min_{\mathbf{z}} \mathbf{z}^H \mathbf{X}^{(\delta)} \mathbf{P}_{\mathbf{Y}} \mathbf{X}^{(\delta)H} \mathbf{z}, \text{ subject to } ||\mathbf{z}||_2^2 = 1 \tag{21}$$

where $\mathbf{P}_{\mathbf{Y}} = \mathbf{I} - \mathbf{Y}^H(\mathbf{Y}\mathbf{Y}^H)^{-1}\mathbf{Y}$ is a projection matrix. The solution, $\mathbf{z}_{opt}$, is given by the eigenvector corresponding to the smallest eigenvalue of $\mathbf{X}^{(\delta)} \mathbf{P}_{\mathbf{Y}} \mathbf{X}^{(\delta)H}$. After the δ-optimization, the Viterbi equalizer in the ensuing stage uses the shortened effective channel and the equalized transmit stream for the detection of each transmit stream.

VI. Computational Complexity

The complexity of the proposed algorithms can be calculated on the basis of the major computational tasks in each algorithm. These tasks include:

1) Finding $\mathbf{z}_{opt}$ or $\mathbf{w}_{opt}$ as the (generalized) eigenvector corresponding to the largest or smallest (generalized) eigenvalue of symmetric matrices, and calculating the corresponding $\mathbf{w}_{opt}$ or $\mathbf{z}_{opt}$ from (15) or (18). Sub-tasks in these calculations are finding $(\mathbf{Y}\mathbf{Y}^H)^{-1}$ or $(\mathbf{X}^{(\delta)}\mathbf{X}^{(\delta)H})^{-1}$ for $\mathbf{P}_{\mathbf{Y}}$ or $\mathbf{P}_{\mathbf{X}}$ depending on the algorithm, and matrix multiplications[3],
2) Detection using the Viterbi detector.

The order of computational complexity for these two parts can be approximated using [28] (Chapters 3 and 8), and turns out to be the same for all three proposed algorithms. Task 1 requires $\mathcal{O}((\nu+1)^3) + \mathcal{O}(M_r^3(G+1)^3) + \mathcal{O}(M_r(G+1)S)$ computations, and Task 2 requires $\mathcal{O}(M^{\nu+1})$ computations. Recall that M denotes the size of the symbol constellation. As stated earlier, the complexity of the LS-$\mathbf{z}$ method can be reduced by using certain gradient descent algorithms, e.g., constrained RLS, for solving constrained least squares optimization problems.

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[3] Note that there are more efficient ways to perform these calculations which do not require explicit computation of matrix inverses.

VII. SIMULATIONS AND DISCUSSIONS

In this section, various simulation results are presented for demonstrating the performance of the proposed algorithms in terms of symbol error rate (SER) and SAINR. With the exclusion of Section VII-C, a single-user 2×2 MIMO system with QPSK modulation is considered. For the purpose of simulations, the channel is assumed to be 3 taps long (i.e. $P = 2$). Further, the different taps of the channel are assumed to have a constant power delay profile and are i.i.d. with distribution $\mathcal{CN}(0, 1)$. A quasi-static channel model is considered, where the channel is assumed to be constant over a frame of symbols of length 100. The simulation results are obtained by averaging the communications performance over a 10000 channel realizations. LabVIEW 7.0 modules for reproducing these simulation results can be downloaded from the LabVIEW MIMO toolkit available at [29]. In the simulations, $\nu + 1 < M_t(L + 1)$.

A. Probability of Symbol Error

We first look at the probability of symbol error performance of the three methods in the absence of cochannel users. For all simulations in this section, the equalizer memory G is chosen to be 4. The effective channel, which would be 7 taps long without any channel shortening, is shortened to a length 4 channel (i.e. $\nu + 1 = 4$). The number of transmit antennas, M_t, is 2.

Note that the shortened channel length is greater than the actual channel length of 3. At this point it is helpful to remember that we use a linear equalizer before the Viterbi equalizer to obtain a good trade-off of performance and complexity between the MLSE and MMSE receivers as in [7]. The effect of the first-stage equalizer is to lengthen the effective channel for the Viterbi equalizer due to convolution. The channel shortening constraint is imposed to shorten the length of this effective channel. The complexity of the optimal MLSE algorithm with these simulation parameters is on the order of $M^{M_t(P+1)} = 4^6$, the complexity of the two-stage approach in [7] without channel shortening is on the order of $M_t M^{L+1} = 2 \times 4^7$, and that of the proposed approach with channel shortening is on the order of $M_t M^{\nu+1} = 2 \times 4^4$. In this scenario, without channel shortening, the approach in [7] turns out to be more complex than the optimal MLSE.

As can be seen from Fig. 2, all three proposed methods yield similar symbol error performance. The SAINR-$\mathbf{w}$ method has the smallest SER among the three methods, followed by the LS-$\mathbf{z}$ method. One can observe that the LS-$\mathbf{z}$ and the SAINR-$\mathbf{z}$ curves are nearly overlapping. Thus, these two methods are, for all practical purposes, equivalent.

We next compare our methods with the MaxPwr method proposed in [22]. The authors use the channel information of the desired user and that of the asynchronous cochannel interferer for interference suppression in SIMO-OFDM channels using channel shortening ideas. For the purpose of comparison with our algorithms, we use their method in our uncoded single carrier MIMO scenario. A 2×2 MIMO system can be viewed as two SIMO channels with the same receiver. To compare the MaxPwr method with our training-based methods, we simulate the MaxPwr method with perfect channel knowledge and with channel estimation (using least-squared error - LSE estimation). From Fig. 2, it is clear that the MaxPwr method has a lower SER at smaller values of input SNR than our methods. The proposed methods, however, have steeper SER curves and their performance is better at input SNRs greater than 12 dB. The matched filter bound for the same scenario is plotted here for reference due to the long simulation time requirements of the optimum MLSE detector.

B. Variation of Parameters

We now look at the variation in performance of the SAINR-$\mathbf{w}$ method with changes in system parameters like the number of equalizer taps and the shortened channel length. The performance metric chosen for the simulations in this section is the SAINR. The shortened channel length is first set to 3 and the number of equalizer taps is varied from 1 to 7. The SAINR increases steadily with the number of equalizer taps as can be seen from Fig. 3. But, as the equalizer length, $G+1$, is increased beyond 5 taps, the increase is negligible. This is because the residual interference due to shortening increases and affects the SAINR.

In Fig. 4, $G+1$ is fixed at 5 and the shortened channel length is varied from 1 to 7, where $\nu+1 = 1$ corresponds to a single tap effective channel and $\nu+1 = 7$ corresponds to the effective channel without shortening. We observe that as the effective channel is shortened, the SAINR reduces. The degradation in SAINR is greater for $\nu + 1 \leq 2$.

Thus an optimum balance needs to be struck between the improvement in receiver performance due to increase in the number of equalizer taps (which adds to the receiver complexity) and the degradation in performance due to excessive shortening of the effective channel length.

C. CCI Suppression

The methods presented in this paper are flexible enough to perform CCI suppression with asynchronous MIMO interferers without requiring their channel information. Their performance is demonstrated in this section in the presence of one cochannel interferer. All channels are modeled as Rayleigh fading channels of length 2. The equalizer is chosen to have 5 taps and the desired user's effective channels are shortened to length 4 channels. For the simulations of the MaxPwr method, an LSE estimate of the interferer's channel is used.

In Fig. 5 and 6, the SAINR-$\mathbf{w}$ method is compared with the MaxPwr method in the presence of one asynchronous cochannel interferer. In this simulation set up, each user has a single transmit antenna and the common receiver has 2 receive antennas. The carrier to interference ratio (CIR), which is the ratio of the transmit signal power of the desired user to that of the undesired interferer, is chosen to be 3 dB. Fig. 5 plots the SER with the difference in propagation delay between the two users. It can be seen that the SER performance of the SAINR-$\mathbf{w}$ method does not change much with the propagation delay, while the MaxPwr method suffers in SER as the delay increases. This is further illustrated in Fig. 6, where the difference in propagation delay between the two users is chosen to be 30 taps. As evident, the symbol error rate achieved by the SAINR-$\mathbf{w}$ method is significantly lower than that obtained by the MaxPwr method. These simulations demonstrate the flexibility of our formulation to deal with asynchronous interferers.

It is important to note that our algorithms are resilient to asynchronicity of CCI due to the fact that they do not require any information about the interferer's channel. To ensure a fair comparison with our algorithm, if the receiver is assumed to lack knowledge of the interferer's training data and thereby its channel, the MaxPwr method would fail to work.

VIII. CONCLUSION

In this paper, a two-stage receiver structure for CAI/CCI cancellation and ISI equalization in frequency-selective MU-MIMO channels was presented. The novelty of the receiver design lies in combining CAI/CCI cancellation with channel shortening to help reduce the receiver complexity. Unlike many previously proposed receivers, this receiver does not require the channel information of cochannel interferers and this enables it to deal with asynchronous users. Simulation results show good symbol error performance for single-user 2×2 MIMO systems with all three methods and reasonable cochannel rejection in a two-user scenario with asynchronous users. Performance can possibly be improved by considering the coloring of the noise due to the space-time equalizer, and incorporating noise whitening constraints in the filter optimization [30]. This approach can be extended to the multicarrier case (MIMO-OFDM) as proposed in [23].

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TABLE I

TABLE OF DIMENSIONS OF IMPORTANT MATRICES, WITH M_t TRANSMIT ANTENNAS, M_r RECEIVE ANTENNAS, P CHANNEL MEMORY, G EQUALIZER MEMORY, AND S SYMBOLS IN A FRAME.

Matrix	Name	Dimensions
$\mathbf{X}$	Input matrix	$M_t(P+G+1) \times S$
$\mathbf{H}$	Channel matrix	$M_r(G+1) \times M_t(P+G+1)$
$\mathbf{Y}$	Output matrix	$M_r(G+1) \times S$
$\mathbf{N}$	Noise matrix	$M_r(G+1) \times S$
$\mathbf{W}$	Equalizer matrix	$M_r(G+1) \times M_t$
$\tilde{\mathbf{Z}}$	Effective channel matrix	$M_t(P+G+1) \times M_t$

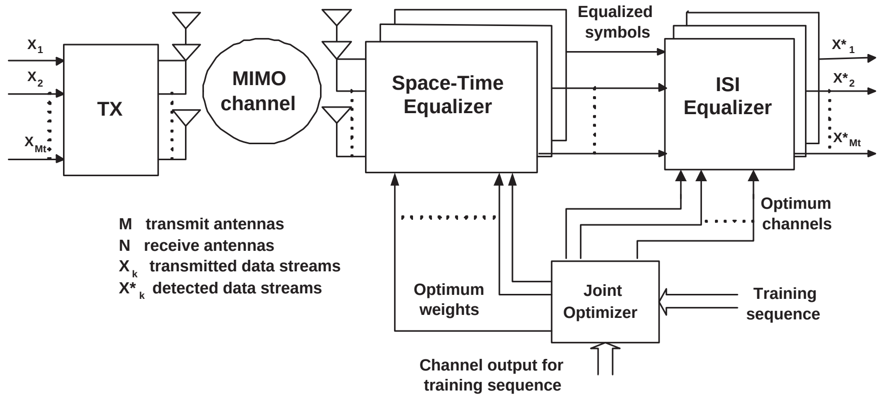

Fig. 1. Schematic for the two-stage receive structure

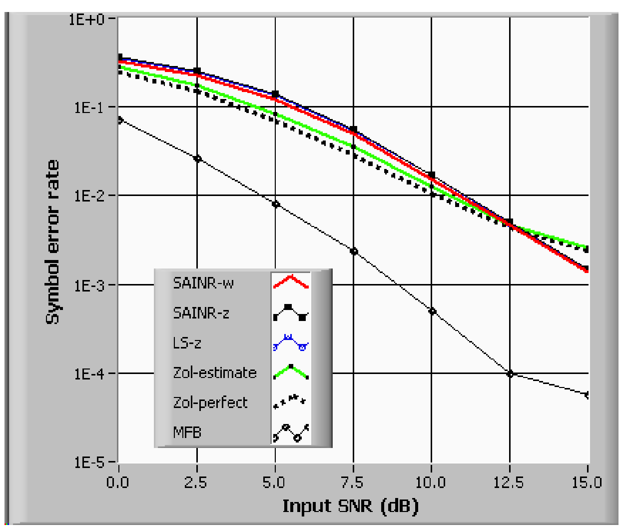

Fig. 2. Comparison of probability of symbol error performance of the three proposed methods with the MaxPwr method, with and without channel estimation

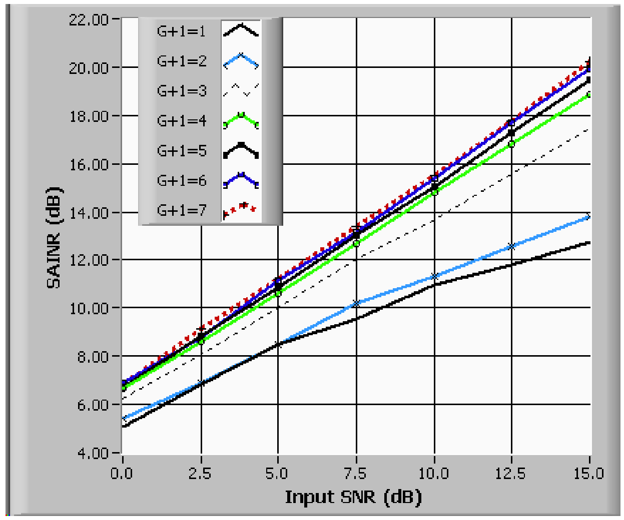

Fig. 3. Variation of SAINR for the SAINR-$\mathbf{w}$ method with change in the number of equalizer taps, $G+1$.

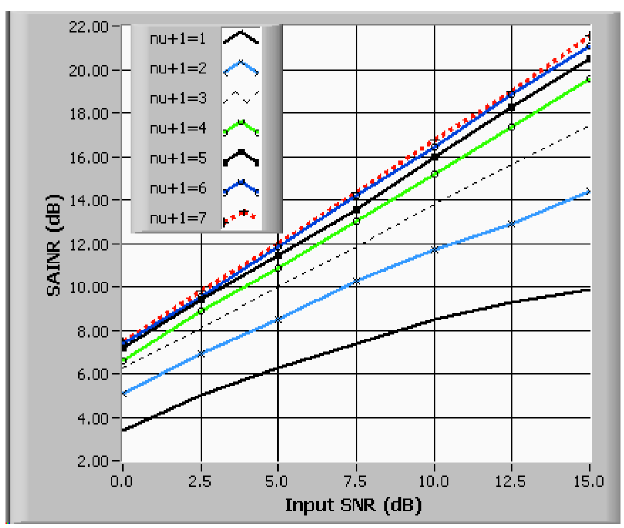

Fig. 4. Variation of SAINR for the SAINR-$\mathbf{w}$ method with change in the length, $\nu+1$, of the shortened channel.

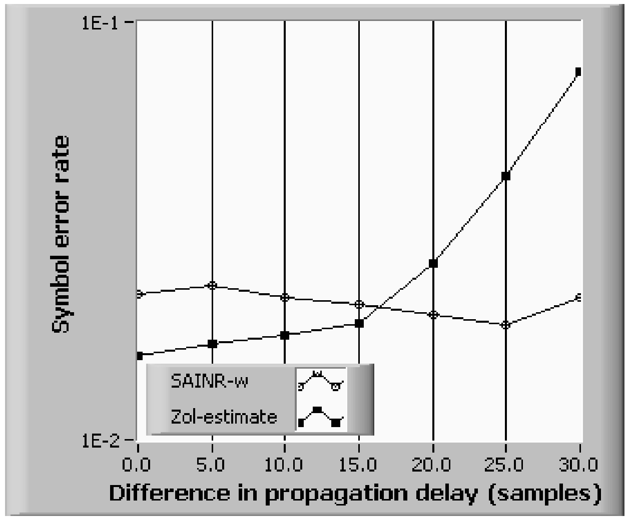

Fig. 5. Comparison of probability of symbol error performance of the desired user with the SAINR-**w** and the MaxPwr methods in the presence of one asynchronous cochannel interferer (with a carrier-to-interference ratio of 3 dB).

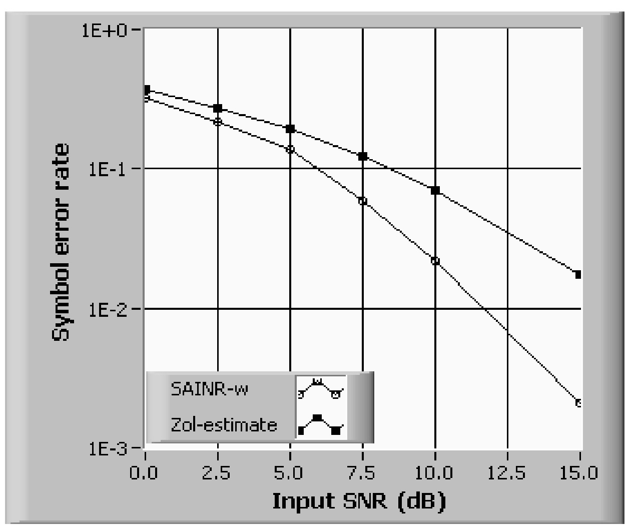

Fig. 6. Comparison of probability of symbol error performance of the desired user with the SAINR-**w** and the MaxPwr methods in the presence of one asynchronous cochannel interferer (with a carrier-to-interference ratio of 3 dB and a propagation delay of 30 samples).