Optimal Downlink OFDMA Subcarrier, Rate, and Power Allocation with Linear Complexity to Maximize Ergodic Weighted-Sum Rates

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Abstract— **In this paper, we propose a resource allocation algorithm for ergodic weighted-sum rate maximization in downlink OFDMA systems. In contrast to most previous research that focused on maximizing instantaneous rates using deterministic optimization techniques, we focus on maximizing ergodic rates using stochastic optimization techniques, which allow us to exploit the temporal dimension, in addition to the frequency and multiuser dimensions. Furthermore, in contrast to most previous algorithms that used greedy suboptimal heuristics with quadratic complexity, we use a dual optimization approach that resulted in a simple subcarrier, rate, and power allocation algorithm that has complexity $O(MK)$ for an M-user, K-subcarrier OFDMA system. Surprisingly, our method is shown to result in duality gaps less than 10^{-4} in scenarios of practical interest, thereby allowing us to claim practical optimality. We present simulation results for a 3GPP-LTE system employing adaptive modulation.**

I. Introduction

Next-generation broadband wireless system standards, e.g. 3GPP-Long Term Evolution (LTE) [1], consider Orthogonal Frequency Division Multiple Access (OFDMA) as the preferred physical layer multiple access scheme, esp. for the downlink. The problem of assigning the subcarriers, rates, and powers to the different users in an OFDMA system has been an area of active research, (see e.g. [2] [3] [4]). In most of the previous work, the formulation and algorithms only consider instantaneous performance metrics. Thus, the temporal dimension is not being exploited when the resource allocation is performed. Instead of considering only instantaneous data rate, we formulate the problem considering user-weighted *ergodic* sum rate. This allows us to exploit all three degrees of freedom in our system, namely time, frequency, and multiuser dimensions. At the same time, we can enforce various notions of fairness through the user weights.

Furthermore, previous research have assumed that algorithms to find the optimal or near-optimal solution to the problem is too computationally complex for real-time implementation. Hence, the main focus of previous research efforts have been on developing heuristic approaches with typical complexities in the order of $\mathcal{O}(MK^2)$. Our approach, on the other hand, is based on a dual optimization framework, which is less complex ($\mathcal{O}(MK)$ per iteration, with less than 10 iterations) and achieves relative optimality gaps that are less than 10^{-4} (i.e. achieving 99.9999% of the optimal solution) in typical scenarios, and thus actually allowing us to claim *practical optimality*. We focus on the discrete rate case in this paper. We also investigated the continuous rate case in [2]. Note that the dual optimization approach was also studied in [3] [4] [5], but their focus has been on instantaneous and continuous rate optimization.

II. System Model

We consider a single OFDMA base station with K-subcarriers and M-users indexed by the set $\mathcal{K} = \{1, \ldots, k, \ldots, K\}$ and $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$ (typically $K \gg M$) respectively. We assume an average transmit power of $\bar{P} > 0$, bandwidth B, and noise density N_0. The received signal vector for the mth user at the nth OFDM symbol is given as

$$\boldsymbol{y}_m[n] = \mathbf{G}_m[n]\mathbf{H}_m[n]\boldsymbol{x}_m[n] + \boldsymbol{w}_m[n] \qquad (1)$$

where $\boldsymbol{y}_m[n]$ and $\boldsymbol{x}_m[n]$ are the K-length received and transmitted complex-valued signal vectors; $\mathbf{G}_m[n]$ is the diagonal gain allocation matrix with diagonal elements $[\mathbf{G}_m[n]]_{kk} = \sqrt{p_{m,k}[n]}$; $\boldsymbol{w}_m[n] \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \boldsymbol{I}_K)$ with noise variance $\sigma_w^2 = N_0 B/K$ is the white zero-mean, circular-symmetric, complex Gaussian (ZMCSCG) noise vector; and $\mathbf{H}_m[n] = \text{diag}\{h_{m,1}[n], \ldots, h_{m,K}[n]\}$ is the diagonal channel response matrix, where

$$h_{m,k}[n] = \sum_{i=1}^{N_t} g_{m,i}[n] e^{-j2\pi\tau_i k\Delta f}. \quad (2)$$

are the complex-valued frequency-domain wireless channel fading random processes, given as the discrete-time Fourier transform of the N_t time-domain multipath taps $g_{m,i}[n]$ with time-delay τ_i and subcarrier spacing Δf. These taps are modeled as stationary and ergodic discrete-time random processes with tap powers $\sigma^2_{m,i}$, which we assume to be independent across the fading paths i and across users m. Since $g_{m,i}[n]$ is stationary and ergodic, so is $h_{m,k}[n]$. Hence, the distribution of $\boldsymbol{h}_m[n]$ is independent of n through stationarity, and we can replace time averages with ensemble averages in the problem formulations through ergodicity. In the subsequent discussion, we shall drop the index n when the context is clear for notational brevity.

We assume[1] that the time domain channel taps are independent ZMCSCG random variables $g_{m,i} \sim \mathcal{CN}(0, \sigma^2_{m,i})$ with total power $\sigma^2_m = \sum_{i=1}^{N_t} \sigma^2_{m,i}$. Then from (2), we have

$$\begin{aligned} \boldsymbol{h}_m &\sim \mathcal{CN}(\mathbf{0}_K, \mathbf{R}_{\boldsymbol{h}_m}) \\ \mathbf{R}_{\boldsymbol{h}_m} &= \mathbf{W}\boldsymbol{\Sigma}_m\mathbf{W}^H \end{aligned} \quad (3)$$

where $\mathbf{W}$ is the $K \times N_t$ DFT matrix with $[\mathbf{W}]_{k,i} = e^{-j2\pi\tau_i k\Delta f}$ and $\boldsymbol{\Sigma}_m = \mathrm{diag}\{\sigma^2_{m,1}, \ldots, \sigma^2_{m,N_t}\}$ is an $N_t \times N_t$ diagonal matrix of the time-domain path powers. Since we also assume that the fading for each user is independent, then the joint distribution of the stacked fading vector for all users $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_M^T]^T$ is likewise a ZMCSCG random vector with distribution $\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}_{KM}, \mathbf{R}_{\boldsymbol{h}})$ where $\mathbf{R}_{\boldsymbol{h}}$ is the $KM \times KM$ block diagonal covariance matrix with $\mathbf{R}_{\boldsymbol{h}_m}$ as the diagonal block elements. This is the distribution over which we shall take the weighted sum rate function in the problem formulations. We let $\boldsymbol{\gamma}_m = [\gamma_{m,1}, \ldots, \gamma_{m,k}]^T$ where $\gamma_{m,k} = |h_{m,k}|^2/\sigma^2_w$ denote the instantaneous channel-to-noise ratio (CNR) with mean $\bar{\gamma}_{m,k} = \sigma^2_m/\sigma^2_w$. Note that $\gamma_{m,k}$ for a particular subcarrier k and different users m are independent but not necessarily identically distributed (INID) exponential random variables; and for a particular user m and different subcarriers k are not independent but identically distributed (NIID) exponential random variables. Throughout the paper, we assume that the transmitter has perfect knowledge of $\boldsymbol{\gamma}_m$ for all users, and that the resource allocation decisions are made known to the users through an error-free control channel.

[1] Although the results of this paper are applicable to any fading distribution, we shall prescribe a particular distribution for the fading channels for illustration purposes.

III. Ergodic Rate Maximization in OFDMA

A. Problem Formulation

The data rate of the kth subcarrier for the mth user can be given by the staircase function

$$R(p_{m,k}\gamma_{m,k}) = \begin{cases} r_0, & \eta_0 \le p_{m,k}\gamma_{m,k} < \eta_1 \\ \vdots, & \vdots \\ r_{L-1}, & \eta_{L-1} \le p_{m,k}\gamma_{m,k} < \eta_L \end{cases} \quad (4)$$

where $\{\eta_l\}_{l\in\mathcal{L}}$, $\mathcal{L} = \{0, \ldots, L-1\}$, are the SNR boundaries which define a particular code-rate and modulation order pair combination that result in r_l data bits per transmission with a predefined target bit error rate (BER), and where $r_l \ge 0$, $r_{l+1} > r_l$, $r_0 = 0$, $\eta_0 = 0$, and $\eta_L = \infty$. Denote by $\boldsymbol{p} = [\boldsymbol{p}_1^T, \cdots, \boldsymbol{p}_K^T]^T$ the vector of powers to be determined, where $\boldsymbol{p}_k = [p_{1,k}, \cdots, p_{M,k}]^T$. Note that determining the power vector consequently determines the subcarrier allocation (zero power means the subcarrier is not allocated) and rate allocation (by (4)). The exclusive subcarrier assignment restriction in OFDMA can then be written as $\boldsymbol{p}_k \in \boldsymbol{\mathcal{P}}_k \subset \mathbb{R}^M_+$,

$$\boldsymbol{\mathcal{P}}_k \equiv \{\boldsymbol{p}_k \in \mathbb{R}^M_+ | p_{m,k}p_{m',k} = 0; \forall m \ne m'\} \quad (5)$$

For notational convenience, we let $\boldsymbol{p} \in \boldsymbol{\mathcal{P}} \equiv \boldsymbol{\mathcal{P}}_1 \times \cdots \times \boldsymbol{\mathcal{P}}_K \subset \mathbb{R}^{MK}_+$ denote the space of allowable power vectors for all subcarriers. Since we assumed perfect CSI, we can consider the power allocation vector $\boldsymbol{p}$ as a function of the realization of the fading CNR $\boldsymbol{\gamma} = [\boldsymbol{\gamma}_1^T, \ldots, \boldsymbol{\gamma}_M^T]^T$.

The ergodic discrete weighted sum rate maximization can then be formulated as

$$\begin{aligned} f^* = \max_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\left\{\sum_{m\in\mathcal{M}} w_m \sum_{k\in\mathcal{K}} R(p_{m,k}\gamma_{m,k})\right\} \\ \text{s.t.} \quad & \mathbb{E}_{\boldsymbol{\gamma}}\left\{\sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} p_{m,k}\right\} \le \bar{P} \end{aligned} \quad (6)$$

B. Dual Optimization Framework

We begin our development by observing that the objective function in (6) is separable across the subcarriers, and is tied together only by the power constraint. In these problems, it is useful to approach the problem using duality principles [6]. The dual problem is defined as

$$g^* = \min_{\lambda\ge 0} \Theta(\lambda) \quad (7)$$

where the dual objective is given by

$$\begin{aligned}
\Theta(\lambda) &= \max_{\boldsymbol{p}\in\boldsymbol{\mathcal{P}}} \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{m\in\mathcal{M}} w_m \sum_{k\in\mathcal{K}} R\left(p_{m,k}\gamma_{m,k}\right) \right\} \\
&\quad + \lambda \left(\bar{P} - \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} p_{m,k} \right\} \right) \\
&= \lambda\bar{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k\in\mathcal{K}} \max_{\boldsymbol{p}_k\in\boldsymbol{\mathcal{P}}_k} \sum_{m\in\mathcal{M}} \right. \\
&\quad \left. \left[w_m R\left(p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k} \right] \right\} \\
&= \lambda\bar{P} + \mathbb{E}_{\boldsymbol{\gamma}} \left\{ \sum_{k\in\mathcal{K}} \max_{m\in\mathcal{M}} \right. \\
&\quad \left. \max_{p_{m,k}\geq 0} \left[w_m R\left(p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k} \right] \right\}
\end{aligned} \tag{8}$$

where the second equality follows from the separability of variables across subcarriers, and the third from the exclusive subcarrier assignment constraint. We have reduced the problem to a per-subcarrier optimization, and since $K \gg M$, we have significantly decreased the computational burden.

Note that $R(p_{m,k}\gamma_{m,k})$ is a discontinuous function; hence, simple differentiation to arrive at the optimal solution is not feasible. However, we can divide the feasible region for $p_{m,k}$ into L segments

$$\mathcal{R}_+^l = \left[\frac{\eta_l}{\gamma_{m,k}}, \frac{\eta_{l+1}}{\gamma_{m,k}} \right), \quad l \in \mathcal{L} \tag{9}$$

for which we have $\forall p_{m,k} \in \mathcal{R}_+^l$,

$$\begin{aligned}
w_m R(p_{m,k}\gamma_{m,k}) - \lambda p_{m,k} &= w_m r_l - \lambda p_{m,k} \\
&\leq w_m r_l - \lambda \frac{\eta_l}{\gamma_{m,k}}
\end{aligned} \tag{10}$$

since λ and $p_{m,k}$ are both non-negative. Thus, there are only L candidate power allocation functions $\tilde{p}_{m,k} \in \left\{ \frac{\eta_0}{\gamma_{m,k}}, \ldots, \frac{\eta_{L-1}}{\gamma_{m,k}} \right\}$ from which we need to choose the maximizer of $w_m r_l - \lambda \frac{\eta_l}{\gamma_{m,k}}$, i.e.

$$\tilde{p}_{m,k} = \frac{\eta_{l^*_{m,k}}}{\gamma_{m,k}} \tag{11}$$

$$l^*_{m,k} \in \arg\max_{l\in\mathcal{L}} w_m r_l - \lambda \frac{\eta_l}{\gamma_{m,k}} \tag{12}$$

This also gives us the rate allocation $\tilde{R}_{m,k} = r_{l^*_{m,k}}$.

A straightforward computation of (12) would require $\mathcal{O}(L)$ complexity. However, if we assume that the discrete rate function (4) is concave[2], we can reduce the complexity of finding the power allocation function by noticing that (12) is equivalent to

$$w_m r_{l^*_{m,k}} - \frac{\lambda \eta_{l^*_{m,k}}}{\gamma_{m,k}} \geq w_m r_l - \frac{\lambda \eta_l}{\gamma_{m,k}} \tag{13}$$

$\forall l \in \mathcal{L} \setminus l^*_{m,k}$,. Thus, for all $\bar{l} > l^*_{m,k}$ and for all $\underline{l} < l^*_{m,k}$, (13) is equivalent to

$$\begin{aligned}
&\frac{r_{\bar{l}} - r_{l^*_{m,k}}}{\eta_{\bar{l}} - \eta_{l^*_{m,k}}} \leq \frac{\lambda}{w_m\gamma_{m,k}} < \frac{r_{l^*_{m,k}} - r_{\underline{l}}}{\eta_{l^*_{m,k}} - \eta_{\underline{l}}} \\
\Leftrightarrow \max_{l>l^*_{m,k}} &\frac{r_l - r_{l^*_{m,k}}}{\eta_l - \eta_{l^*_{m,k}}} \leq \frac{\lambda}{w_m\gamma_{m,k}} < \min_{l<l^*_{m,k}} \frac{r_{l^*_{m,k}} - r_l}{\eta_{l^*_{m,k}} - \eta_l}
\end{aligned}$$

Since the slope $\Delta r/\Delta\eta$ is non-increasing for a concave function, (12) is equivalent to

$$l^*_{m,k} = \left\{ l \in \mathcal{L} \,\middle|\, \frac{\lambda}{w_m\gamma_{m,k}} \in \left[\frac{r_{l+1} - r_l}{\eta_{l+1} - \eta_l}, \frac{r_l - r_{l-1}}{\eta_l - \eta_{l-1}} \right) \right\} \tag{14}$$

where with slight abuse of notation, we define $(r_0 - r_{-1})/(\eta_0 - \eta_{-1}) \equiv \infty$. Geometrically, we can consider $\lambda/w_m\gamma_{m,k}$ as a slope value for which we are looking for an interval of consecutive slope values for which it belongs. Since the set of rates and SNR region boundaries r_l and η_l are predefined in a communications system, we can store the set of slopes into a lookup table, thereby reducing the computational complexity of finding the optimal power allocation to a single table lookup operation.

We can now write (7) as

$$g^* = \min_{\lambda\geq 0} \lambda\bar{P} + \sum_{k\in\mathcal{K}} \mathbb{E}_{\boldsymbol{\gamma}_k} \left\{ g_k(\boldsymbol{\gamma}_k, \lambda) \right\} \tag{15a}$$

$$g_k(\boldsymbol{\gamma}_k, \lambda) = \max_{m\in\mathcal{M}} \left\{ g_{m,k}(\gamma_{m,k}, \lambda) \right\} \tag{15b}$$

$$g_{m,k}(\gamma_{m,k}, \lambda) = w_m r_{l^*_{m,k}} - \lambda \frac{\eta_{l^*_{m,k}}}{\gamma_{m,k}} \tag{15c}$$

It is important to note that (15c), despite the negative term, is always non-negative, since both r_0 and η_0 are equal to zero, and thus $g_{m,k}(\gamma_{m,k}, \lambda) \geq 0$.

C. Numerical Evaluation of the Expected Dual

Computing the expectation in (15a) in a straightforward manner involves an $M-$dimensional integral over the joint pdf of the $M-$length fading vector $\boldsymbol{\gamma}_k$, which is typically too complex to solve using direct numerical

[2]Concavity for this discontinuous staircase function simply means that the slopes when "connecting the dots" of the edges of the staircase are non-increasing.

integration techniques (e.g. Gaussian quadrature) except for small M, e.g. 2 or 3, since this requires $\mathcal{O}(N^M)$ computations where N is the number of function evaluations required for a one-dimensional integral with the same accuracy [7]. However, if we can somehow compute a closed-form expression for the pdf of the per-subcarrier dual function $g_k(\gamma_k, \lambda)$ (15b), then we can reduce the expectation to just a one-dimensional integral that is solvable in $\mathcal{O}(MN)$. Since $\gamma_{m,k}$ for different ms are INID, then (15c) is likewise INID for different ms. Thus, (15b) is the largest order statistic of INID random variables $g_{m,k}$, which has the following closed-form pdf [8, Sec. 5.2]

$$f_{g_k}(g_k) = \prod_{m\in\mathcal{M}} F_{g_{m,k}}(g_k)\left(\sum_{m\in\mathcal{M}} \frac{f_{g_{m,k}}(g_k)}{F_{g_{m,k}}(g_k)}\right) \quad (16)$$

where $F_{g_{m,k}}(g_{m,k})$ and $f_{g_{m,k}}(g_{m,k})$ are the cdf and pdf of $g_{m,k}$, given as (see [9] for a derivation)

$$F_{g_{m,k}}(g_{m,k}) = u(g_{m,k})F_{\gamma_{m,k}}(s_1) + \sum_{l\in\mathcal{L}\backslash 0}\left[F_{\gamma_{m,k}}\left(\min\left(h_l(g_{m,k}), s_{l+1}\right)\right) - F_{\gamma_{m,k}}(s_l)\right]^+ \quad (17)$$

$$f_{g_{m,k}}(g_{m,k}) = \delta(g_{m,k})F_{\gamma_{m,k}}(s_1) + \sum_{l\in\mathcal{L}\backslash 0} \mathbf{1}\left(h_l(g_{m,k}) \in \mathcal{S}\right) f_{\gamma_{m,k}}\left(h_l(g_{m,k})\right)\frac{h_l^2(g_{m,k})}{\lambda\eta_l} \quad (18)$$

where $[x]^+ \equiv \max(x, 0)$, $h_l(g_{m,k}) = \lambda\eta_l/[w_m r_l - g_{m,k}]^+$, $\mathcal{S} \equiv [s_l, s_{l+1})$ with $s_l = \lambda(\eta_l - \eta_{l-1})/\left(w_m(r_l - r_{l-1})\right)$, and $\mathbf{1}(x)$, $u(x)$ and $\delta(x)$ are the indicator, unit step, and Kronecker delta functions, respectively. Fig. 1 shows an example of the distribution functions for $w_m = 1$, $\lambda = 1$, $\bar{\gamma} = 20$ dB, and discrete rate function given in Section IV. We also superimposed empirical curves generated using a Monte-Carlo generation for verification, which are indistinguishable from the analytical results. Finally, using (17)-(18) in the maximal order statistic formula (16), and then in (15a), the dual problem is then

$$g^* = \min_{\lambda\geq 0} \lambda\bar{P} + \sum_{k\in\mathcal{K}}\int_0^\infty g_k f_{g_k}(g_k)dg_k \quad (19)$$

D. Optimal Rate, Subcarrier, and Power Allocation

Using duality arguments (see e.g. [6, Prop. 5.1.2]), the dual objective function in (19) can be shown to be convex in the single variable λ.Unfortunately, it is not continuously differentiable due to the presence of the $[x]^+$ function in (17). Hence, we resort to derivative-free single-dimensional search methods that only need function evaluations, e.g. Golden-section search [7]. The optimal subcarrier, rate, and power allocation is then determined using λ^* as

$$m_k^* = \arg\max_{m\in\mathcal{M}} w_m r_{l^*_{m,k}} - \lambda^*\frac{\eta_{l^*_{m,k}}}{\gamma_{m,k}} \quad (20)$$

$$R^*_{m,k} = \begin{cases} r_{l^*_{m,k}}, & m = m_k^* \\ 0, & m \neq m_k^* \end{cases} \quad (21)$$

$$p^*_{m,k} = \begin{cases} \frac{\eta_{l^*_{m,k}}}{\gamma_{m,k}}, & m = m_k^* \\ 0, & m \neq m_k^* \end{cases} \quad (22)$$

where $l^*_{m,k}$ is given by (14) with $\lambda = \lambda^*$.

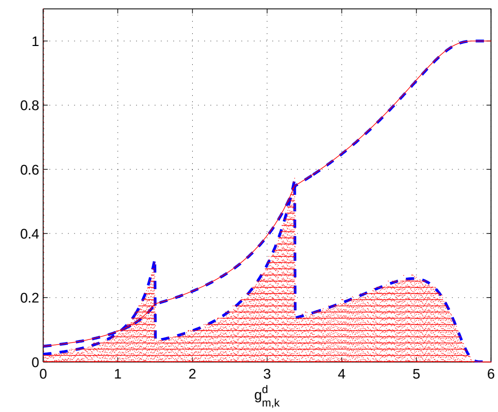

Fig. 1. Analytical (thick dashed lines) and empirical (thin solid lines) CDF (17) and PDF (18) of (15c).

IV. NUMERICAL RESULTS

We consider an OFDMA system roughly based on a 3GPP-LTE downlink [1], with 128 subcarriers, 76 used subcarriers, 1.25 MHz bandwidth, 1.92 MHz sampling frequency, and a cyclic prefix length of 6 samples. We assume a Grey-coded square 2^{r_l}-QAM modulation scheme, with rate set $r_l \in \{0, 2, 4, 6\}$ bits and SNR thresholds $\eta_l \in \{-\infty, 9.97, 16.96, 23.19\}$ dB computed using BER $\approx 0.2\exp\left[\frac{-1.6p_{m,k}\gamma_{m,k}}{2^{r_l}-1}\right]$ [10] with a BER constraint of 10^{-3}. Note that we assume that channel coding is not present in this case for simplicity, but since the framework merely needs the SNR thresholds and rate values, our results also apply to the coded case. We simulate the frequency-selective Rayleigh fading channel using the ITU-Vehicular A channel model [1]. We generate 10000 IID channel realizations per data point, where for each user's channel realization $\boldsymbol{h}_m$ (2), we generate a complex Gaussian random vector with N_t independent entries, each with variance corresponding to the power delay profile for the corresponding path.

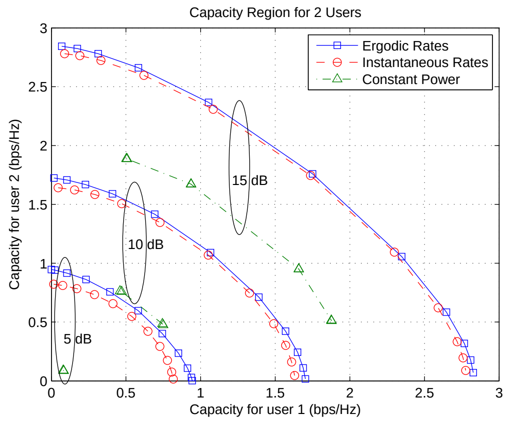

Fig. 2. 2-User Capacity Region for ergodic and instantaneous discrete rate maximization.

In Figure 2, we compare the capacity region for 2 users in various SNRs of our ergodic discrete-rate maximization algorithm with instantaneous maximization [3][4] and constant power allocation. We see that the gain of ergodic maximization is more pronounced for low SNRs and more disparate user weights. This observation is analogous to previous studies in adaptive modulation, e.g. [10], which concluded that the exploitation of the additional temporal dimension through the ergodic formulation is most useful when other degrees of freedom have been significantly curtailed. Observe also that a large loss is incurred by the constant power allocation case. This is due to the big loss of freedom in the rate allocation (limited to just 4 rates in contrast to an infinite number of rates in the continuous case), which when coupled with constant power allocation results in a huge loss in performance.

Table I shows the average number of function evaluations N to numerically compute the integral in (15a) with an error of 10^{-6}, the number of iterations I to search for the optimum λ in (19), and the relative optimality gaps[3] [6] [9] for the ergodic and instantaneous discrete rate allocation algorithms. We see that the relative optimality gaps are virtually zero, which allows us to claim optimality of the algorithms for all practical purposes.

Table II shows the complexity order of the different resource allocation algorithms. Only the ergodic rate algorithms require initialization. If we use the average numbers given in Table I, the ergodic rate algorithm is less complex than the instantaneous rate algorithm per symbol on average, as long as the rate of change of the channel fading statistics (roughly at the rate of change of slow fading, e.g. Log-normal shadowing) is much lower than the rate of change of the actual channel realizations (roughly at the rate of fast fading, e.g. Rayleigh fading), such that the initialization is performed less often.

[3]This is a measure of how far we are from the optimal solution, where a gap of 0 means we have attained the optimal solution.

TABLE I

RELEVANT METRICS FOR THE RESOURCE ALLOCATION ALGORITHMS

Metric	SNR	Ergodic	Inst.
No. of Fun. Eval. (N)	5 dB	62.09	–
	10 dB	91.55	–
	15 dB	133.0	–
No. of Iterations (I)	5 dB	9.818	17.24
	10 dB	10.55	17.20
	15 dB	9.909	17.30
Relative Gap ($\times 10^{-4}$)	5 dB	.8711	3.602
	10 dB	.9507	1.038
	15 dB	.5322	.3996

TABLE II

COMPLEXITY FOR THE RESOURCE ALLOCATION ALGORITHMS.

Algorithm	Initialization	Runtime
Ergodic Rates	$\mathcal{O}(INML)$	$\mathcal{O}(MK\log(L))$
Instantaneous Rates	–	$\mathcal{O}(IMK\log(L))$
Constant Power	–	$\mathcal{O}(MK\log(L))$

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