Sum Capacity of Multiuser MIMO Broadcast Channels with Block Diagonalization

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Abstract

The sum capacity of a Gaussian broadcast MIMO channel can be achieved with dirty paper coding (DPC). However, algorithms that approach the DPC sum capacity do not appear viable in the forseeable future, which motivates lower complexity interference suppression techniques. Block diagonalization (BD) is a linear precoding technique for downlink multiuser MIMO systems. With perfect channel knowledge at the transmitter, BD can eliminate other users' interference at each receiver. In this paper, we study the sum capacity of BD with and without receive antenna selection. We analytically compare BD without receive antenna selection to DPC for a set of given channels. It is shown that 1) if the user channels are orthogonal to each other, then BD achieves the same sum capacity as DPC; 2) if the user channels lie in the same subspace, then the gain of DPC over BD can be upper bounded by the minimum of the number of transmit and receive antennas. These observations also hold for BD with receive antenna selection. Further, we study the ergodic sum capacity of BD with and without receive antenna selection in a Rayleigh fading channel. Simulations show that BD can achieve a significant part of the total throughput of DPC. An upper bound on the ergodic sum capacity gain of DPC over BD is proposed for easy estimation of the gap between the sum capacity of DPC and BD without receive antenna selection.

Index Terms

MIMO, capacity, multiuser, broadcast channels, precoding, dirty paper coding

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I. INTRODUCTION

Although the capacity results for point-to-point Multiple-input-multiple-output (MIMO) systems are well understood [1], [2], [3], only recently has the capacity region of the multiuser MIMO Gaussian broadcast channels (BC) been discovered. It was conjectured that the MIMO BC capacity region is achieved with dirty paper coding (DPC) [4] and subsequently proven in [5]. The sum capacity, which is defined as the maximum aggregation of all users' data rates, can be obtained by iterative water-filling algorithms [6], [7].

Although the sum capacity of a Gaussian MIMO BC channel is achievable with DPC, a practical coding scheme that approaches the DPC sum capacity is still unavailable. Several nonlinear and linear algorithms have been proposed in [8], [9], [10]. These algorithms, however, are typically too complicated for cost-effective implementations. An alternative linear precoding technique for downlink multiuser MIMO systems, generally named Block Diagonalization (BD), was proposed in [11], [12], [13], [14]. With BD, each user's data is multiplied by a linear precoding matrix before transmission. The precoding matrix for every user lies in the null space of all other users' channels. Thus, provided that prefect channel state information is available at the base station, zero inter-user interference is achievable at every receiver, thereby enabling a simple receiver structure to be used. Hence, BD is a potentially realizable precoding method for a MIMO broadcast channel, although it is suboptimal for the sum capacity.

The sum capacity gain of DPC vs. TDMA for has been studied in [15], [16]. In this paper, we focus on the sum capacity gain of DPC over BD. We define BD's sum capacity to be the maximum total throughput over all possible user sets. Hence the TDMA sum capacity, where the transmitter only sends data to the user with the largest channel capacity, is a lower bound on BD's sum capacity. We also propose to jointly optimize the precoding and post-processing matrices for sum capacity. While the optimal solution is difficult to obtain, we analyze the sum capacity of BD with receive antenna selection as a special case of the joint optimization.

II. SYSTEM MODEL AND BACKGROUND ON BLOCK DIAGONALIZATION

Consider a downlink multiuser MIMO system with K users, where N_t denotes the number of transmit antennas at base station and $N_{r,j}$ $(\leq N_t)$ denotes the number of receive antennas for the jth user. The transmitted symbol of user j is denoted as a N_j-dimensional vector $\mathbf{x}_j$, which is multiplied by a $N_t \times N_j$ precoding matrix $\mathbf{T}_j$. At receiver j, a $M_j \times N_{r,j}$ ($M_j \leq N_{r,j}$) matrix $\mathbf{R}_j$ is applied to the received signals. Hence, the post-processed received signal $\mathbf{y}_j$ for user j is

$$\mathbf{y}_j = \mathbf{R}_j \left(\mathbf{H}_j \mathbf{T}_j \mathbf{x}_j + \sum_{k=1,k\neq j}^{K} \mathbf{H}_j \mathbf{T}_k \mathbf{x}_k + \mathbf{v}_j \right) = \mathbf{R}_j \mathbf{H}_j \mathbf{T}_j \mathbf{x}_j + \mathbf{R}_j \sum_{k=1,k\neq j}^{K} \mathbf{H}_j \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_j \mathbf{v}_j \tag{1}$$

where $\mathbf{v}_j$ denotes the additive white Gaussian noise (AWGN) vector for user j with variance $E[\mathbf{v}_j \mathbf{v}_j^*] = \sigma^2 \mathbf{I}$, and $()^*$ denotes the matrix conjugate transpose. Matrix $\mathbf{H}_j \in \mathbb{C}^{N_{r,j} \times N_t}$ denotes the channel transfer matrix from the base station to the jth user, with each entry following an i.i.d. complex Gaussian distribution $\mathcal{CN}(0,1)$, which ensures rank $(\mathbf{H}_j) = \min(N_{r,j}, N_t)$ for all j with probability one. We further assume that the channels $\mathbf{H}_j$ experienced by different users are statistically independent due to users' different locations.

The key idea of BD is to design $\mathbf{T}_j \in \mathbb{U}(N_t, N_j)$ and $\mathbf{R}_j^T \in \mathbb{U}(N_{r,j}, M_j)$, such that

$$\mathbf{R}_i \mathbf{H}_i \mathbf{T}_j = 0 \quad \text{for all } i \neq j \;\text{ and } 1 \leq i, j \leq K, \tag{2}$$

where $()^T$ denotes the matrix transpose and $\mathbb{U}(n,k)$ represents the set of $n \times k$ ($n \geq k$) matrices with orthonormal columns. Thus, the post-processed received signal for user j is reduced to

$$\mathbf{y}_j = \mathbf{R}_j \mathbf{H}_j \mathbf{T}_j \mathbf{x}_j + \mathbf{R}_j \sum_{k=1,k\neq j}^{K} \mathbf{H}_j \mathbf{T}_k \mathbf{x}_k + \mathbf{R}_j \mathbf{v}_j = \mathbf{R}_j \mathbf{H}_j \mathbf{T}_j \mathbf{x}_j + \mathbf{R}_j \mathbf{v}_j. \tag{3}$$

For a fixed set of $\{\mathbf{R}_j\}_{j=1}^K$, let $\widetilde{\mathbf{H}}_j = [(\mathbf{R}_1\mathbf{H}_1)^T \;\cdots\; (\mathbf{R}_{j-1}\mathbf{H}_{j-1})^T \;\; (\mathbf{R}_{j+1}\mathbf{H}_{j+1})^T \;\cdots\; (\mathbf{R}_K\mathbf{H}_K)^T]^T$. To satisfy the constraint in (2), $\mathbf{T}_j$ shall be in the null space of $\widetilde{\mathbf{H}}_j$.

In the rest of the paper, we assume $N_{r,k} = N_r$ for $k = 1, 2, \cdots, K$. The results in this paper can be easily extended to the general case.

III. Sum Capacity of Block Diagonalization with Receiver Antenna Selection

Notice that the precoding matrices $\{\mathbf{T}_j\}_{j=1}^K$ can be determined based on $\{\mathbf{H}_j\}_{j=1}^K$ and $\{\mathbf{R}_j\}_{j=1}^K$, i.e. $\mathbf{T}_j$ can be any set of basis in the null space of $\widetilde{\mathbf{H}}_j$. Ideally, the sum capacity can be obtained by jointly optimizing $\{\mathbf{R}_j\}_{j=1}^K$ and the users' transmit signal covariance matrices $\{\mathbf{Q}_j\}_{j=1}^K$ in the following problem

$$\max_{M_j, \mathbf{R}_j, \mathbf{Q}_j} \sum_{j=1}^{K} \log \left| \mathbf{I} + \frac{1}{\sigma^2} \mathbf{R}_j \mathbf{H}_j \mathbf{T}_j \mathbf{Q}_j \mathbf{T}_j^* \mathbf{H}_j^* \mathbf{R}_j^* \right| \tag{4}$$

$$
\begin{aligned}
\text{subject to}\quad & \mathbf{R}_j^T \in \mathbb{U}(N_r, M_j) \quad \text{for all } j \\
& \mathbf{R}_i \mathbf{H}_i \mathbf{T}_j = 0 \quad \text{for all } i \neq j \\
& 0 \leq M_j \leq N_r \quad \text{for all } j \\
& \sum_{j=1}^{K} \mathrm{Tr}(\mathbf{Q}_j) \leq P \\
& \mathbf{Q}_j \geq 0 \quad \text{for all } j
\end{aligned}
$$

where $\mathbf{R}_j \mathbf{H}_j \mathbf{T}_j$ denotes the effective channel for user j, $\mathbf{Q}_j = E[\mathbf{x}_j \mathbf{x}_j^*]$ is user j's input covariance matrix of size $N_j \times N_j$, and P denotes the total transmit power available at the base station. The optimization over $\mathbf{Q}_j$ ensures the best signal covariance for user j. The maximization over $\mathbf{R}_j$, as well as its dimension M_j, ensures that the total throughput is maximized. If $M_j < N_{r,j}$, the post-processing matrix $\mathbf{R}_j$ reduces the dimensionality of the received signal for user j, therefore decreasing its own throughput. However, this in turn makes it easier for other users to satisfy the zero inter-user interference constraint and achieve higher capacity. If $M_j = N_{r,j}$, then the zero-forcing constraint in (2) is equivalent to $\mathbf{H}_j \mathbf{T}_j = 0$. Thus, the post-processing does not affect transmission if it is square. Further, notice that $M_j = 0$ for unscheduled users.

Given fixed $\mathbf{R}_j, \forall j = 1, \ldots, K$, the optimization problem in (4) reduces to a standard waterfilling problem over the eigenvalues of the equivalent channels. The general optimization problem in (4), however, is difficult to solve, especially the maximization over $\{\mathbf{R}_j\}_{j=1}^K$. The difficulty primarily comes from the zero inter-user interference requirement. In [14], an iterative algorithm was proposed to optimize $\{\mathbf{R}_j\}_{j=1}^K$ and $\{\mathbf{T}_j\}_{j=1}^K$ so that the aggregate effective channel energy is maximized. The sum capacity, however, is not directly optimized in [14]. In this paper, we consider a set of special $M_j \times N_r$ matrices $\mathbf{R}_j$ (for $j = 1, 2, \cdots, K$) that are formed by taking M_j rows from $\mathbf{I}_{N_r}$ [17]. For example, if $M_j = 2$ and $N_r = 3$, $\mathbf{R}_j$ must be in the following set:

$$
\mathcal{R}^{(2,3)} = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} \tag{5}
$$

where $\mathcal{R}^{(m,n)}$ denotes the set of matrices formed by taking m rows from $\mathbf{I}_n$. These special $\mathbf{R}_j$ matrices correspond to receiver antenna selection for user j, where a subset of receive antennas are used. The motivation of studying this special $\mathbf{R}_j$ are:

1) Since the matrices $\{\mathbf{R}_j\}_{j=1}^K$ and $\{\mathbf{T}_j\}_{j=1}^K$ are designed at the base station, the post-processing matrices $\{\mathbf{R}_j\}_{j=1}^K$ need to be conveyed to the users, which is a system overhead that should

be kept low. To successfully convey the post-processing matrices to the users, much less overhead (in the number of bits) is required for this specially formed $\mathbf{R}_j$ than a general $M_j \times N_r$ matrix. For example, $\log_2\left(\sum_{M_j=0}^{N_r} |\mathcal{R}^{(M_j,N_r)}|\right) = N_r$ bits are sufficient to convey $\mathbf{R}_j$ to user j, where $|\mathcal{R}^{(M_j,N_r)}|$ denotes the cardinality of set $\mathcal{R}^{(M_j,N_r)}$.

2) With this special $\mathbf{R}_j$, user j can select M_j receive antennas to use. Hence, user selection and receive antenna selection can be combined to optimize the total throughput of all users. If $\mathbf{R}_j = \mathbf{I}_{N_r}$ for those users scheduled for transmission and $\mathbf{R}_j = \emptyset$ (i.e. $M_j = 0$) for those unscheduled users, then the generalized block diagonalization [14] reduces to the BD algorithm without post-processing presented in [11], [12].

3) With the additional constraint that $\mathbf{R}_j \in \mathcal{R}^{(M_j,N_r)}$ for $j = 1, 2, \cdots, K$, the optimization problem in (4) is solvable by exhaustively searching over all possible sets of $\{\mathbf{R}_j\}_{j=1}^K$. For each set of $\{\mathbf{R}_j\}_{j=1}^K$, the corresponding $\{\mathbf{T}_j\}_{j=1}^K$ can be found according to the SVD outlined in Section II. Further, with zero inter-user interference brought by BD, the optimal $\{\mathbf{Q}_j\}_{j=1}^K$ can be easily obtained by the water-filling algorithm with an overall transmit power constraint [11].

IV. BD vs. DPC: Sum Capacity for a Given Set of Channels

In this section, we compare the sum capacity achieved by BD without receive antenna selection with the sum capacity achieved by DPC. Since BD without receive antenna selection is a special case of BD with receive antenna selection, i.e. $\mathbf{R}_j = \mathbf{I}_{N_r}$ for $j = 1, 2, \cdots, K$, the results in this section also hold for BD with receive antenna selection.

For a multiuser MIMO system, let $\mathcal{K} = \{1, 2, \cdots, K\}$ denote the set of user indices. Assume all user sets are ordered and let $\mathcal{A}_i \in \mathcal{K}$ be the ith set. Let $\overline{\mathbf{H}}_j = \mathbf{H}_j\mathbf{T}_j$ denote the effective channel after precoding for user $j \in \mathcal{A}_i$, then the total throughput achieved with BD applied to the user set $\mathcal{A}_i$ with total power P can be expressed as

$$C_{BD|\mathcal{A}_i}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2) = \max_{\{\mathbf{Q}_j:\, \mathbf{Q}_j \geq 0,\, \sum_{j\in\mathcal{A}_i} \mathrm{Tr}(\mathbf{Q}_j) \leq P\}} \sum_{j\in\mathcal{A}_i} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\overline{\mathbf{H}}_j\mathbf{Q}_j\overline{\mathbf{H}}_j^*\right| \tag{6}$$

where $\mathbf{Q}_j = E[\mathbf{x}_j\mathbf{x}_j^*]$ is user j's input covariance matrix of size $N_j \times N_j$. Let $\mathcal{A}$ be the set containing all possible user sets, i.e. $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \cdots\}$. The sum capacity of BD is defined as

$$C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) = \max_{\mathcal{A}_i \in \mathcal{A}} C_{BD|\mathcal{A}_i}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2). \tag{7}$$

The sum capacity of a multiuser Gaussian broadcast channel is achieved with DPC [18],

$$C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) = \max_{\{\mathbf{S}_j:\, \mathbf{S}_j \geq 0, \sum_{j=1}^{K} \mathrm{Tr}(\mathbf{S}_j) \leq P\}} \log \left| \mathbf{I} + \frac{1}{\sigma^2} \sum_{j=1}^{K} \mathbf{H}_j^* \mathbf{S}_j \mathbf{H}_j \right| \tag{8}$$

where $\mathbf{S}_j$ of size $N_r \times N_r$ is the signal covariance matrix for user j in the dual multiple access channel [18]. In this section, we are interested in the gain of DPC over BD in terms of sum capacity. Analogous to [15], we define the ratio of DPC to BD as

$$G(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) \triangleq \frac{C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)}{C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)}. \tag{9}$$

The gain is dependent on channel realizations $\{\mathbf{H}_k\}_{k=1}^K$, total power, and noise variance. In the next theorem, we give a bound on $G(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)$ that is valid for any $\{\mathbf{H}_k\}_{k=1}^K$, P, and σ^2, based on Theorem 3 in [15].

Theorem 1: The sum capacity gain of DPC over BD is lower bounded by 1 and upper bounded by the minimum of N_t and K, i.e.

$$1 \leq G(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) \leq \min\{N_t, K\} \tag{10}$$

Proof: Theorem 3 in [15] states that $\frac{C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)}{C_{TDMA}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)} \leq \min\{N_t, K\}$ where

$$C_{TDMA}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) = \max_{k \in \mathcal{K}} \max_{\{\mathbf{Q}_k:\, \mathbf{Q}_k \geq 0, \mathrm{Tr}(\mathbf{Q}_k) \leq P\}} \log \left| \mathbf{I} + \frac{1}{\sigma^2} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^* \right|. \tag{11}$$

According to the definition, BD is superior to TDMA in terms of sum capacity. Also, it is obvious that BD is inferior to the optimum DPC. Thus, Theorem 1 is immediately obtained. □

Although the bound in Theorem 1 holds for any N_t, N_r, K, $\{\mathbf{H}_i\}_{i=1}^K$, P, and σ^2, it is generally a loose bound. If $N_r \leq N_t$, $K \leq \lfloor \frac{N_t}{N_r} \rfloor$, and $\{\mathbf{H}_k\}_{k=1}^K$ are mutually orthogonal, i.e. $\mathbf{H}_i \mathbf{H}_j^* = 0$ for $i \neq j$, then $C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) = C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)$. Further, if $N_r \leq N_t$ and the row vector spaces of all user channels are the same, i.e. $\mathrm{span}(\mathbf{H}_1) = \mathrm{span}(\mathbf{H}_2) = \cdots = \mathrm{span}(\mathbf{H}_K)$, then $G(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) \leq \min\{N_r, K\}$, which is the same as TDMA. For the general case where the user channels partially overlap, BD may be superior to TDMA since multiple users may be supported simultaneously.

V. BD vs. DPC: Ergodic Sum Capacity in Rayleigh Fading Channels

In this section, we derive a bound on the gain of DPC's ergodic capacity over BD in Rayleigh fading channels. Let $\overline{\mathbf{H}}_j = \mathbf{H}_j\mathbf{T}_j$ be the $N_r \times N_j$ equivalent channel for user j after precoding. With the assumption that $\mathbf{H}_j$ for $j = 1, 2, \cdots, K$ are mutually independent and the elements in $\mathbf{H}_j$ are i.i.d. complex Gaussian, the effective channel $\overline{\mathbf{H}}_j$ still follows an i.i.d. complex Gaussian distribution [12]. Hence the ergodic capacity of user j can be evaluated with the eigenvalue distribution of $\overline{\mathbf{H}}_j\overline{\mathbf{H}}_j^*$ [1], [19], [20].

We first derive a lower bound on the ergodic sum capacity with BD. Let $\mathcal{A}_i = \{1, 2, \cdots, i\}$ be the set of the first i users, for $i = 1, 2, \cdots, I$ where $I = \min\left\{K, \left\lfloor \frac{N_t}{N_r} \right\rfloor\right\}$. For $i \leq \min\left\{K, \left\lfloor \frac{N_t}{N_r} \right\rfloor\right\}$, when the elements in $\{\mathbf{H}\}_{k=1}^K$ follow i.i.d. complex Gaussian distribution, we have the following lower bound on the ergodic sum capacity of BD

$$
\begin{aligned}
E\left[C_{BD}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2)\right] &\overset{(a)}{\geq} E\left[\max_{\{\mathbf{Q}_j:\, \mathbf{Q}_j \geq 0,\, \sum_{j\in\mathcal{A}_i} \mathrm{Tr}(\mathbf{Q}_j) \leq P\}} \sum_{j=1}^{i} \log\left|\mathbf{I} + \frac{\overline{\mathbf{H}}_j\mathbf{Q}_j\overline{\mathbf{H}}_j^*}{\sigma^2}\right|\right] && (12)\\
&= \sum_{j=1}^{i} E\left[\sum_{n=1}^{N_r} \log\left|1 + \frac{P_{j,n}}{\sigma^2}\overline{\lambda}_{j,n}^2\right|\right] && (13)\\
&\overset{(b)}{\geq} \sum_{j=1}^{i} E\left[\sum_{n=1}^{N_r} \log\left|1 + \frac{P}{iN_r\sigma^2}\overline{\lambda}_{j,n}^2\right|\right] \\
&\overset{(c)}{=} i\, N_r E\left[\log\left|1 + \frac{P}{iN_r\sigma^2}\overline{\lambda}_{i,1}^2\right|\right] \triangleq \overline{C}_{BD}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2) && (14)
\end{aligned}
$$

where $\overline{\lambda}_{j,n}^2$ are nth unordered eigenvalues of $\overline{\mathbf{H}}_j\overline{\mathbf{H}}_j^*$ and $\overline{\mathbf{H}}_j$ is of size $N_r \times (N_t - (i-1)N_r)$. Inequality (a) holds because the RHS of (12) assumes all i users are simultaneously transmitting for all channel realizations. Inequality (b) holds because the RHS of (14) assumes equal power is allocated to every non-zero eigenmode. Equality (c) holds because $\overline{\lambda}_{j,n}$ has the same probability density function for $j = 1, 2, \cdots, i$ and $n = 1, 2, \cdots, N_r$, which is given in [1]. Thus, we can lower bound the ergodic sum capacity with BD by

$$
E\left[C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)\right] \geq \max_{i\in\{1,2,\cdots,I\}} \overline{C}_{BD}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2). \qquad (15)
$$

Second, we upper bound the Ergodic Sum Capacity of DPC. The sum capacity of a K-user broadcast channel with DPC is upper bounded if the receivers are allowed to cooperate [18].

Let $\mathbf{H} = [\mathbf{H}_1^* \ \mathbf{H}_2^* \ \cdots \ \mathbf{H}_K^*]^*$, $N = \max\{N_t, KN_r\}$ and $M = \min\{N_t, KN_r\}$, then

$$E\left[C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)\right] \leq E\left[\max_{\{\mathbf{Q}:\ \mathbf{Q}\geq 0, \mathrm{Tr}(\mathbf{Q})\leq P\}} \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{HQH}^*\right|\right]$$
$$= \sum_{m=1}^{M} E\left[\log\left(1 + \frac{P_m}{\sigma^2}\lambda_m^2\right)\right] \leq M \int_{\sigma^2/\Gamma_0}^{\infty} \log\left(\frac{\Gamma_0\alpha_1}{\sigma^2}\right) p_{N,M}(\alpha_1)d\alpha_1 \triangleq \overline{C}_{coop}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2) \tag{16}$$

where $\mathbf{Q} = E(\mathbf{xx}^*)$ and $\mathbf{x}$ is the sum transmitted signal from the base station antenna array, λ_m^2 is mth unordered eigenvalue of $\mathbf{H}^*\mathbf{H}$ and $\alpha_1 \triangleq \lambda_1^2$; $p_{N,M}(\alpha_1)$ is the distribution for α_1, which can be found in [1]. The parameter Γ_0 is optimized so that the ergodic sum capacity is maximized with the average power constraint, i.e. $M\int_{\sigma^2/\Gamma_0}^{\infty}\left(\Gamma_0 - \frac{\sigma^2}{\alpha}\right)p_{N,M}(\alpha)d\alpha = P$. Details on the inequality (16) can be found in [21].

With the above two bounds, we can directly upper bound the ergodic sum capacity gain of DPC over BD as

$$\frac{E\left[C_{DPC}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)\right]}{E\left[C_{BD}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)\right]} \leq \frac{\overline{C}_{coop}(\mathbf{H}_{1,\cdots,K}, P, \sigma^2)}{\max\limits_{i\in\{1,2,\cdots,I\}} \overline{C}_{BD}(\mathbf{H}_{\mathcal{A}_i}, P, \sigma^2)}. \tag{17}$$

VI. NUMERICAL RESULTS

In this section, we compare the sum capacity achieved by 1) DPC implemented with the iterative water-filling algorithm [7], 2) BD with receive antenna selection (BD w RxAS), and 3) BD without receive antenna selection (BD w/o RxAS). Monte Carlo simulations show that in Rayleigh fading channels, BD achieves a significant part of the ergodic sum capacity of DPC. And the bound in (17) is tight for medium to high SNRs or when $K \leq \lfloor \frac{N_t}{N_r} \rfloor$.

Fig. 1 shows the ergodic sum capacity of DPC vs. BD under different SNRs, with $N_t = 10$, $N_r = 2$, and $K = 5$. In the low SNR regime, BD achieves almost the same sum capacity as DPC. As SNR goes to infinity, the sum capacity of both DPC and BD increase with the same slope. Essentially, the ratio of the sum capacity of BD and DPC equals one in asymptotically low and high SNR regimes [15]. Fig. 2 shows the gain of DPC over BD from the curves in Fig. 1, as well as the bound on the gain evaluated from (17). As the SNR increases, the bound in (17) gets tighter. For low SNR, the bound in (17) is loose mainly because 1) the lower bound on BD assumes an equal power allocation to all non-zero eigenvalues; 2) the cooperative upper bound on DPC is also loose in low SNR. The bound in the low SNR regime is, however, less interesting because it has been proven in [15] that the sum capacity of BD equal that of DPC for asymptotically low SNRs.

Fig. 3 shows the ergodic sum capacity of DPC vs. BD for different N_t, with $N_r = 2$ and $K = 3$. As the number of transmit antenna increases, the sum capacity of BD gets closer to the sum capacity of DPC. Fig. 4 shows the gain of DPC over BD from the curves in Fig. 3, with $SNR = 20$ dB. It is observed that the bound from (17) is fairly tight for $N_t > KN_r$, which corresponds to a practical under-loaded system.

Fig. 5 shows the ergodic sum capacity of DPC vs. BD for different numbers of users, with $N_t = 10$ and $N_r = 2$. For small numbers of users, BD achieves almost the same sum capacity as DPC. As the number of users increases, DPC exhibits higher performance than BD. Fig. 6 shows the gain of DPC over BD from the curves in Fig. 5, with $SNR = 20$ dB. For small numbers of users, the bound from (17) is very tight. For larger numbers of users, the bound from (17) loosens, which is due to the fact that the lower bound on the sum capacity of BD in (15) does not take multiuser diversity into consideration.

VII. CONCLUSIONS

In this paper, we compare the sum capacity of BD with and without receive antenna selection to that of DPC. For a given set of channel realizations, the sum capacity gain of DPC over BD can be generally bounded by $\min\{N_t, K\}$, where N_t and K are the number of transmit antennas and the number of users, respectively. The ergodic sum capacity gain of DPC over BD is also studied in a Rayleigh fading channel. Simulations show that BD can achieve a significant part of the total throughput of DPC. An upper bound on the ergodic sum capacity gain of DPC over BD is proposed. The bound is very tight for medium to high SNRs or when $K \leq \lfloor \frac{N_t}{N_r} \rfloor$, which is useful in estimating how far away BD is from being optimal in terms of ergodic sum capacity.

REFERENCES

[1] I. E. Telatar, “Capacity of multi-antenna Gaussian channels,” *European Trans. on Telecommunications*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[3] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Sel. Areas in Communications*, vol. 21, no. 5, pp. 684–702, Jun. 2003.

[4] M. Costa, "Writing on dirty paper," *IEEE Trans. on Info. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[5] H. Weingarten, Y. Steinberg, and S. Shamai (Shitz), "The capacity region of the Gaussian MIMO broadcast channel," in *Proc. IEEE ISIT*, Jun. 2004, p. 174.

[6] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi, "Iterative water-filling for Gaussian vector multiple-access channels," *IEEE Trans. on Info. Theory*, vol. 50, no. 1, pp. 145–152, Jan. 2004.

[7] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum power iterative water-filling for multi-antenna Gaussian broadcast channels," *IEEE Trans. on Info. Theory*, vol. 51, no. 4, pp. 1570–1580, Apr. 2005.

[8] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Trans. on Info. Theory*, vol. 48, no. 6, pp. 1250–1276, Jun. 2002.

[9] M. Airy, A. Forenza, R. W. Heath Jr., and S. Shakkottai, "Practical costa precoding for the multiple antenna broadcast channel," in *Proc. IEEE Globecom*, Dec. 2004, vol. 6, pp. 3942–3946.

[10] M. Stojnic, H. Vikalo, and B. Hassibi, "Rate maximization in multi-antenna broadcast channels with linear preprocessing," in *IEEE Globecom*, Dec. 2004, vol. 6, pp. 3957–3961.

[11] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[12] L. U. Choi and R. D. Murch, "A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach," *IEEE Trans. on Wireless Communications*, vol. 3, no. 1, pp. 20–24, Jan. 2004.

[13] K. K. Wong, R. D. Murch, and K. B. Letaief, "A joint-channel diagonalization for multiuser MIMO antenna systems," *IEEE Trans. on Wireless Communications*, vol. 2, no. 4, pp. 773–786, Jul. 2003.

[14] Z. Pan, K. K. Wong, and T. S. Ng, "Generalized multiuser orthogonal space-division multiplexing," *IEEE Trans. on Wireless Communications*, vol. 3, no. 6, pp. 1969–1973, Nov. 2004.

[15] N. Jindal and A. Goldsmith, "Dirty-paper coding versus TDMA for MIMO broadcast channels," *IEEE Trans. on Info. Theory*, vol. 51, no. 5, pp. 1783–1794, May 2005.

[16] M. Sharif and B. Hassibi, "Scaling laws of sum rate using time-sharing, DPC, and beamforming for MIMO broadcast channels," in *Prof. IEEE ISIT*, Jun. 2004, p. 175.

[17] R. W. Heath Jr. and D. J. Love, "Multi-mode antenna selection for spatial multiplexing with linear receivers," *IEEE Trans. on Signal Processing*, vol. 53, no. 8, part 2, pp. 3042–3056, Aug. 2005.

[18] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. on Info. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[19] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*, John Wiley & Sons, Inc., 1982.

[20] A. Edelman, *Eigenvalue and Condition Numbers of Random Matrices*, Ph.D. thesis, MIT, May 1989.

[21] Z. Shen, R. W. Heath Jr., J. G. Andrews, and B. L. Evans, "Comparison of space-time water-filling and spatial water-filling for MIMO fading channels," in *Proc. IEEE Globecom*, Dec. 2004, vol. 1, pp. 431–435.

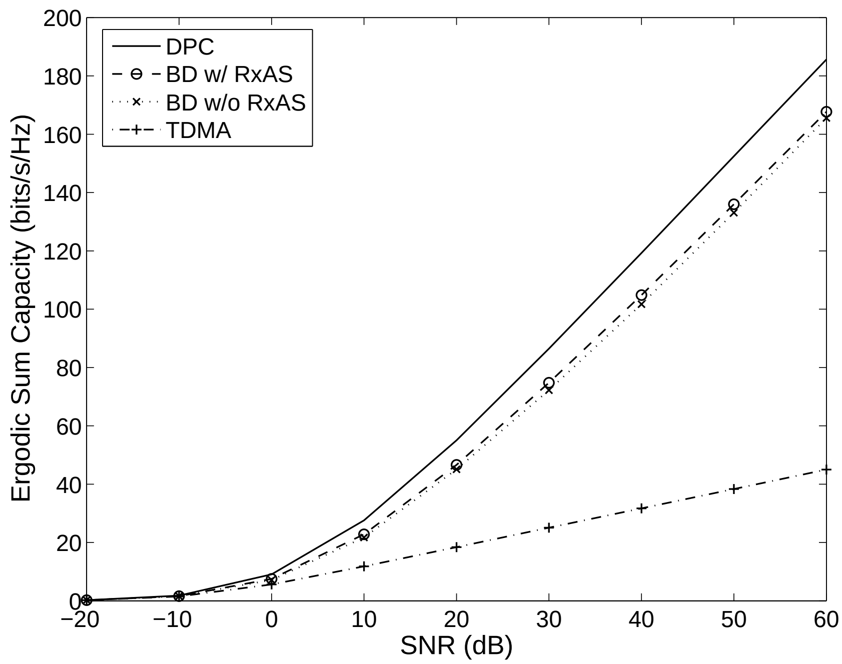

Fig. 1. Ergodic sum capacity of DPC vs. BD in Rayleigh fading channels. $N_t = 10$, $N_r = 2$, $K = 5$.

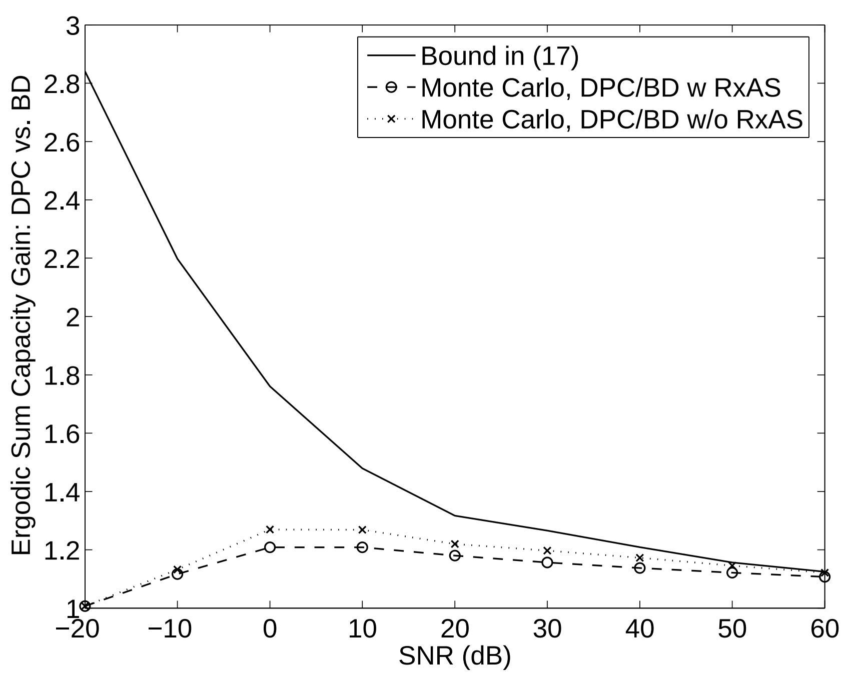

Fig. 2. Ergodic sum capacity gain of DPC over BD in Rayleigh fading channels. $N_t = 10$, $N_r = 2$, $K = 5$.

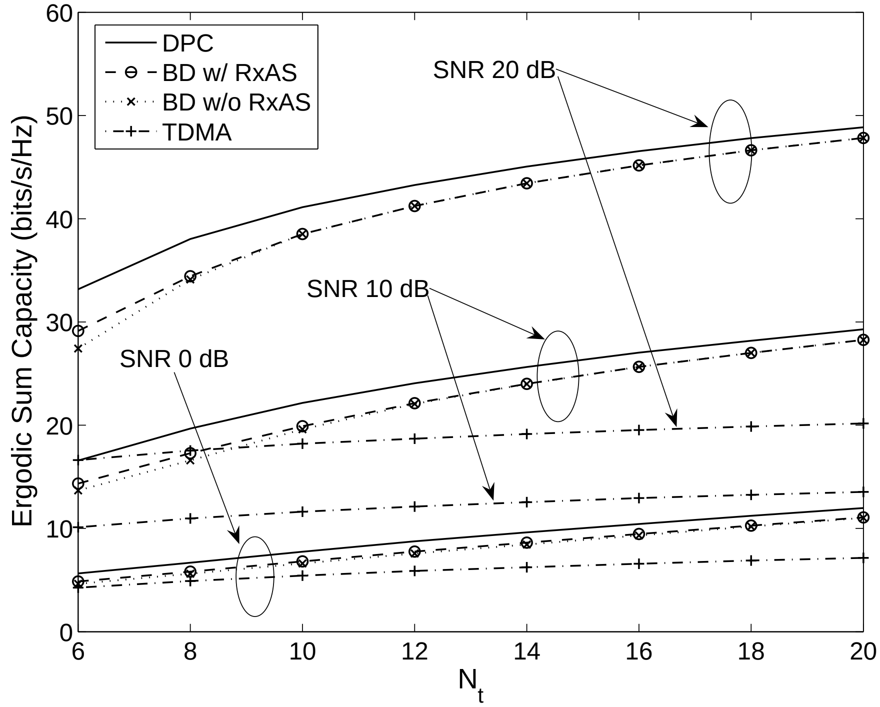

Fig. 3. Ergodic sum capacity of DPC vs. BD in Rayleigh fading channels. $N_r = 2$, $K = 3$.

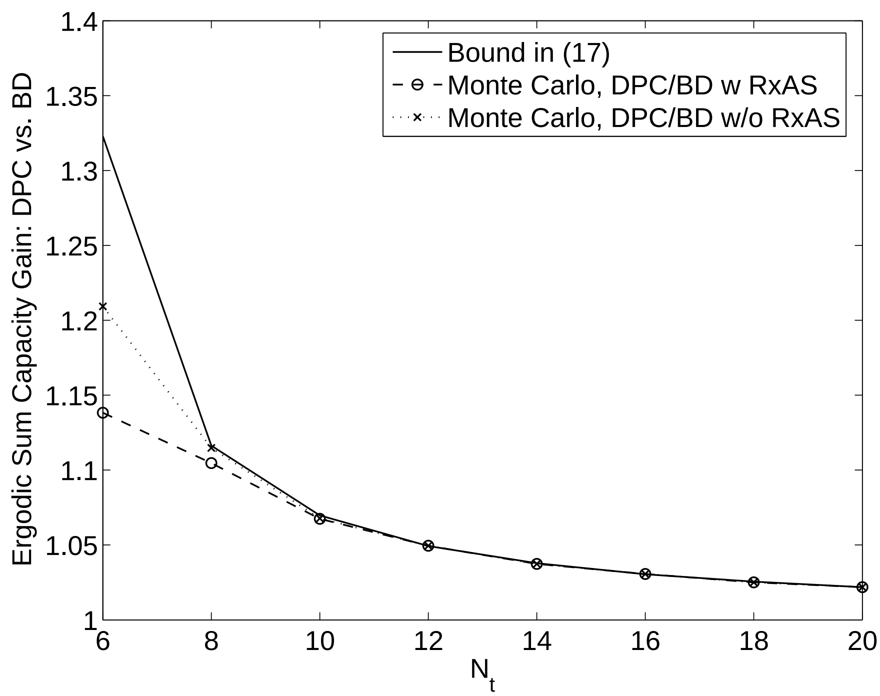

Fig. 4. Ergodic sum capacity gain of DPC over BD in Rayleigh fading channels. $N_r = 2$, $K = 3$, $\mathrm{SNR} = 20$ dB.

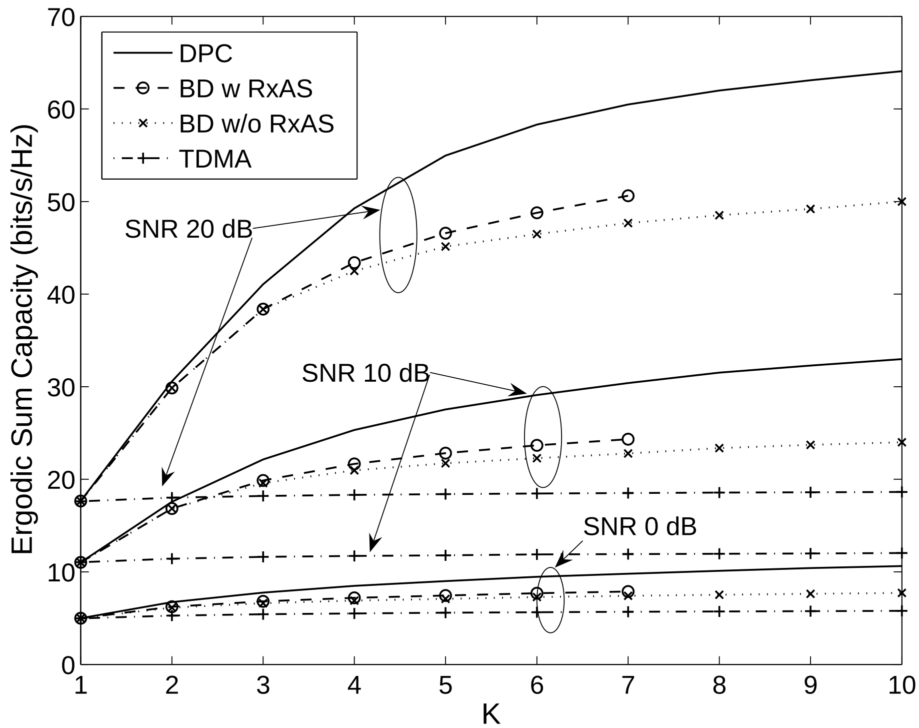

Fig. 5. Ergodic sum capacity of DPC vs. BD in Rayleigh fading channels. $N_t = 10$, $N_r = 2$.

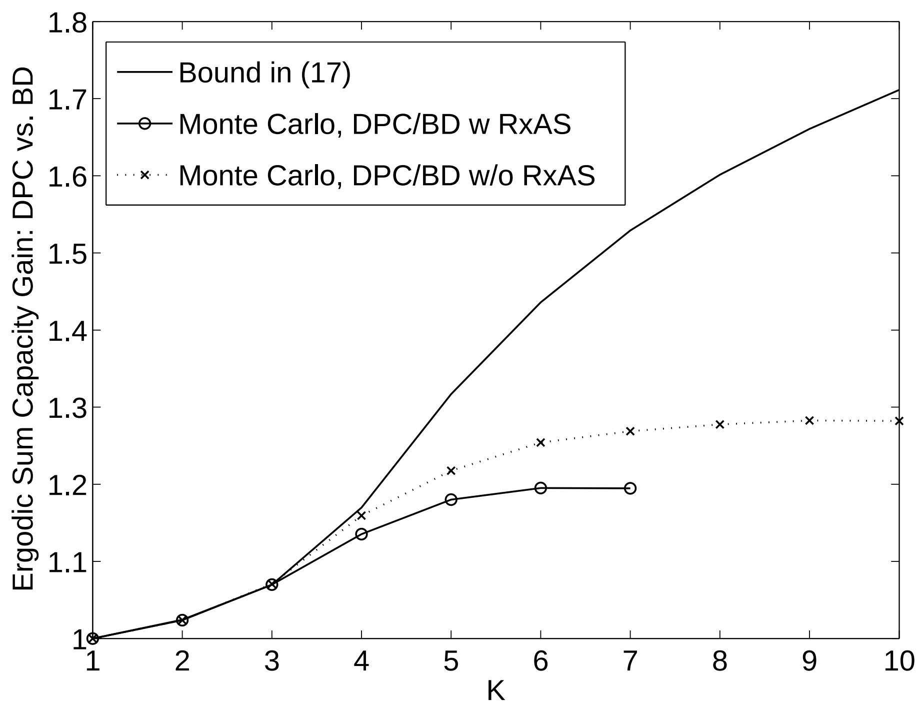

Fig. 6. Ergodic sum capacity gain of DPC over BD in Rayleigh fading channels. $N_t = 10$, $N_r = 2$, $\text{SNR} = 20$ dB.