Using Higher Order Cyclostationarity to Identify Space-Time Block Codes

Marcus R. DeYoung[†,‡], Robert W. Heath, Jr.[†], and Brian L. Evans[†]
{deyoung, rheath, bevans}@ece.utexas.edu

[†] Dept. of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX, 78712
[‡] Zeta Associates, Inc. 10302 Eaton Place, Suite 500, Fairfax, VA 22030

***Abstract*—Research in cognitive radios has renewed interest in tools, such as spectrum estimation and modulation identification, to characterize the radio frequency (RF) environment. The use of multiple antennas for multiple-input multiple-output (MIMO) communications presents a new challenge in detecting and classifying signals. In this paper, we propose a cyclostationarity-based statistical test to detect space-time block codes, focusing on the two transmitter Alamouti space-time block code (STBC). Our test exploits a new characterization of the Alamouti code using fourth order cyclic frequencies. The test requires only a single receive antenna, and does not require any symbol synchronization.**

I. Introduction

Cognitive radios are expected to help fill under-utilized bandwidth by sensing for holes in the spectrum and transmitting in a manner that does not interfere with other signals [1]. Along with spectrum estimation, blind modulation classification– determining the modulation type of signal with unknown parameters– helps present a more complete picture with which an intelligent software defined radio may optimize its transmission scheme [2]. Current research indicates that cognitive radios should be able to operate in both white space (no signal energy present) and gray space (signal energy present) by classifying the signals present and transmitting in a way that avoids interference [3].

Modulation classification algorithms have been extensively studied for interference characterization and surveillance. Prior work focuses on single antenna modulations, such as M-QAM, FSK, and PSK. Emerging wireless systems are using multiple transmit and receive antennas. These multiple-input multiple-output (MIMO) communications systems provide large benefits in increased spectral efficiency, thereby allowing more information to be transmitted in a smaller amount of costly bandwidth [4]. There are many different types of MIMO communication algorithms, such as space-time block coding, spatial multiplexing, and beamforming. Modulation classification methods are needed for each of these methods for cognitive radios to be able to operate in environments with MIMO communication.

In this paper, we propose a classification method for the Alamouti space-time block code [5]. The Alamouti code is a MIMO modulation technique for two transmit antennas, which is used to increase spatial diversity in wireless links. It is the most extensively employed MIMO technology, being already deployed in WCDMA and included in several WiMax profiles. We derive the fourth order cyclic frequency of the Alamouti space-time block code, present a statistical test for its presence with only a single receive antenna in a flat-fading environment, and provide simulation results illustrating the effectiveness of the proposed algorithm.

M. DeYoung and B. Evans were supported in part by an equipment grant from Intel Corporation. R. W. Heath was supported in part by the National Science Foundation under grant CCF-514194.

II. Cyclostationarity

The theory of cyclostationarity has received much attention in the communications world due to its relatively accurate stochastic description of digital and analog communications signals. Cyclostationary properties are often used when little can be assumed about the signal or noise environment, such as with modulation identification [6], spectrum sensing [7], and blind channel identification[8], [9]. A cyclostationary process is one whose statistics vary periodically in time. A process is said to exhibit k-th order cyclostationarity if there exists a nonlinear transform of the k-th order that produces finite strength sinusoid components [10]. Practically speaking, this means there is a k-th order periodicity inherent in the statistics of the signal.

The k-th order time varying moment $m_{kx}(t;\tau)$ of a signal $x(t)$ can be defined as a Fourier expansion of the k-th order cyclic moment $\mathcal{M}_{kx}$ as [10]

$$\begin{aligned} m_{kx}(t;\tau) = &\quad \mathbb{E}\{x(t)x^{(*)}(t+\tau_1)\ldots x^{(*)}(t+\tau_{k-1})\} \\ \mathcal{M}_{kx} = &\quad \lim_{T\to\infty}\textstyle\sum_{t=0}^{T-1} m_{kx}(t;\tau)e^{-j\alpha t} \end{aligned}$$

where $(*)$ denotes an optional conjugation and α is a cycle frequency. Similarly, the time-varying cumulant c_{kx} can be represented by a Fourier series representation of its cyclic covariance $\mathcal{C}_{kx}(\alpha;\tau)$ as

$$c_{kx}(t;\tau) = \sum_{\alpha} C_{kx}(\alpha;\tau)e^{j\alpha t}$$

$$\mathcal{C}_{kx}(\alpha;\tau) = \lim_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1} c_{kx}(t;\tau)e^{-j\alpha t}.$$

Note that the second order cumulant is just the covariance; the second and third order cumulants are both equivalent to their respective moments. Also, note that at $\alpha = 0$, the cyclic moments and cumulants are equivalent to their respective standard (non-cyclic) forms.

III. System Model

Consider an unknown, sampled, continuous-time communications signal that is digitally modulated and employs pulse shaping. Let a digitally modulated signal in a flat fading channel oversampled by a factor of $P = \frac{T}{T_s}$ be represented as

$$y[n] = h[n]e^{j2\pi f_e Pn} \sum_k s[k]g_{tx}[n - kT_s] + v[n]$$

where f_e is the frequency offset, $s[k]$ is a zero-mean, i.i.d. symbol sequence, $g_{tx}[n]$ is the transmitter pulse-shape, $\frac{1}{T_s}$ is the symbol rate, and $v[n]$ is additive white Gaussian noise (AWGN). Using the cyclostationary properties of the digital modulation, we can find the symbol rate by detecting a cyclic frequency at $\alpha_0 = \frac{T}{T_s}$ [11]. We will assume for simplicity that there is no frequency offset ($f_e = 0$), but the results still hold for imperfect carrier recovery [12].

The received signal from a MIMO transmission with two transmitters, one receiver, and no frequency offset may be represented as

$$\begin{aligned} y[n] =& h_1[n] \sum_i s_1[i]g_{tx}[n - iT_s] + \\ & h_2[n] \sum_k s_2[k]g_{tx}[n - kT_s] + v[n] \end{aligned}$$

where $s_1[n]$ and $s_2[n]$ are the source symbols from the two transmitters.

Channel impairments caused by multipath may impact the performance proposed algorithm. Frequency selective fading induces additional repetition into the received signal, which could increase the false alarm rate for signals other than the Alamouti space-time block code. Flat fading presents problems when the scaling is very asymmetric between channels. The main focus of this paper is on the presence and detectability of the unique fourth-order cyclic frequencies. Therefore, we will employ the simplest channel model in the subsequent simulations– unit channel gains in additive white gaussian noise– and leave the detailed analysis of the effects of channel impairments as an avenue for further research.

IV. Statistical Test for Alamouti Code

Following from the concepts of symbol rate detection, we derive a statistical test to distinguish the 2×2 Alamouti STBC from spatial multiplexing or a single transmitter. The Alamouti code is given by [5]

$$\mathcal{X} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}.$$

If we simplify and consider perfect sampling at the rate of one sample per symbol, then the received samples of Alamouti coded symbols may be written as

$$\begin{aligned} y[n] =& h_1[n]s_1[n] + h_2[n]s_2[n] + v[n] \\ y[n+1] =& -h_1[n+1]s_2^*[n] \\ & + h_2[n+1]s_1^*[n] + v[n]. \end{aligned}$$

The cyclic autocorrelation is the same for Alamouti coded data as it is for a single emitter or even multiple independent streams, as in a spatial multiplexing transmission mode. Therefore, the second order cyclostationary techniques can still be used to find the symbol rate, but provide no useful distinction between Alamouti coded data and a single transmitter or multiple independent transmitters. However, the fourth order cyclic cumulant is different with Alamouti coded signals, providing the basis for the following proposed statistical test. A summary of the derivation is provided in the Appendix.

A. Fourth order Cyclic Frequency for Alamouti Code

The fourth order cyclic cumulant is defined as [13]

$$\begin{aligned} &\mathcal{C}_{4x}(\alpha;\tau_1,\tau_2,\tau_3) = \mathcal{M}_{4x}(\alpha;\tau_1,\tau_2,\tau_3) - \\ &\sum_{\beta \in \mathcal{A}_2^m} \mathcal{M}_{2x}(\alpha-\beta;\tau_1)\mathcal{M}_{2x}(\beta;\tau_1)e^{j\beta\tau_2} + \\ &\mathcal{M}_{2x}(\alpha-\beta;\tau_2)\mathcal{M}_{2x}(\beta;\tau_1-\tau_3)e^{j\beta\tau_3} + \\ &\quad \mathcal{M}_{2x}(\alpha-\beta;\tau_3)\mathcal{M}_{2x}(\beta;\tau_2-\tau_1)e^{j\beta\tau_1} \end{aligned} \tag{1}$$

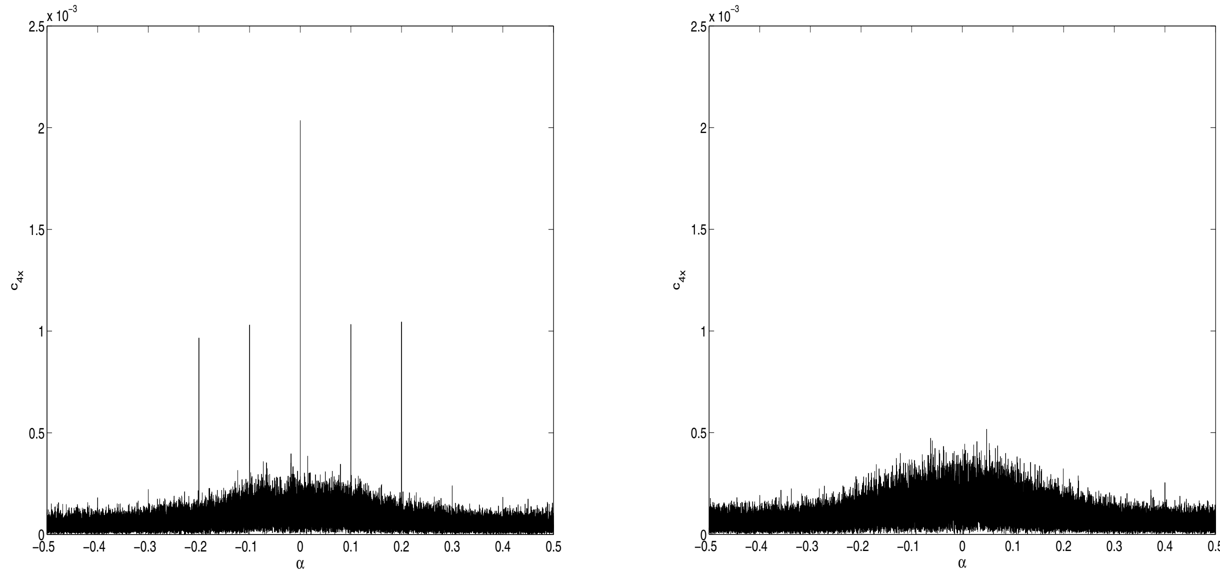
where $\mathcal{A}_2^m$ is the set of second order cyclic frequencies. Fortunately, we can see that $\forall \beta \in \mathcal{A}_2^m$ where $(\alpha - \beta) \notin \mathcal{A}_2^m$, the second order terms go to zero and $\mathcal{C}_{4x}(\alpha;\tau_1,\tau_2,\tau_3) = \mathcal{M}_{4x}(\alpha;\tau_1,\tau_2,\tau_3)$. The derivation for the fourth order cyclic frequency of the Alamouti code is provided in the Appendix. We find a cycle at $\alpha_1 = \pm\frac{1}{2P} = \pm\frac{T}{2T_s}$. We expect there to be a relation at half the symbol rate since the symbols at time $t+1$ are related to those at time t every other set of symbols. Space-time block codes with larger dimensions also have corresponding distinguishing features with even higher order cyclic cumulants. For example, it can be shown that the 4×4 quasi-orthogonal space-time block code defined in [14] has spectral lines at $\pm\frac{T}{4T_s}$ when using 8-th order cyclic cumulants.

B. Statistical Test

Determining the symbol rate will be the first step in the algorithm presented in Section IV. Assuming that the symbol rate is known, we now check the presence of a cyclic frequency at $\alpha_1 = \frac{T}{2T_s}$. Dandawate and Giannakis provide a method to detect the presence of a cyclic frequency in a k-th order cyclic cumulant [13]. The authors define the sample estimator for the fourth order cyclic cumulant as the true cumulant plus error as

$$\hat{C}_{4x}(t;\tau) \triangleq c_{4x}(t;\tau) + \epsilon_{4x}^{(T)}(\alpha;\tau)$$

where $\epsilon_{4x}^{(T)}(\alpha;\tau)$ is the estimation error (goes to zero as $T \to \infty$). With this formulation, we map cycle detection to a binary



(a) $\hat{\mathcal{C}}_{4x}$, 2×2 Alamouti

(b) $\hat{\mathcal{C}}_{4x}$, Two independent streams

Fig. 1. Sample fourth order cyclic cumulants as a function of α with $\tau = (0, P, 0, P)$ and the delayed versions conjugated. The modulation is QPSK, $\frac{\mathcal{E}_s}{N_o} = 5$ dB, $\frac{T}{T_s} = \frac{1}{5}$, $N = 20,000$ symbols.

hypothesis test as shown below

$$\begin{aligned}
\mathbf{H_0} :& \alpha \notin A_4 \forall \{\tau_n\}_{n=1}^{N} \Rightarrow c_{4x}(\hat{t}; \tau) = \epsilon_{4x}^{(T)}(\alpha; \tau) \\
\mathbf{H_1} :& \alpha \in A_4 \text{for some} \{\tau_n\}_{n=1}^{N} \\
& \Rightarrow \hat{c}_{4x}(t; \tau) = c_{4x}(t; \tau) + \epsilon_{4x}^{(T)}(\alpha; \tau)
\end{aligned}$$

where A_4 is the set of fourth-order cyclic frequencies. It is shown that $\lim_{T\to\infty} \sqrt{T}\epsilon_{kx}^{(T)}$ converges in distribution to a multivariate normal distribution with zero mean and asymptotic covariance Σ_{kc}. This leads to the definition of a test statistic for a k-th order cycle $\mathcal{T}_{kc}$ as

$$\mathcal{T}_{kc} \triangleq T\hat{\mathbf{c}}_{kx}(t; \tau)\hat{\Sigma}_{kc}^{-1}\hat{\mathbf{c}}_{kx}(t; \tau)^T$$

where $\hat{\mathbf{c}}_{kx}(t; \tau)$ is a vector of the real and imaginary portions of the sample cyclic cumulant for a fixed set of lags $\{\tau_1, ...\tau_N\}$ and $\hat{\Sigma}_{kc}$ is the sample covariance of this vector computed with the sample spectral correlation functions. It is shown that $\mathcal{T}_{kc}$ asymptotically converges to a χ^2 random variable without the presence of a cycle ($\mathbf{H_0}$), and a Gaussian random variable with higher mean ($\mathbf{H_1}$) if the cycle exists. Therefore, a desired false alarm rate (P_F) or probability of detection (P_D) can be computed for a given cyclic frequency. The reader is referred to [13] for a detailed explanation. The following procedure may be used detect the presence of the fourth order cycles induced by a 2×2 Alamouti block code.

1) Estimate $\alpha_0 = \frac{T}{T_s}$ using the cyclic autocovariance test from [11].
2) Compute the desired probability of detection P_D according to the χ^2 distribution.
3) Compute $\hat{\mathbf{c}}_{4x}(t; \tau...)$ for $\pm\alpha_1 = \pm\frac{1}{2}\alpha_0$ and compare to the threshold.

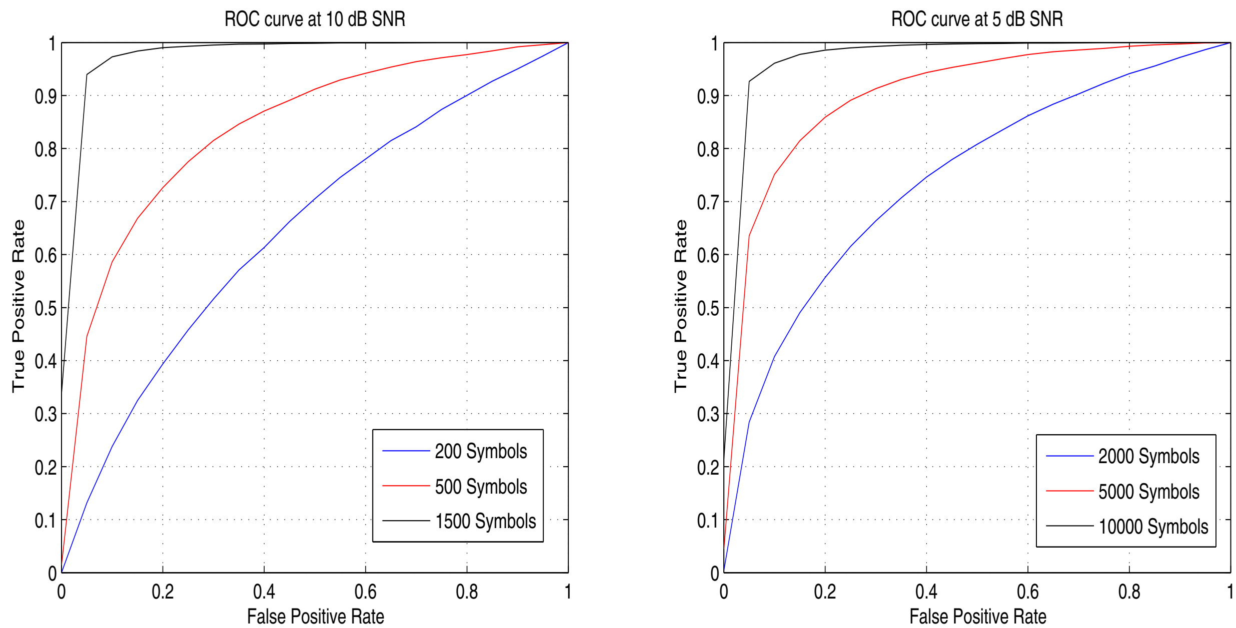
The complexity of the algorithm is fundamentally a function of the sample cyclic cumulant computation, and the computation of the sample covariance. For the Alamouti code, only a fourth order sample cyclic cumulant at a single frequency is required, and may be computed as

$$\hat{C}_{4x}(\alpha; \tau) = \frac{1}{N}\sum_{n=0}^{N-1} x[n]x^*[n+P]x[n]x^*[n+P]e^{-j\alpha t}$$

The complexity of this operation is $\mathcal{O}(N)$. The reader is referred to [13] for details on the computation of the 2×2 sample covariance matrix $\hat{\Sigma}_{4c}$. The elements are linear combinations of the real and imaginary portions of the sample second order cyclic spectrum, which may be estimated by using a smoothed periodogram approach. This operation is $\mathcal{O}(NW_n)$, where W_n is the length of the smoothing window. Thus, overall the test takes $\mathcal{O}(NW_n)$ multiplications.

V. Simulation Results

To demonstrate the effectiveness of our proposed test, we run the algorithm defined in Section IV on both the symbols generated by the Alamouti code and those from a spatial multiplexing scheme. In these simulations, the spatial multiplexing system is two independent streams of data from the two transmitters. We assume a flat-fading environment, and



(a) 10dB Aggregate Received SNR

(b) 5dB Aggregate Received SNR

Fig. 2. Receiver operating characteristic (ROC) curves for the proposed test at 5 and 10 dB (aggregate) SNR. The curves illustrate the tradeoffs between false alarm rates and attainable detection rates in additive white Gaussian noise. The modulation is QPSK with a square root-raised cosine pulse. The data is oversampled by a ratio of $P = 4$.

for simplicity we use unit channel gains. A square root-raised cosine pulse with excess bandwidth of 50% is employed at the transmitter. The data was oversampled by a factor of $P = 4$. A Kaiser window with parameter 10 and length of 101 taps is used to smooth the spectral correlation function. The received signal to noise ratio $\frac{\mathcal{E}_{\mathbf{s}}}{N_0}$ is defined such that $\mathcal{E}_{\mathbf{s}} = \mathcal{E}_{s_1} + \mathcal{E}_{s_2}$.

In Figure 2, we plot simulated receiver operating characteristic (ROC) curves for the proposed test statistic $\mathcal{T}_{4c}$ for the cyclic frequency $\frac{1}{2P}$ for the Alamouti code against two independent streams received with one antenna. From these plots, we illustrate the effects of received SNR and number of symbols used on accurate detection. Figure 2(a) shows curves with 10 dB of SNR received on the single receive antenna using 200, 500, and 1500 symbols. Figure 2(b) shows similar curves when 5 dB SNR is received. In each case, we can get to very high probability of detection with very low probability of error. However, we note that at 10 dB aggregate SNR we can achieve this with 1500 symbols, but at 5 dB aggregate SNR we need approximately 10000 symbols to achieve similar results. Thus, it is clear from these simulations that the detection ability of the statistical test is dependent on both the signal to noise ratio and the number of symbols included in the test. This is one of the main problems inherent with a higher order statistical test. As the order of the cyclostationarity increases, the number of samples required to obtain an accurate estimate increases greatly.

VI. Conclusions

In this paper, we derived unique fourth order cyclic statistics for the Alamouti space-time block code, and proposed a statistical test based on the work of Dandawate and Giannakis. Through simulations, we illustrated the effectiveness of the test with a relatively low number of samples at higher SNR, as well as the drawback of requiring many samples to achieve accurate results in lower SNR environments. Finding lower complexity algorithms that perform well in with low SNR is an area for further research.

It is important to note that this test does not necessarily uniquely identify the Alamouti code. In the case of second order symbol rate detection, it was seen that by weighting the observations we can achieve better results [11]. Any signal that has a fourth order cyclic frequency at $\frac{T}{2T_s}$ will be identified using this test, including possibly higher order (e.g. 4×4) block codes. Therefore, to accurately distinguish between many types of modulation, we must include even higher order statistical tests. As the order increases, the number of samples required rises as well, so it remains an area of further research to find more efficient tests. Another factor that makes realistic processing difficult is fading. Flat fading provides a challenge if the matrix is very unbalanced (e.g. $h_1 = 1, h_2 = 0.05$). Frequency selective fading is even more difficult, since multiple scaled versions of the same symbols will be received and more cycles will be observed. One more avenue for further research is the use of multiple receive antennas. It may be possible to use selection combining to increase the effective SNR and thus increase the accuracy of the test or reduce the number of samples needed.

APPENDIX

A sketch of the derivation for the cyclic frequency at $2P$, where P is the oversampling factor, is now provided. From Section IV, we see that we only need to look at the fourth order moment $m_{4x}(t;\tau_1,\tau_2,\tau_3)$ for the Alamouti code. Let $x[i]$ represent the received symbol, which is $h[0]s_1[i]+h[1]s_2[i]$ at time i and $-h[0]s_2^*[i]+h[1]s_1^*[i]$ at time $i+1$. Note that $x[i]$ and $x[i+1]$ are not (necessarily) independent. However, in the case of a single transmitter or multiple antenna transmission streams sending independent data, $x[i]$ is independent from $x[i+1]\ \forall i$. For notational simplicity, shortening the pulse shape g_{tx} to g, let

$$\mathcal{G}_4^n(\kappa_1,\kappa_2,\kappa_3,\kappa_4) \triangleq g[n-\kappa_1]g^*[n-\kappa_2]g[n-\kappa_3]g^*[n-\kappa_4]. \quad (2)$$

Ignoring the noise term (which has a zero fourth order moment if Gaussian), we write out the fourth order moment with $\tau = P$ as

$$\begin{aligned} m_{4y}(n;\tau) =\ & \mathbb{E}\{y[n]y^*[n+\tau]y[n]y^*[n+\tau]\} \\ =& \mathbb{E}\{(\sum_i x[i]g[n-iP])(\sum_j x^*[j]g^*[n+\tau-jP]) \\ & (\sum_k x[k]g[n-kP])(\sum_l x^*[l]g^*[n+\tau-lP])\} \\ =& \sum_i\sum_j\sum_k\sum_l \mathbb{E}\{x[i]x^*[j]x[k]x^*[l]\}\times \\ & \mathcal{G}_4^n(iP,jP-\tau,kP,lP-\tau) \end{aligned} \quad (3)$$

following from the definition of $\mathcal{G}_4^n$ in (2). We then further expand (3) using the property that, for Alamouti coded data, $x[i]$ is independent from $x[i+k], k\geq 2$

$$\begin{aligned} =& \sum_i \mathbb{E}\{x[i]x^*[i]x[i]x^*[i]\}\mathcal{G}_4^n(iP,iP-\tau,iP,iP-\tau) \\ &+\sum_i \mathbb{E}\{x[i]x^*[i]x[i]x^*[i+1]\} \\ & \times\mathcal{G}_4^n(iP,iP-\tau,iP,(i+1)P-\tau) \\ &+\ldots \\ &+\sum_i \mathbb{E}\{x[i+1]x^*[i+1]x[i+1]x^*[i]\} \\ &\times\mathcal{G}_4^n((i+1)P,(i+1)P-\tau,(i+1)P,iP-\tau) \end{aligned} \quad (4)$$

When $x[n]$ is a single transmitter with has an independent source stream, only the first summation in (4) is nonzero since $x[n]$ is independent from $x[n+1]\forall n$. This implies there is not a fourth order cyclic frequency at $2P$ for a single transmitter, or for a spatial multiplexing stream of independently modulated data from multiple antennas. To illustrate the $2P$ period in the Alamouti coded signal, we see that the expansion, starting with the second term, is

$$\begin{aligned} & m_{4y}(n+kP;\tau) = \sum_i \mathbb{E}\{x[i]x^*[i]x[i]x^*[i+1]\}\times \\ & \mathcal{G}_4^n((i-k)P,(i-k)P-\tau,(i-k)P,(i-k+1)P-\tau) \\ & \quad +\ldots \end{aligned} \quad (5)$$

$$\begin{aligned} &= \sum_{j=(k-i)} \mathbb{E}\{x[j]x^*[j]x[j]x^*[j+1]\} \\ & \quad \mathcal{G}_4^n(jP,jP-\tau,jP,(j+1)P-\tau)+\ldots \\ &= m_{4y}(n;\tau) \text{ if } k=\pm 2z, z\in\mathbb{Z}. \end{aligned} \quad (6)$$

Due to the structure of the Alamouti space-time block code, $x[i]$ and $x[i+1]$ are dependent when i is even and independent when i is odd, assuming zero-indexed data. Note that in equation (5), if k is a multiple of 2, we are guaranteed independence between $x[i]$ and $x[i+k]$. Therefore, we see that $m_{4x}(n+2P;\tau) = m_{4y}(n;\tau)$, which shows that Alamouti coded signals have a fourth order periodicity of length $2P$.

REFERENCES

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, pp. 201–220, Feb. 2005.

[2] O. Dobre, A. Abdi, Y. Bar-Ness, and W. Su, "Cyclostationarity-based blind classification of analog and digital modulations," in *Proc IEEE. Military Communications Conf.*, Oct. 2006, pp. 1–7.

[3] A. Mody, S. Blatt, D. Mills, T. Mcelwain, N. Thammakhoune, J. Niedzwiecki, M. Sherman, C. Myers, and P. Fiore, "Recent advances in cognitive communications," *IEEE Communications Magazine*, vol. 45, pp. 54–61, Oct. 2007.

[4] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311–335, March 1998.

[5] S. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1451–1458, Oct. 1998.

[6] K. Kim, I. A. Akbar, K. K. Bae, J.-S. Um, C. M. Spooner, and J. H. Reed, "Cyclostationary approaches to signal detection and classification in cognitive radio," *IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks*, pp. 212–215, Apr. 2007.

[7] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," *Proc. IEEE Asilomar Conference on Signals, Systems and Computers*, vol. 1, pp. 772– 776, Nov. 2004.

[8] R.W. Heath, Jr. and G.B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems," *IEEE Transactions on Signal Processing*, vol. 47, pp. 848–856, Mar. 1999.

[9] H. Bolcskei and R.W. Heath, Jr. and A.J. Paulraj, "Blind channel identification and equalization in OFDM-based multiantenna systems," *IEEE Transactions on Signal Processing*, vol. 50, pp. 96–109, Jan. 2002.

[10] W. Gardner and C. Spooner, "The cumulant theory of cyclostationary time-series, Part I: Foundation," *IEEE Transactions on Signal Processing*, vol. 42, pp. 3387–3408, Dec. 1994.

[11] L. Mazet and P. Loubaton, "Cyclic correlation based symbol rate estimation," *Proc. IEEE Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 1008–1012, Oct. 1999.

[12] P. Ciblat, P. Loubaton, E. Serpedin, and G. Giannakis, "Asymptotic analysis of blind cyclic correlation-based symbol-rate estimators," *IEEE Transactions on Information Theory*, vol. 48, pp. 1922–1934, July 2002.

[13] A. Dandawate and G. Giannakis, "Statistical tests for presence of cyclostationarity," *IEEE Transactions on Signal Processing*, vol. 42, pp. 2355–2369, Sept. 1994.

[14] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Transactions on Communications*, vol. 49, pp. 1–4, Jan. 2002.