MIMO Receiver Design in the Presence of Radio Frequency Interference

Kapil Gulati*, Aditya Chopra*, Robert W. Heath Jr.*, Brian L. Evans*, Keith R. Tinsley† and Xintian E. Lin†
* The University of Texas at Austin, Austin, Texas 78712
Email: {gulati,chopra,rheath,bevans}@ece.utexas.edu
†Intel Corporation, Santa Clara, CA 95054
Email: {keith.r.tinsley, eddie.x.lin}@intel.com

***Abstract*—Multi-input multi-output (MIMO) receivers have been designed and their communication performance analyzed under the assumption of additive Gaussian noise. Wireless transceivers, however, may be affected by radio frequency interference (RFI) that is well modeled using non-Gaussian impulsive statistics. In this paper, we consider the problem of receiver design for a two transmit, two receive antenna MIMO system in the presence of RFI. First, we show that RFI is well modeled using a bivariate Middleton Class A model and validate the model with measured data. Using this RFI model, we demonstrate that conventional MIMO receivers experience significant degradation in communication performance. Then we derive the maximum likelihood (ML) receiver assuming bivariate Middleton Class A noise. Furthermore, we develop a parameter estimation method for this noise model and propose two sub-optimal ML receivers with reduced computational complexity. Simulations show significant improvement in symbol error rate performance of the proposed techniques over receivers designed assuming additive Gaussian noise.**

I. Introduction

Wireless transceivers deployed on a computation platform (e.g. laptop computer) are greatly affected by the RFI generated by the clocks and buses on the platform itself [1]. RFI is a combination of independent radiation events. It is well modeled using non-Gaussian impulsive statistics. In [2], it was demonstrated that conventional MIMO receivers designed assuming additive Gaussian noise, such as zero-forcing, maximum likelihood and space-time block coded systems, show degradation in communication performance in the presence of a mixture of Gaussian and impulsive noise. Accurate noise modeling is thus needed to design MIMO receivers in the presence of RFI.

Several RFI models are available for single antenna systems. Middleton Class A, B and C noise models [3] are perhaps the most common statistical-physical model. They have been shown to accurately model the non-linear phenomenon governing electromagnetic interference (EMI), while explicitly including a Gaussian component to account for the thermal noise present at the receiver [3]. Extending the Middleton models to multi-antenna systems, though, is not straightforward. Many authors (e.g. [4], [5]) have used a weighted sum of multivariate Gaussian densities to approximate the distribution of a Middleton Class A model for multi-antenna systems. These models, however, are not derived based on physical principles governing EMI. In [6], an extension to the Middleton Class A model was proposed for two antenna systems based on statistical physical principles. In this paper, we restrict our attention to 2×2 MIMO systems using this bivariate Middleton Class A noise model.

MIMO receiver design and space-time block coding in presence of spatially uncorrelated Middleton Class A noise was studied in [7]. The assumption of spatially uncorrelated noise, however, may not be valid since the interfering sources are external to the receiver elements. In [4], an adaptive MIMO receiver was proposed where a mixture of multivariate Gaussian distributions was used to approximate the multivariate Middleton Class A noise. While this work captures the spatial correlation in the noise, it is based on an approximation of the RFI model that is not derived based on physical principles.

In this paper, we consider the problem of receiver design for a 2×2 MIMO system in the presence of RFI modeled as a bivariate Middleton Class A model [6]. A parameter estimator for the RFI model is proposed based on the method of moments. The bivariate Middleton Class A model is demonstrated to model RFI better than the Gaussian model using measured RFI data acquired from a laptop embedded wireless transceiver. We demonstrate a degradation in communication performance of conventional receivers in presence of RFI to motivate the problem of receiver design. A maximum likelihood (ML) receiver is then derived that assumes the knowledge of the model parameters. Further, we propose two suboptimal approximations to the optimal ML receiver with lower computational complexity and without prior knowledge of the model parameters. Simulations show that the proposed receivers provide a significant improvement in symbol error rate performance over conventional receivers.

II. System Model

Consider a wireless communication system where both the transmitter and receiver are equipped with two antennas. The discrete-time baseband MIMO channel model can be expressed as

$$\mathbf{Y} = \sqrt{\frac{E_s}{2}}\mathbf{HS} + \mathbf{N} \tag{1}$$

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where $\mathbf{Y}$ is the $2 \times T$ matrix of received signals, T being the length of the transmitted data block, $\mathbf{H}$ is an 2×2 channel matrix with independent and identical distributed (*i.i.d.*) complex Gaussian entries with mean zero and unit variance, E_s is the total transmit energy, $\mathbf{S}$ is the $2 \times T$ transmitted data block and $\mathbf{N}$ is the $2 \times T$ matrix representing additive noise.

The additive noise is modeled using the bivariate Middleton Class A model, which represents narrowband RFI (i.e. noise spectrum bandwidth is less than the receiver bandwidth) and explicitly includes a Gaussian component. Since we assume that RFI is a sum of independent transmission events [3], the noise observations will be temporally independent and identically distributed (*i.i.d.*). Further, the real and imaginary components of the noise ($\mathbf{N} = \mathbf{n}_R + j\mathbf{n}_I$) are assumed to be *i.i.d.* with each having the joint spatial distribution [6]

$$f_{\mathbf{n}}(n_1, n_2) = \frac{e^{-A}}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2}} + \frac{\left(1 - e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2}} \tag{2}$$

where $|\cdot|$ denotes the determinant function and for $m = 0, 1$,

$$\mathbf{K}_m = \begin{bmatrix} (c_m)^2 & \kappa c_m \hat{c}_m \\ \kappa c_m \hat{c}_m & (\hat{c}_m)^2 \end{bmatrix} \tag{3}$$

$$(c_m)^2 = \frac{\frac{m}{A} + \Gamma_1}{1 + \Gamma_1}, \ (\hat{c}_m)^2 = \frac{\frac{m}{A} + \Gamma_2}{1 + \Gamma_2}. \tag{4}$$

This model is uniquely determined by the following four parameters:

- A is the overlap index. It is the product of the average number of emission events impinging on the receiver per second and mean duration of a typical interfering source emission, $A \in [10^{-2}; 1]$ in general [8].
- Γ_1, Γ_2 are the ratio of the Gaussian to the non-Gaussian component intensity at each antenna, $\Gamma_1, \Gamma_2 \in [10^{-6}, 1]$ in general [8].
- κ is the correlation coefficient between the observations at the two antennas.

III. Estimation of Noise Model Parameters

In this section, we propose an estimator for the bivariate Middleton Class A model parameters based on the method of moments. For the joint probability density function given in (2), the moment generating function can be expressed as

$$M_{\mathbf{n}}(\mathbf{t}) = e^{-A} e^{\frac{\mathbf{t}^T\mathbf{K}_0\mathbf{t}}{2}} + \left(1 - e^{-A}\right) e^{\frac{\mathbf{t}^T\mathbf{K}_1\mathbf{t}}{2}} \tag{5}$$

where, $\mathbf{t} = [t_1 \ \ t_2]^T$. The following moments can be derived

$$m_2^{(1)} = \mathbb{E}(n_1^2) = e^{-A}(c_0)^2 + \left(1 - e^{-A}\right)(c_1)^2 \tag{6}$$

$$m_4^{(1)} = \mathbb{E}(n_1^4) = 3e^{-A}(c_0)^4 + 3\left(1 - e^{-A}\right)(c_1)^4 \tag{7}$$

$$m_6^{(1)} = \mathbb{E}(n_1^6) = 15e^{-A}(c_0)^6 + 15\left(1 - e^{-A}\right)(c_1)^6 \tag{8}$$

$$m_2^{(2)} = \mathbb{E}(n_2^2) = e^{-A}(\hat{c}_0)^2 + \left(1 - e^{-A}\right)(\hat{c}_1)^2 \tag{9}$$

$$m_4^{(2)} = \mathbb{E}(n_2^4) = 3e^{-A}(\hat{c}_0)^4 + 3\left(1 - e^{-A}\right)(\hat{c}_1)^4 \tag{10}$$

$$m_6^{(2)} = \mathbb{E}(n_2^6) = 15e^{-A}(\hat{c}_0)^6 + 15\left(1 - e^{-A}\right)(\hat{c}_1)^6 \tag{11}$$

$$m_1^{(1,2)} = \mathbb{E}(n_1 n_2) = \kappa e^{-A} c_0 \hat{c}_0 + \kappa\left(1 - e^{-A}\right) c_1 \hat{c}_1 \tag{12}$$

where $\mathbb{E}(.)$ represents the expected value. Estimates for c_0, c_1 and $\hat{c}_0, \hat{c}_1$ can then be obtained using (6), (7), (8) and (9), (10), (11) respectively (see Appendix). Using these estimates in (4), parameter estimators for A, Γ_1, Γ_2 and κ can be expressed as

$$\widetilde{A} = \frac{1}{2}\left[\frac{1 - (\widetilde{c_0})^2}{(\widetilde{c_1})^2 - (\widetilde{c_0})^2} + \frac{1 - \left(\widetilde{\hat{c}_0}\right)^2}{\left(\widetilde{\hat{c}_1}\right)^2 - \left(\widetilde{\hat{c}_0}\right)^2}\right] \tag{13}$$

$$\widetilde{\Gamma_1} = \frac{(\widetilde{c_0})^2}{1 - (\widetilde{c_0})^2}, \quad \widetilde{\Gamma_2} = \frac{\left(\widetilde{\hat{c}_0}\right)^2}{1 - \left(\widetilde{\hat{c}_0}\right)^2} \tag{14}$$

$$\widetilde{\kappa} = \frac{m_1^{(1,2)}\left((\widetilde{c_0})^2 - (\widetilde{c_1})^2\right)}{\widetilde{c_0}\widetilde{\hat{c}_0}\left(m_2^{(1)} - (\widetilde{c_1})^2\right) - \widetilde{c_1}\widetilde{\hat{c}_1}\left(m_2^{(1)} - (\widetilde{c_0})^2\right)} \tag{15}$$

where $(\widetilde{\cdot})$ denotes the estimated value of the corresponding parameters.

IV. Measured Data Fitting

Two antenna measurements of RFI deployed on a laptop computer were obtained from Intel Corporation representing actual interfering emissions. A set of $50,000$ baseband noise samples were used to generate a sample probability density function (pdf) to compare with the bivariate Middleton Class A model and the Gaussian model. The noise was assumed to be broadband, i.e. noise bandwidth greater than the receiver bandwidth, and the the radio was used to listen to the platform noise only (no data communication was being carried out). No further information was provided. We do not expect the pdf of the measured data to perfectly match the bivariate Middleton Class A density function because of two primary reasons: a) the measured RFI is broadband while the Middleton Class A models narrowband interference, and b) the measured noise exhibited occasional clipping due to the saturation of the radio frequency front-end during large impulses. We expect, however, the bivariate Middleton Class A model to approximate the measured data distribution considerably better than the Gaussian model.

We used the parameter estimators proposed in Section III for the bivariate Middleton Class A model. The Kullback-Leibler (KL) divergence [9] is used to quantify the closeness of two probability distribution functions, where a KL divergence of zero indicates an exact match of the densities. The empirical probability density of the measured data was estimated using kernel smoothing density estimators [10]. The KL divergence of the empirical density was computed as 1.004 from the estimated bivariate Middleton Class A density and 1.682 from an equi-power Gaussian density. Hence, the measured RFI data was modeled better by the bivariate Middleton Class A model as compared to the Gaussian model. Fig. 1 compares the marginal densities at one antenna and provides a visual justification for the same.

V. Receiver Design

In this section, we consider receiver design for a 2×2 MIMO system in the presence of RFI modeled as a bivariate

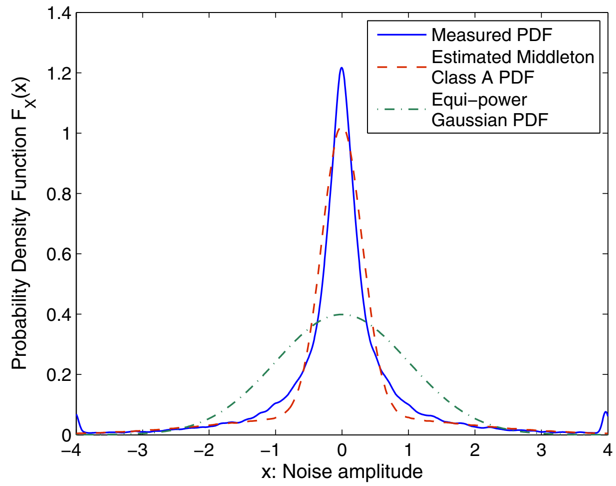

Fig. 1: Marginal PDF of measured data compared with the marginals of the estimated bivariate Middleton Class A PDF (estimated parameters $\widetilde{A} = 0.313, \widetilde{\Gamma_1} = 0.105, \widetilde{\Gamma_2} = 0.101, \widetilde{\kappa} = -0.085$) and an equi-power Gaussian PDF at one antenna.

Middleton Class A process. The transmitted signal is generated from a codebook $\mathcal{C}$ and it is assumed that all codewords have equal probability of transmission. Since all the transmitted signal vectors are equally likely, the maximum likelihood (ML) receiver is a bit error rate optimal detector for this system. We derive the ML receiver and analyze its symbol error rate performance through simulations. Further, two additional detection schemes are proposed, which approximate the likelihood function for the Class A distribution. This approximation is required to reduce the computational complexity of the detector and yields lower complexity sub-optimal receivers.

A. Maximum Likelihood Receiver

The ML receiver uses the following detection rule

$$\hat{\mathbf{c}}_{ML} = \arg \underbrace{\max}_{\mathbf{s}\in\mathcal{C}} \{L(\mathbf{s}|\mathbf{y})\}. \tag{16}$$

Using the joint probability density function $f_{\mathbf{n}}(n_1, n_2)$ of the Middleton noise model given in (2), the ML estimate can be expressed as the maximizing argument of the likelihood function $L(\mathbf{s}|\mathbf{y})$, where

$$L(\mathbf{s}|\mathbf{y}) = \left\{ \frac{e^{-A}}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2}} + \frac{\left(1-e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}} e^{\frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2}} \right\} \tag{17}$$

where $\mathbf{n} = \mathbf{y} - \mathbf{H}\mathbf{s}$.

Note that the Middleton noise model is defined for only real sample observations. For a communication system using complex signals, we assume that the real and the imaginary components of the signal are independent and the noise vectors are *i.i.d.*. The optimal and sub-optimal receivers discussed in this section can then be easily extended to a passband transmission system and the relative communication performance and computational complexity between different receivers will remain the same.

The computational complexity of the ML receiver for Middleton Class A noise is higher than its Gaussian counterpart since the likelihood function given by (17) can no longer be expressed as just a fraction of the minimum distance. Furthermore, the optimum receiver requires perfect knowledge of the noise model parameters defined in Section II.

B. Sub-optimal Receivers

For the ML receiver derived in Section V-A, the likelihood function, (17), to be maximized was found to be a sum of two exponential functions. This is unlike ML detection in Gaussian noise, where the likelihood function consists of only one exponential function and hence the log-likelihood function can be expressed as the distance between the received signal and candidate points in the constellation. The distance metric, however, cannot be attained for ML detection in presence of bivariate Middleton Class A noise, thereby resulting in increased complexity of evaluating the exponential functions for all possible candidate solutions. Thus, as a first step, the goal of a lower complexity receiver is to approximate the cost function so that the computational complexity of the decoder is similar to that of the ML decoder for additive Gaussian noise.

Taking the logarithm of the likelihood function in (17),

$$l(\mathbf{s}|\mathbf{y}) = \frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2} + \ln\left(\Lambda_0\right) + \ln\left(1 + \frac{\Lambda_1 e^{\frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2}}}{\Lambda_0 e^{\frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2}}}\right) \tag{18}$$

$$= \frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2} + \ln\left(\Lambda_0\right) + \phi\left(\frac{-\mathbf{n}^T\left(\mathbf{K}_1^{-1} - \mathbf{K}_0^{-1}\right)\mathbf{n}}{2} + \ln\left(\frac{\Lambda_1}{\Lambda_0}\right)\right) \tag{19}$$

where the constants Λ_0, Λ_1 and the function $\phi(z)$ are defined as

$$\Lambda_0 = \frac{\left(e^{-A}\right)}{2\pi|\mathbf{K}_0|^{\frac{1}{2}}}, \quad \Lambda_1 = \frac{\left(1-e^{-A}\right)}{2\pi|\mathbf{K}_1|^{\frac{1}{2}}} \tag{20}$$

$$\phi(z) = \ln\left(1 + e^z\right) \quad \forall z \in \mathcal{R}. \tag{21}$$

The suboptimal receivers proposed are based on approximating the function $\phi(z)$ using a piecewise linear function. This approximation allows the receiver to evaluate the log-likelihood function for each candidate point using computations similar to that of a Gaussian ML decoder. Since the log-likelihood function is being approximated, the receiver is no longer optimal, although simulations show that both suboptimal receivers show communication performance close to the optimal receiver. Two approximations to the function $\phi(z)$, as shown in Fig. 2, are described as follows.

1) Two-Piece Linear Approximation: The two piece linear approximation of $\phi(z)$ is given by (22). Using this approximation, the log-likelihood function evaluation in (19) reduces to the detection criterion given by (23).

$$\phi_1(z) = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases} \tag{22}$$

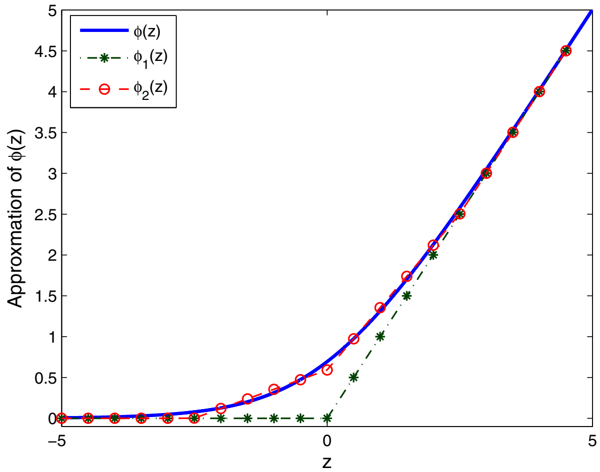

Fig. 2: Two-piece ($\phi_1(z)$) and four-piece ($\phi_2(z)$) linear approximations of $\phi(z) = \ln(1 + e^z)$.

This detection rule is equivalent to evaluating the arguments of both the exponential terms and choosing the log-likelihood approximation as the argument with the lower magnitude.

$$l(\mathbf{s}|\mathbf{y}) \approx \begin{cases} \frac{-\mathbf{n}^T\mathbf{K}_0^{-1}\mathbf{n}}{2} + \ln(\Lambda_0) & \frac{\mathbf{n}^T\left(\mathbf{K}_1^{-1}-\mathbf{K}_0^{-1}\right)\mathbf{n}}{2} > \ln\left(\frac{\Lambda_1}{\Lambda_0}\right) \\ \frac{-\mathbf{n}^T\mathbf{K}_1^{-1}\mathbf{n}}{2} + \ln(\Lambda_1) & \frac{\mathbf{n}^T\left(\mathbf{K}_1^{-1}-\mathbf{K}_0^{-1}\right)\mathbf{n}}{2} < \ln\left(\frac{\Lambda_1}{\Lambda_0}\right) \end{cases} \quad (23)$$

2) Four-Piece Linear Approximation: We can also approximate $\phi(z)$ as the concatenation of four lines with different slopes, constrained by the fact that the overall approximating function is continuous. The approximating function $\phi_2(z)$ is given in (24) where $\alpha = 0.236$ and $\gamma = 2.507$ are chosen so as to minimize the mean square error $\int_{-\infty}^{\infty} \left|\hat{\phi}_2(z) - \phi(z)\right|^2 dz$. Replacing $\phi(.)$ with $\phi_2(.)$ in (19) yields the detection rule for the second reduced complexity sub-optimal rule.

$$\phi_2(z) = \begin{cases} 0 & \text{if } z < -\gamma \\ \alpha z + \alpha\gamma & \text{if } -\gamma \le z < 0 \\ (1-\alpha)z + \alpha\gamma & \text{if } 0 \le z < \gamma \\ z & \text{if } z \ge \gamma \end{cases} \quad (24)$$

The optimal ML detector assumes perfect knowledge of the noise parameters at the receiver. The sub-optimal receivers, however, do not assume *à priori* knowledge of the noise model parameters and were implemented to use the estimated parameters using the methods developed in Section III.

C. Complexity Analysis

In this subsection, we compare the complexity of the optimal and proposed suboptimal receivers in the presence of bivariate Middleton Class A noise. Computational complexity is quantified in terms of the number of quadratic form evaluations $\left(\mathbf{x}^T\mathbf{Q}\mathbf{x}\right)$, exponential function evaluations (e^x) and comparisons required to calculate the likelihood function. Table I shows the complexity costs of the receivers for decoding a M-QAM modulated signal.

TABLE I: Complexity analysis for decoding a M-QAM modulated signal in the presence of bivariate Middleton Class A noise.

Receiver	Quadratic Forms	Exponentials	Comparisons
Optimal ML	$2M^2$	$2M^2$	0
Suboptimal ML (Four-piece)	$2M^2$	0	$3M^2$
Suboptimal ML (Two-piece)	$2M^2$	0	M^2

The proposed sub-optimal ML receivers remove exponential function evaluations at the expense of adding comparisons to the optimal ML receiver scheme. Exponential function evaluations have inherently higher cost than comparison operations in terms of computational complexity (for truncated Taylor series implementation) or memory requirements (for lookup table implementations).

VI. Results

The 2×2 MIMO system discussed in Section II was simulated to observe the performance of MIMO receivers in presence of RFI. The symbol error rate performance of the optimal and the proposed sub-optimal maximum likelihood (ML) receivers was compared to the ML detector for Gaussian Noise. The symbol error rates were calculated as an average for 10^8 transmitted symbols. 4-QAM modulation was used for spatial multiplexing while 16-QAM modulation was used for Alamouti transmission strategy. Parameters of the bivariate Middleton Class A noise used in simulations were $A = 0.1$, $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$ and $\kappa = 0.4$.

A. Performance Analysis of Conventional MIMO Receivers in Presence of RFI

Fig. 3 shows a communication performance degradation in typical MIMO receivers, in the presence of additive bivariate Middleton Class A noise. Energy of the Middleton Class A model is a sum of the Gaussian and impulsive components of the noise. At low signal-to-noise ratio (SNR), the communication performance is sensitive to the Gaussian component of the Middleton Class A noise, which has lower energy than an equi-powered Gaussian noise. Hence, typical MIMO receivers exhibit better communication performance in RFI compared to Gaussian noise at low SNR. At high SNR, the receivers become sensitive to the impulsive component of RFI, which is absent in a Gaussian noise environment, thereby causing severe degradation of communication performance in RFI.

B. Performance of Proposed ML Detectors in Presence of RFI

A 2×2 MIMO system operating in spatial multiplexing mode was simulated to compare the performance of the proposed and traditional receivers. Fig. 4 shows the symbol error performance of the ML receiver for Gaussian distributed noise and proposed ML receivers for bivariate Middleton Class A noise. The communication performance of the proposed receivers is significantly better than the conventional ML receiver designed for Gaussian noise. For a vector symbol error rate of 10^{-2}, the proposed receivers show approximately 8 dB improvement in the signal-to-noise ratio (SNR). Table

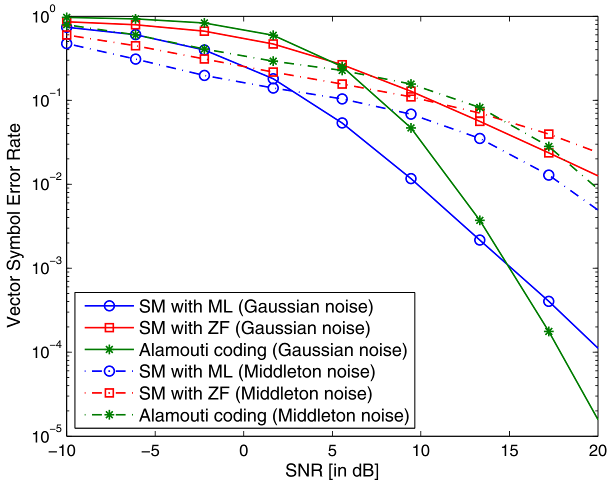

Fig. 3: Communication performance of conventional MIMO receivers in presence of Gaussian and bivariate Middleton Class A noise with parameters $A = 0.1$, $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$ and $\kappa = 0.4$. (Here SM and ZF stand for Spatial Multiplexing and Zero Forcing respectively).

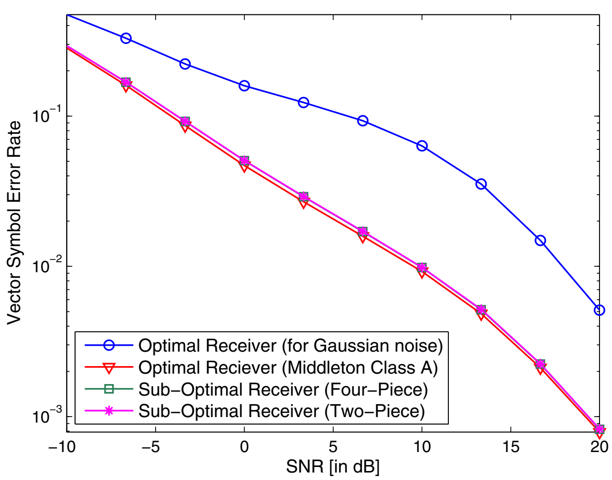

Fig. 4: Comparison of communication performance for optimal and sub-optimal receivers in presence of Middleton Class A noise with parameters $A = 0.1$, $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$ and $\kappa = 0.4$.

TABLE II: Improvement in communication performance of the proposed receivers over conventional Gaussian maximum likelihood receiver at a vector symbol error rate of 10^{-2} in the presence of bivariate Middleton Class A noise with parameters $\Gamma_1 = 0.01$, $\Gamma_2 = 0.1$, $\kappa = 0.4$ and different values of A.

A	Noise Characteristic	Improvement
0.01	Highly impulsive	≈ 14 dB
0.1	Moderately impulsive	≈ 8 dB
1	Near Gaussian	≈ 0.5 dB

II lists the improvement in SNR for different impulsive characteristics of the noise, achieved by varying the parameter A of the bivariate Middleton Class A model. Furthermore, the sub-optimal receivers perform nearly as well as the optimal receiver while operating with lower computational complexity and using estimated model parameters.

Appendix

For $n = 0, 1$ define

$$a^{(n)} = 15\left[3\left(m_2^{(n)}\right)^2 - m_4^{(n)}\right] \tag{25}$$

$$b^{(n)} = 3\left[m_6^{(n)} - m_2^{(n)}m_4^{(n)}\right] \tag{26}$$

$$c^{(n)} = 5\left(m_4^{(n)}\right)^2 - 3m_2^{(n)}m_6^{(n)} \tag{27}$$

Then, the following estimates can be obtained

$$(\widetilde{c_0})^2 = \frac{-b^{(1)} + \sqrt{\left(b^{(1)}\right)^2 - 4a^{(1)}c^{(1)}}}{2a^{(1)}} \tag{28}$$

$$\left(\widetilde{\hat{c}_0}\right)^2 = \frac{-b^{(2)} + \sqrt{\left(b^{(2)}\right)^2 - 4a^{(2)}c^{(2)}}}{2a^{(2)}} \tag{29}$$

$$(\widetilde{c_1})^2 = \frac{m_4^{(1)} - 3m_2^{(1)}(\widetilde{c_0})^2}{3\left(m_2^{(1)} - (\widetilde{c_0})^2\right)}, \quad \left(\widetilde{\hat{c}_1}\right)^2 = \frac{m_4^{(2)} - 3m_2^{(2)}\left(\widetilde{\hat{c}_0}\right)^2}{3\left(m_2^{(2)} - \left(\widetilde{\hat{c}_0}\right)^2\right)} \tag{30}$$

where $\widetilde{(\cdot)}$ denotes the estimated value of the corresponding parameter.

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