Low Complexity EM-based Decoding for OFDM Systems with Impulsive Noise

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***Abstract*—Modern OFDM systems such as cellular LTE and powerline communications experience additive impulsive noise emitted from their environment. OFDM modulation has been shown to provide resilience to impulsive noise due to its code diversity. However, typical OFDM receivers designed under the Gaussian noise assumption will lead to suboptimal performance due to the dependence in noise statistics across subcarriers resulting from the FFT operation. As a result, optimal detection of OFDM symbols becomes prohibitive due to its exponential complexity. We consider the design of a practical class of OFDM receivers that are constrained to perform independent detection on each subcarrier. In this paper, we propose an EM based low-complexity iterative decoding algorithm for OFDM systems in impulsive noise environments that preserves the independent decoding across subcarriers. Then we validate its performance under typical impulsive noise conditions based on noise traces collected from wireless and powerline platforms. Our proposed method achieves a gain between $2-7$dB over the conventional OFDM receiver depending on the SNR range.**

I. Introduction

Communication transceivers in powerline communication (PLC) and wireless networks suffer from uncoordinated interference from other users and non-communication sources such as microwave ovens and switching power supplies. The typical additive white Gaussian noise (AWGN) assumption is inadequate for capturing the statistical properties of such interference and leads to suboptimal receivers. Different statistical models for impulsive noise have been proposed in order to help in designing receivers that mitigate the impulsive noise. Middleton class-A model, Symmetric Alpha Stable, and the more general Gaussian mixture model have been shown to accurately model interference in uncoordinated wireless networks [1], [2], [3], [4] and PLC networks [5].

Orthogonal Frequency Division Multiplexing (OFDM) modulation is used to combat multipath and increase the data rates. Due to these advantages, OFDM modulation has been adopted in many modern wireless communication standards, such as IEEE802.11n and LTE, and recent PLC standards, such as PRIME and G3. In addition, [6] explored the impulse resilient properties of OFDM systems by viewing them as time-codes. The design of OFDM receiver in impulsive noise can be classified into two subcategories according to the assumed impulsive noise structure. First, non-parametric models do not assume any particular model for impulsive noise and treat it as a sparse vector. A compressed-sensing based non-parametric approach has been proposed in [7], while a more general Sparse Bayesian Learning approach was given in [8]. On the other hand, parametric models assume a given impulsive noise model and design the receiver based on its statistical properties. In [6], a non-iterative time-domain MMSE-based receiver was proposed to increase performance under the constraint of preserving independent decoding across subcarriers. In this method an estimate of the time-domain signal is computed, followed by the conventional OFDM receiver. Iterative receivers have been proposed in [9], [6], [10], [11]. In particular, in [9], [10] an iterative scheme with a threshold limiter applied at each iteration is used to improve performance of OFDM under impulsive noise. However, the threshold selection is ad-hoc and poses difficulty for practical systems. In addition, [10] and [11] propose an iterative scheme based on the turbo-decoding principle that comes close to the derived PEP bound. However, this scheme requires a complex receiver that may not be implementable on current computational platforms. It was shown in [8], that parametric approaches outperform non-parametric models if the assumed model provides a good match for the underlying impulsive noise.

As a result, in this paper we propose a parametric EM-based low-complexity iterative decoding algorithm for OFDM systems in impulsive noise environments that preserves independent decoding across subcarriers. We also fit the parametric models to collected data from wireless and PLC receivers and use the obtained parameters in out simulation results to validate our proposed method.

II. Impulsive Noise Enviroments

Various statistical physical models have been proposed to capture the non-Gaussian nature of the interference in uncoordinated wireless and PLC networks [1], [4], [2], [5]. This section will describe the used parametric models in more details and then provide typical values for the parameters encountered in practice.

A. *Statistical-Physical Noise Models*

The main statistical-physical models for modeling impulsive noise are the Gaussian mixture, the Middleton's Class A, and the Symmetric Alpha-Stable. A random variable W has a

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TABLE I
PARAMETERS OF FITTED 2-TERM GAUSSIAN

Dataset Number	π_1	π_2	$\frac{\sigma_2^2}{\sigma_1^2}$
Wireless Environment			
1	0.75	0.25	13.7
2	0.56	0.44	23.4
Powerline Environment			
1	0.89	0.11	198
2	0.87	0.13	140

Gaussian mixture distribution if its probability density function (pdf) is a weighted sum of a set of Gaussian distributions[1] $\Omega = \left\{\mathcal{N}_c\left(\mu_i, \sigma_i^2\right)\right\}_{i=1}^K$ given by

$$f(w) = \sum_{k=1}^{K} \pi_k \cdot \mathcal{N}_c(w; 0, \sigma_k^2), \tag{1}$$

where $\mathcal{N}_c(w; 0, \sigma_k^2)$ denotes the complex Gaussian distribution with zero mean and variance σ_k^2, and π_k is the mixing probability of the k-th Gaussian component. The mixing vector $\boldsymbol{\pi} = [\pi_1 \cdots \pi_K]$ can be interpreted as a discrete *pdf* of a latent random variable S that controls which component of Ω is drawn to generate a sample of W. In other words, given that $S = s$, W is a Gaussian random variable with variance σ_s^2 i.e. $f\left(w|s\right) = \mathcal{N}_c\left(w; 0, \sigma_s^2\right)$. In this paper, we refer to the knowledge of S as "Noise State Information" (NSI).

Middleton's Class A model, characterized by the parameters (A, Γ), is a special case of a Gaussian mixture distribution with $\pi_k = e^{-A}\frac{A^(k-1)}{(k-1)!}$ and $\sigma_k^2 = \frac{(k-1)/A+\Gamma}{1+\Gamma}$ as $K \to \infty$. In practice, only the first few significant terms are retained [3]. On the other hand, a Symmetric Alpha-Stable (SαS) distribution does not have a closed form pdf and is characterized by its characteristic function $\Phi(\omega) = e^{j\delta\omega - \gamma|\omega|^\alpha}$, where (α, δ, γ) are defining parameters. For receiver design, a $S\alpha S$ RV can be approximated by a Gaussian mixture RV [1]. As a result, we restrict ourself to Gaussian mixture distributions and particularly to 2-term Gaussian mixtures, also known as $\epsilon-$contaminated Gaussian, due to their tractability for parameter estimation.

B. Impulsive Noise in Wireless and PLC receivers

The statistical fit of the interference to Gaussian mixture models has been analyzed for wireless receivers in [1], [2] and for PLC systems in [5]. In this section, we fit various collected datasets from wireless transceivers and PLC systems to the tractable 2-term Gaussian mixture model. The wireless dataset was provided by Intel Corporation and was collected by connecting a scope to a laptop receiver. The PLC dataset was collected from the 40-100kHz range in a residential complex in Austin, TX. Table I shows some typical values for the estimated model parameters that will be used to set a practical range for simulations in Section VIII.

[1]In general, the first Gaussian component $\mathcal{N}_c\left(0, \sigma_1^2\right)$ represents the background thermal noise.

III. SYSTEM MODEL

We consider the simplified OFDM system, described by the discrete-time baseband model

$$\mathbf{y} = \underbrace{\sqrt{\rho}\mathbf{F}^*\mathbf{x}}_{\mathbf{u}} + \mathbf{w} \tag{2}$$

where $\mathbf{y} = [y_1 \cdots y_N]^T$ is the received signal with N being the FFT block length (number of subcarriers), ρ is signal power, $\mathbf{X}$ is the $N \times 1$ frequency-domain transmitted symbols, and $\mathbf{w} = [w_1 \cdots w_N]^T$ is the $N \times 1$ additive noise vector. The FFT operation is represented by the $N \times N$ matrix $\mathbf{F}$ where $(\cdot)^*$ represents the Hermitian operator. The $N \times 1$ vector $\mathbf{u}$ is the time-domain transmitted OFDM signal. This simplified model serves as a fair comparison with single carrier systems and provides insight into OFDM's impulse resilience properties. The noise is assumed to be temporally independent and identically distributed (*i.i.d.*) Gaussian mixture random vector. Thus, the probability density function (*pdf*) of $\mathbf{w}$ is the product form of (1) given by

$$f\left(\mathbf{w}\right) = \prod_{i=1}^{N}\sum_{j=1}^{K} \pi_j \mathcal{N}_c\left(w_i; 0, \sigma_j^2\right). \tag{3}$$

IV. OPTIMAL OFDM DETECTION IN IMPULSIVE NOISE

The problem of detecting an OFDM symbol for the system model given in (2) can be formulated as

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} f(\mathbf{y}|\mathbf{x}) = \arg\max_{\mathbf{x}} f_{\mathbf{w}}\left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right). \tag{4}$$

In (4), each of the vector components of $\mathbf{w}$ depends on $\left(y_j - \sqrt{\rho}\left[\mathcal{F}^\dagger\mathbf{x}\right]_j\right)$ which is a function of all components of $\mathbf{x}$. On top of that, there is no efficient code representation for $\mathbf{F}$ which would reduce the decoding complexity. As a result, an exhaustive search would be required to solve this problem. Conventional OFDM receivers, designed under the Gaussian noise assumption, circumvent this problem by computing the following statistic

$$\Psi = \mathbf{F}\mathbf{y} = \sqrt{\rho}\mathbf{x} + \underbrace{\mathbf{F}\mathbf{w}}_{\mathbf{z}}. \tag{5}$$

When $\mathbf{w}$ is Gaussian, the transformed noise $\mathbf{z}$ would still have a product form *pdf* across subcarriers because $\mathbf{F}$ is unitary and preserves the Gaussian statistics of $\mathbf{w}$ and thereby the independence between the noise vector samples in the Fourier domain . As a result, Ψ is a sufficient statistic and decoding can be performed independently across subcarriers. However, for the noise model in (3), the transformed noise $\mathbf{z}$ has dependent components which means that detection across subcarriers can not be decoupled as in the Gaussian case. This leads to the same exhaustive search as in (4).

V. LOW COMPLEXITY SUBOPTIMAL DECODERS

Due to the dependance of noise samples in the Fourier domain, optimal detection can not be performed independently across subcarriers and has an exponential complexity in the number of subcarriers N (ranging from 64 to 1024 for modern communication systems). This makes optimal detection based on (4) impractical on current computational platforms. In addition, many communication system assume independent decoding across subcarriers. As a result, it desirable to design algorithms that will improve performance under such a constraint. Two observations, employed by [6] to simplify the problem, are : 1) the noise $\mathbf{w}$ is *i.i.d.* in time, and 2) the time-domain signal $\mathbf{u} = [u_1 \cdots u_N]^T$ in (2) can be approximated as being *i.i.d.* in time and $u_j \sim \mathcal{N}_c(0, \rho), \forall j$ by the Central Limit Theorem. [6] then proceeds to find the MMSE estimate, $\hat{\mathbf{u}}$, of $\mathbf{u}$ with NSI and without NSI, followed by hard detection on $\mathbf{F}\hat{\mathbf{u}}$. Since the proof is not explicitly given in [6], we provide it here for completeness.

A. MMSE Estimation with NSI

When NSI is available, the noise at time j is Gaussian with variance $\sigma_{s_j}^2$. The NSI is given by vector $\mathbf{s} = [s_1 \cdots s_N]^T$ where s_j represents the state of the noise at the time instance j (see Section II-A). Let Λ be a matrix function of $\mathbf{s}$ given by

$$\Lambda(\mathbf{s}) = \text{diag}\{1/\sigma_{s_1}, \cdots, 1/\sigma_{s_N}\}. \tag{6}$$

Multiplying (2) by $\Lambda(\mathbf{s})$, we obtain

$$\Lambda(\mathbf{s})\,\mathbf{y} = \Lambda(\mathbf{s}) \underbrace{\mathbf{u}}_{\sqrt{\rho}\mathbf{F}^*\mathbf{x}} + \underbrace{\Lambda(\mathbf{s})\,\mathbf{w}}_{\mathbf{n}}|\mathbf{s} \tag{7}$$

where $\mathbf{n}$ is now a Gaussian vector with identity covariance matrix. However, independent detection across subcarriers would introduce intersymbol interference (ISI) in the frequency domain since $\mathbf{F}\Lambda(\mathbf{s})\,\mathbf{F}^* \neq \mathbf{I}_N$. Since $\mathbf{u}$ and $\mathbf{n}$ are Gaussian, the MMSE estimate of $\mathbf{u}$ is also the Linear MMSE estimate given by

$$\hat{\mathbf{u}}(\mathbf{y}, \mathbf{s}) = \text{diag}\left\{\frac{\rho}{\rho + \sigma_{s_1}^2}, \cdots, \frac{\rho}{\rho + \sigma_{s_N}^2}\right\}\mathbf{y}. \tag{8}$$

At any time instant j, (8) multiplies the observation by $\frac{\rho}{\rho+\sigma_{s_j}^2}$. This scaling reflects the reliability of the received sample based on the noise state it was received under. The implementation complexity of this estimator is low, however the assumption of having NSI at the receiver does not hold in most cases.

B. MMSE Estimation without NSI

When NSI is not present at the receiver, (2) can not be normalized as in (7) and the resulting MMSE estimator $\hat{\mathbf{u}} = [\hat{u}_1 \cdots \hat{u}_N]^T$ of $\mathbf{u}$ is a nonlinear function of $\mathbf{y}$. It can be shown that the MMSE estimate is given by

$$\hat{u_j} = \frac{\mathrm{E}_s\left[\frac{\rho}{(\rho+\sigma_s^2)^2}\exp\left(-\frac{\|y_j\|^2}{\rho+\sigma_s^2}\right)\right]}{\mathrm{E}_s\left[\frac{1}{\rho+\sigma_s^2}\exp\left(-\frac{\|y_j\|^2}{\rho+\sigma_s^2}\right)\right]} \cdot y_j \tag{9}$$

where the index j is dropped from the expectation. The proof is given in the Appendix.

VI. The EM Algorithm

The EM algorithm is an iterative algorithm used to compute the ML estimate of a desired parameter $b \in \mathcal{B}$ given some observed data $y \in \mathcal{Y}$. In particular, it solves the following optimization

$$\hat{b} = \arg\max_{b \in \mathcal{B}} f(y|b) \tag{10}$$

where $f(y|b)$ is the conditional density of y given b. In order to achieve this, it treats this problem as incomplete data estimation problem where the missing data α simplifies the evaluation of $f(y, \alpha|b)$. The EM algorithm uses the likelihood function of the complete data in a two-step procedure as follows:

1) E-step: Compute $Q\left(b|\hat{b}^i\right) = \mathrm{E}_\alpha\left[\log f(y, \alpha|b)\,|y, \hat{b}^i\right]$
2) M-step: Solve $\hat{b}^{i+1} = \arg\max_{b \in \mathcal{B}} Q\left(b|\hat{b}^i\right)$

Given the right initial conditions, the estimate $\hat{b}^i$ will converge to a stationary point. In general, the solution of (10) can be obtained by an appropriate choice of the initial value. In communication systems, the EM algorithm has been widely applied to sequence and channel estimation problems. In [12], the authors give a detection-specific framework for applying EM to sequence estimation problems in communication systems.

VII. Proposed EM-based Detection Algorithm

The ML estimate of the transmitted vector $\mathbf{x}$ is given by

$$\hat{\mathbf{x}} = \arg\max_{\mathbf{x}} f(\mathbf{y}|\mathbf{x}). \tag{11}$$

In Section V-A, NSI reduced the complexity of the MMSE estimation from a non-linear function to a linear function of $\mathbf{y}$. This suggests that the latent vector of noise states $\mathbf{s}$ could be an appropriate choice for unobserved data in an EM-implementation. Thus, we choose $(\mathbf{y}, \mathbf{s})$ as our complete data and formulate the E-step accordingly. The likelihood of the complete data can be written as

$$f(\mathbf{y}, \mathbf{s}|\mathbf{x}) = f(\mathbf{y}|\mathbf{s}, \mathbf{x})\,f(\mathbf{s}|\mathbf{x}) = f(\mathbf{y}|\mathbf{s}, \mathbf{x})\,f(\mathbf{s}) \tag{12}$$

where the second equality follows from the fact that $\mathbf{x}$ and $\mathbf{s}$ are independent (transmission is not adapted to noise state). Since $f(\mathbf{s})$ is not a function of $\mathbf{x}$, it will not have an effect on the M-step and can be ignored. Given that $\mathbf{y}$ is Gaussian given $\mathbf{s}$ and $\mathbf{x}$, the E-step can be expressed as

$$\begin{aligned} Q\left(\mathbf{x}|\hat{\mathbf{x}}^i\right) &= \mathrm{E}_{\mathbf{s}}\left\{\log f(\mathbf{y}|\mathbf{s}, \mathbf{x})\,|\mathbf{y}, \hat{\mathbf{x}}^i\right\} \\ &\overset{(1)}{=} \mathrm{E}_{\mathbf{s}}\left\{-\left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right)^* \Lambda_{\mathbf{s}}^{-1}\left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right)|\mathbf{y}, \hat{\mathbf{x}}^i\right\} \\ &\overset{(2)}{=} -\left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right)^* \mathrm{E}_{\mathbf{s}}\left\{\Lambda_{\mathbf{s}}^{-1}|\mathbf{y}, \hat{\mathbf{x}}^i\right\}\left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right) \end{aligned}$$

where $\Lambda_{\mathbf{s}} = \text{diag}\left\{\sigma_{s_1}^2, \cdots, \sigma_{s_N}^2\right\}$ is the covariance matrix of $\mathbf{y}$ given $\mathbf{s}$ and $\mathbf{x}$. The term $\mathrm{E}_{\mathbf{s}}\left\{\Lambda_{\mathbf{s}}^{-1}|\mathbf{y}, \hat{\mathbf{x}}^i\right\}$ is a diagonal matrix as well with diagonal entries $\frac{1}{\gamma_j^i}, \forall j \in \{1, \cdots, N\}$ given by

$$\frac{1}{\gamma_j^i} = \sum_{s_j=1}^{K} \frac{1}{\sigma_{s_j}^2} f\left(s_j|\mathbf{y}, \hat{\mathbf{x}}^i\right) = \sum_{s_j=1}^{K} \frac{\pi_{s_j}}{\sigma_{s_j}^2} \frac{f\left(y_j|s_j, \hat{\mathbf{x}}^i\right)}{f\left(y_j|\hat{\mathbf{x}}^i\right)} \tag{13}$$

where the second equality follows from the application of Bayes rule and substituting for the corresponding probabilities. The term $f\left(y_j|\hat{\mathbf{x}}^i\right)$ is a constant with respect to s_j and can be computed as the normalization constant for the distribution $f\left(s_j|\mathbf{y}, \hat{\mathbf{x}}^i\right)$ as follows

$$f\left(y_j|\hat{\mathbf{x}}^i\right) = \sum_{s_j=1}^{K} \pi_{s_j} f\left(y_j|s_j, \hat{\mathbf{x}}^i\right).$$

As a result, the only term that requires non-linear computation is $f\left(y_j|s_j, \hat{\mathbf{x}}^i\right) = \frac{1}{\pi\sigma_{s_j}^2} e^{-|y_j-\sqrt{\rho}[\mathbf{F}^*\hat{\mathbf{x}}^i]_j|^2/\sigma_{s_j}^2}$ which can be implemented using a look-up table. Let $\Gamma_{\mathbf{y},\hat{\mathbf{x}}^i} = \text{diag}\{\gamma_1^i, \cdots, \gamma_N^i\}$, then the M-step can be written as

$$\hat{\mathbf{x}}^{i+1} = \arg\min_{\mathbf{x}} \left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right)^* \Gamma_{\mathbf{y},\hat{\mathbf{x}}^i}^{-1} \left(\mathbf{y} - \sqrt{\rho}\mathbf{F}^*\mathbf{x}\right) \quad (14)$$

where $\max$ was replaced by $\min$ by removing the minus sign. The objective in (14) can be interpreted as resulting from the system given by (2) where the noise vector $\mathbf{w}$ consists of Gaussian random variables each with a different variance given by $\gamma_j, \forall j$. In other words, this problem is similar to the problem in Section V-A with perfect noise state information (NSI) where the states are specified by $\Gamma_{\mathbf{y},\hat{\mathbf{x}}^i}$. Thus, taking the FFT will just lead to ICI as described in Section V-A. The exact solution of (14) still requires an exponential search over $\mathbf{x}$. However; by formulating the problem as an EM problem, we transformed the highly non-linear objective of (11) into a quadratic objective given in (14). In addition, the problem was transformed from detection with no NSI (highly non-linear) into multiple iterations of detection with perfect NSI (with linear MMSE estimate). As a result, we approximate the solution of (14) by taking the MMSE estimate of the OFDM symbol in the time domain using the NSI followed by hard detection similar to the method given in Section V-A. As a result, the new step is given by

$$\hat{\mathbf{x}}^{i+1} \approx \left[\mathbf{F}\hat{\mathbf{u}}^{\mathbf{i+1}}\right] \quad (15)$$

where $[\cdot]$ denotes hard detection and $\hat{u}^{i+1}$ is given by its Linear MMSE estimate as follows

$$\hat{u}^{i+1} = \text{diag}\left\{\frac{\rho}{\rho+\gamma_1^i}, \cdots, \frac{\rho}{\rho+\gamma_N^i}\right\}\mathbf{y}. \quad (16)$$

The choice of the initial value $\hat{\mathbf{x}}^0$ for the EM algorithm has a big effect on the convergence rate and converging value. Two possible initial points are: 1) the result of the typical OFDM receiver (taking an FFT followed by hard decision), and 2) taking the result of the MMSE receiver without NSI described in Section V-B. The former is computationally more tractable since it involves only an FFT operation while the latter might provide a better estimate and lead to lower number of iterations. This is explored further in the results section.

VIII. Results

We simulate the OFDM system given in (2) using Monte-Carlo simulations. The communication performance of the discussed algorithms is compared for $N = 1024$ with 4-QAM modulation in the presence of a 2-term Gaussian mixture model (2-term Gaussian also called the ϵ-contaminated Gaussian usually suffices in practice [3], [1]). The symbol error rate (SER) of the proposed iterative method is given for the conventional OFDM and single carrier (SC) receivers and for the non-iterative estimator-correlator receivers with NSI and without NSI. The noise parameters are set to typical values given in Table I. In particular, we set $\pi = \begin{bmatrix}0.9 & 0.1\end{bmatrix}$, $\sigma_1^2 = 1$, and $\sigma_2^2 = 150$. SNR is interpreted as the signal power to the second moment of the impulsive noise which exists in closed form for Gaussian mixture distributions.

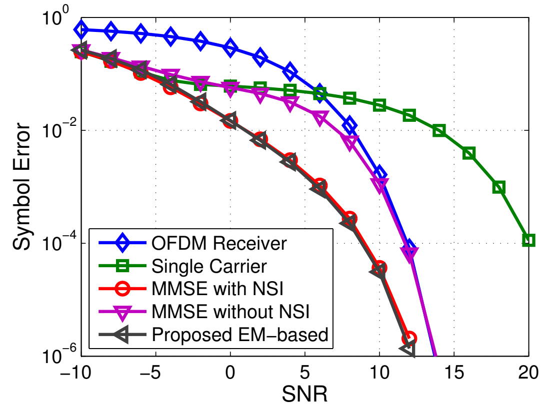

Fig. 1. Communication performance of the low-complexity receivers in the presence of impulsive noise ($\pi_1 = 0.9$, $\pi_2 = 0.1$, $\sigma_1^2 = 1$ and $\sigma_2^2 = 100$). The proposed method has a gain of around 6dB in the moderate SNR region over the next best implementable algorithm.

A. Single Carrier vs. Conventional OFDM

Fig. 1 shows the communication performance degradation between single carrier (SC) systems and conventional OFDM receivers (using FFT followed by hard detection). It is noticed that the single carrier system performs better at low SNR till around 6dB. After that the conventional OFDM system considerably outperforms the SC system with gains up to 7.5dB at SER=10^{-4}. This can be explained by the fact that at low SNR the occurring impulses have a much larger energy than the signal. In SC systems, this translates to losing the symbol exposed to the impulse. However, in OFDM systems the high energy of the impulse is spread across the whole OFDM symbol which results in losing the whole OFDM block. As a result, the SC performs better at low SNR. The opposite occurs at high SNR where the amplitude of the impulse is spread across the whole OFDM symbol without affecting it, while the SC system still suffers from the single symbols errors as in the previous case. This is the basis for the OFDM impulse resilience ability which is the result of the time diversity it provides when viewed as a time-code.

B. Performance of the Proposed Method

The communication performance of the non-itarative MMSE methods described in Section V and the proposed iterative method based on the EM algorithm are shown in Fig. 1. The non-iterative method with NSI provides the lower

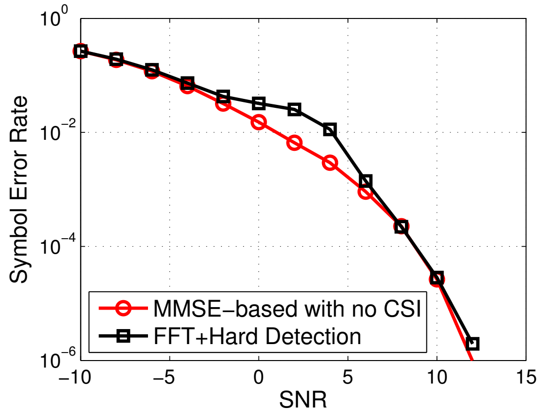

Fig. 2. Communication performance for different initial values of $\mathbf{x}^0$ with 10 iterations of the EM algorithm. The initial value obtained by the MMSE-based detector with no NSI provides a slightly better performance for additional computational complexity than the conventional OFDM receiver.

bound on the achievable performance using these time-domain MMSE based class of algorithms. However, the perfect NSI at the receiver assumption is not valid in most cases and this algorithm is impractical. The iterative algorithm is allowed to run for a maximum of 10 iterations. It is seen that the proposed EM-based algorithm provides a gain of ranging from 2dB 7dB over the non-iterative MMSE without NSI. Further, the proposed method, which is an approximate ML detector, achieves the lower bound for the MMSE based methods with perfect NSI at almost the same computational complexity. The effect of the choice of the initial condition on the performance of the proposed EM-based algorithm is given in Fig. 2. The MMSE-based detector with no NSI provides only a slight improvement at 10 iterations of the EM algorithm.

IX. Computational Complexity and Limitations

The computational complexity of the proposed algorithm is analyzed in terms of the number of exponential evaluations and FFT operations it requires per subcarrier. For each subcarrier k, the proposed algorithm has to compute γ_k given in (13) for each iteration. This requires K (number of Gaussian components, usually 2) scalar exponential evaluations that could be implemented in a lookup table. In addition to that, at the end of each iteration a FFT operation of size N has to be performed. Although the proposed method leads to significant performance gain for typical impulsive environments such as the ones measured in Section II-B, it can fail in impulsive noise with extreme amplitudes that are for example 30dB higher above the noise floor. Such scenarios are not common since in many cases very high impulses are clipped by the receiver reducing their amplitude. This degradation in performance is due to the approximation made in (15) by which the EM algorithm loses its monotonic increase in likelihood.

Appendix

The system model of (2) can be expressed as

$$y_j = u_j + w_j \qquad j = 1, \cdots, T.$$

The MMSE estimate of u_j given y_j is

$$\hat{u}_j = \mathrm{E}[u_j|y_j] = \int_{\mathbb{C}^N} u_j f\left(u_j|y_j\right) \mathrm{d}u_j. \tag{17}$$

Using Bayes rule and summing over all noise state realizations $s_j \in \mathcal{S}$ with $\Pr[s_j] = \pi_j$, we obtain

$$\begin{aligned} f\left(u_j|y_j\right) &= \frac{\sum_{\mathcal{S}} \pi_j f\left(u_j|y_j, s_j\right) f\left(y_j|s_j\right)}{\sum_{\mathcal{S}} \pi_j f\left(y_j|s_j\right)} \\ &= \frac{\mathrm{E}_s[f\left(u_j|y_j, s_j\right) f\left(y_j|s_j\right)]}{\mathrm{E}_s[f\left(y_j|s_j\right)]} \end{aligned} \tag{18}$$

Substituting (18) in (17) and interchanging the order of integration and expectation

$$\hat{u}_j = \frac{\mathrm{E}_s[f\left(y_j|s_j\right) \cdot \int u_j f\left(u_j|y_j, s_j\right) \mathrm{d}u_j]}{\mathrm{E}_s[f\left(y_j|s_j\right)]}.$$

Given the noise state s_j, y_j is a sum of two independent Gaussian vectors and therefore Gaussian with covariance $\rho + \sigma_{s_j}^2$. On the other hand, $\int u_j f\left(u_j|y_j, s_j\right) \mathrm{d}u_j = \mathrm{E}[u_j|y_j, s_j]$ is the LMMSE estimate of u_j given in (8). Thus,

$$\hat{u}_j = \frac{\mathrm{E}_s\left[\frac{1}{\rho+\sigma_s^2} \exp\left(-\frac{\|y_j\|^2}{\rho+\sigma_s^2}\right) \cdot \frac{\rho}{\rho+\sigma_{s_j}^2} y_j\right]}{\mathrm{E}_s\left[\frac{1}{\rho+\sigma_s^2} \exp\left(-\frac{\|y_j\|^2}{\rho+\sigma_s^2}\right)\right]}$$

which simplifies to (9).

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