

# Cyclostationary Noise Mitigation in Narrowband Powerline Communications

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**Abstract**—Future Smart Grid systems will intelligently monitor and control energy flows in order to improve the efficiency and reliability of power delivery. This monitoring and control requires low-delay, highly reliable, two-way communication between customers, local utilities and regional utilities. Narrowband powerline communication (NB-PLC) systems operating in the 3–500 kHz band have been standardized to enable these two-way communication links. In NB-PLC systems, additive non-Gaussian noise/interference is primary limitation to the communication performance. From field trials, the dominant source of this non-Gaussian noise/interference is cyclostationary. In this paper, we address the problem of cyclostationary noise mitigation in NB-PLC systems and other orthogonal frequency division multiplexing (OFDM) systems. The contributions of this paper include developing a parametric noise estimation algorithm based on switching linear autoregressive (AR) process, and a simple adaptive noise whitening approach that can be immediately integrated into the conventional OFDM transceiver structure to improve its performance. In our simulations, the proposed noise whitening method achieves up to 3dB SNR gain over conventional OFDM systems at SNRs higher than -3dB.

## I. INTRODUCTION

The electrical grid today has evolved to a new paradigm, the Smart Grid. Compared to traditional grids that carries one-way flow of power from generators to customers, the Smart Grid uses two-way flows of energy and information to create an intelligent energy delivery network. Various technologies have emerged to facilitate data communication throughout the grid, and in particular between local utilities and customers. Applications of such “last mile” communications include smart metering and real-time energy management. Powerline communications (PLC) over the medium-voltage (MV) and low-voltage (LV) lines have been attractive as a no-new-wire solution to the last mile communications. Recently there has been a lot of interest in developing high data rate narrowband (NB) PLC systems, which operates in the 3–500 kHz band to provide data rates up to 800 kbps. These systems generally employ orthogonal frequency division multiplexing (OFDM) to combat multi-path frequency selective channels. Examples of OFDM-based NB PLC systems are specified in the industry-developed standards PRIME and G3, and recent international standards ITU-T G.hnem and IEEE P1901.2.

Despite of its costless deployment, the power lines, originally designed for electricity transfer, is a hostile environment for data communication systems. Powerline noise is one of the primary impairments for NB PLC systems. Typical noise sources include electronic devices connected to the supply

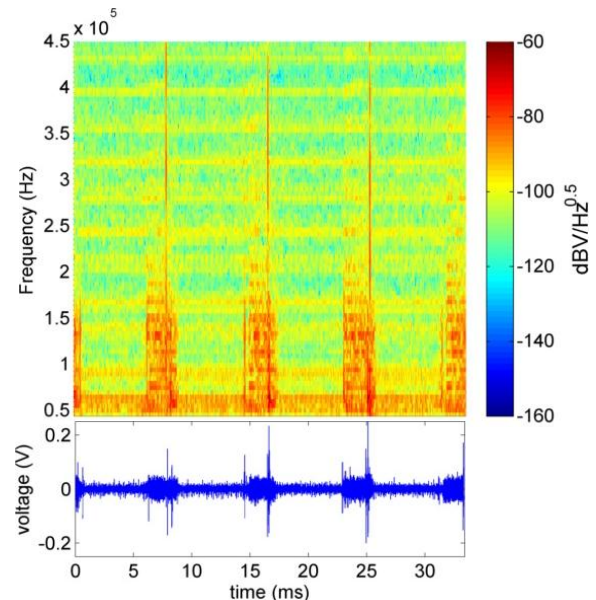


Fig. 1. A noise trace captured at a low voltage field site. The noise exhibits cyclostationary features both in time and frequency domain.

network (e.g. microwave oven, brush motors, light dimmers and other devices with switching modes) and nonlinear circuits in switching power supplies and silicon-controlled rectifiers. The aggregated noise deviates significantly from the general assumption of additive white Gaussian noise (AWGN).

Recent study has shown that the dominant component of the additive noise in NB PLC is so-called synchronous cyclostationary noise [1][2][6]. Such noise exhibits periodically varying statistics, with the period synchronous to half the AC cycle (Fig. 1). The periodic bursts of noise can reach as high as 30dB above the noise floor, which lowers the signal-to-noise ratio (SNR) over a significant portion of the spectrum during 10–50% of the period. In OFDM-based PLC systems, the presence of cyclostationary noise dramatically decreases the achievable data rate, since reliable transmission is limited to those subcarriers and temporal durations with relatively good SNR. Therefore, in this work we seek to mitigate cyclostationary noise by exploiting the noise structure.

Various models have been proposed to characterize the temporal and spectral structures of cyclostationary noise in NB PLC. [1] captures the temporal variation of noise power by a white Gaussian process with periodically evolving variance,

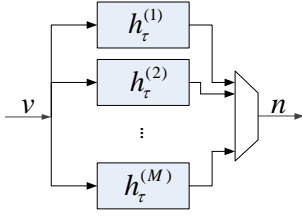


Fig. 2. An linear periodically time-varying system model for cyclostationary noise in NB PLC.

i.e.  $n_k \sim \mathcal{N}(0, \sigma_k^2)$ , where  $\sigma_k^2$  is a periodic function of a parametric form. The temporal correlation, or frequency selective spectrum, of the noise is introduced by passing  $n_k$  through a linear time-invariant (LTI) filter, where a constant spectral shape during the entire period is assumed. In order to better describe the spectral variation within a period, a more general noise model based on linear periodically time-varying (LPTV) systems is proposed in [2] (Fig. 2), and has been accepted to IEEE P1901.2 standard [6]. The model partitions each period into  $M$  segments. Noise within each segment is assumed to be a stationary Gaussian process, whose temporal correlation is defined by an LTI spectral shaping filter. More specifically, let  $S_i$  and  $\{h_\tau^{(i)}\}$  denote the set of discrete time instances and the filter's impulse response in the  $i$ -th segment of a period, the cyclostationary noise can be generated from the AWGN input  $v_k \sim \mathcal{N}(0, 1)$  by

$$n_k = \sum_{i=1}^M 1_{k \in S_i} \sum_{\tau} h_\tau^{(i)} v_{k-\tau}. \quad (1)$$

In this paper, we address the problem of cyclostationary noise mitigation in OFDM-based NB PLC systems. To make the parameter estimation more tractable, we further parameterize the LTI impulse responses  $\{h_\tau^{(i)}\}_{i=1}^M$  by autoregressive (AR) filters, and hence the LPTV system by periodically switching AR process. The switching states and AR parameters are then estimated from the observed noise trace by Bayesian learning. Based on the estimated switching AR model, we develop a pre-filtering method that mitigates cyclostationary noise by spectral whitening.

## II. MODEL PARAMETERIZATION

Injected by nonlinear electrical circuits, cyclostationary noise in NB PLC is essentially a nonlinear random vibration that has also been widely observed in mechanical and biological systems (e.g. rotating motors, brain waves, etc.). Such vibrations in continuous time are generally modeled by nonlinear differential equations. In discrete time, various time series models have been proposed [3][4] to capture the complex nonlinear behaviors. In particular, nonlinear AR processes, i.e. AR processes with time-varying coefficients, have been attractive, since parameter estimation can usually be transformed into a linear regression problem.

In this section, we propose using a periodically switching AR process with AWGN input as an approximation to the cyclostationary noise model in (1). A periodically switching

AR process is an AR process whose coefficients switch among a set of possible states periodically, i.e.

$$n_k = \sum_{\tau=1}^R a_\tau^{(z_k)} n_{k-\tau} + b^{(z_k)} v_k, \quad v_k \sim \mathcal{N}(0, 1) \quad (2)$$

where  $z_k \in \{1, 2, \dots, M\}$  is the periodic state sequence, and  $\{a_\tau^{(i)}, b^{(i)}\}$  are the AR coefficients corresponding to the  $i$ -th state. Compare (2) to (1), we simply approximate the each of the  $M$  LTI filters  $\{h_\tau^{(i)}\}_{\tau=-\infty}^{\infty}$  by a linear AR filter with  $R+1$  parameters  $\{a_\tau^{(i)}, b^{(i)}\}$ . The approximation simplifies parameter estimation from the observed noise trace, since conditioned on the state sequence, estimation of  $\{a_\tau^{(i)}, b^{(i)}\}$  converts to a set of standard linear regression problems. As demonstrated in our simulation results, the AR approximation closely matches the noise model in (1).

## III. PARAMETER ESTIMATION

Since data transmission in NB PLC is bursty, the receiver can listen to the cyclostationary noise when the transmitter is silent. One can identify the switching AR model from one period (half the AC cycle) of noise samples. The model identification problem includes estimating the number of states  $M$ , the state sequence  $\{z_k\}$ , the filter order  $R$ , and the filter coefficients corresponding to each state. In certain special cases, the number of states and the state sequence might be inferable by visually inspecting the spectrogram such as the one in Fig. 1. In most situations, where visual inspection easily produces ambiguity, the assumption that the number of states is known and fixed has to be relaxed.

A Bayesian nonparametric inference framework has been developed in [5] to estimate switching AR models with unknown number of states. The inference assumes that the evolution of the state sequence follows a hidden Markov model (HMM), which is a valid assumption if only one period of noise is considered. The HMM is endowed with an infinite state space and hence defined by an infinite-dimensional transition probability matrix, i.e.

$$z_k | z_{k-1} \sim \pi_{z_{k-1}}, \quad (3)$$

where  $\pi_i$  is an infinite-dimensional vector of transition probabilities, whose  $j$ -th element is  $\pi_{ij} = P(z_k = j | z_{k-1} = i)$ .

A sticky hierarchical Dirichlet process (HDP) prior is imposed on the set of transition probability measures  $\{G_i\}_{i=1}^{\infty}$ .  $G_i$  is a discrete probability measure with an infinite collection of atoms

$$G_i = \sum_{j=1}^{\infty} \pi_{ij} \delta_{\theta_j}, \quad i = 1, \dots, \infty \quad (4)$$

where  $\theta_j \triangleq \{a_1^{(j)}, \dots, a_R^{(j)}, b^{(j)}\}$  denotes the AR parameters of state  $j$ , and  $\delta_{\theta^*} \triangleq 1_{\theta=\theta^*}$ . The sticky HDP prior provides a distribution of the set  $\{G_i\}_{i=1}^{\infty}$ . The weights in each  $G_i$ , i.e.  $\pi_i$ , are independently sampled via a stick-breaking construction, denoted by  $\pi_i \sim GEM(\alpha + \kappa)$ ,

$$\pi_{ij} = \beta_j \prod_{l=1}^{j-1} (1 - \beta_l), \quad \beta_j \sim Beta(1, \alpha + \kappa). \quad (5)$$

All  $G_i$ 's share the same support  $\{\theta_j\}_{j=1}^{\infty}$ , which are drawn from a global Dirichlet process, i.e.

$$G_0 = \sum_{j=1}^{\infty} \lambda_j \delta_{\theta_j} \quad (6)$$

$$\lambda = \frac{\alpha\omega + \kappa\delta_i}{\alpha + \kappa} \quad (7)$$

where  $\delta_i$  is a vector of all zeros except for a one in its  $i$ -th element, and  $\omega \sim GEM(\gamma)$ .  $\{\theta_j\}_{j=1}^{\infty}$  are drawn from an inverse-Wishart matrix-normal prior

$$\begin{aligned} \mathbf{a}^{(j)} &\sim \mathcal{MN}(M, V, K) \\ (b^{(j)})^2 &\sim \mathcal{IW}(n_0, S_0), \end{aligned} \quad (8)$$

with  $\mathcal{MN}(M, V, K)$  denoting a matrix-normal distribution with mean matrix  $M$  and left and right covariances  $K^{-1}$  and  $V$ , and  $\mathcal{IW}(n_0, S_0)$  an inverse-Wishart prior with  $n_0$  degrees of freedom and scale matrix  $S_0$ .

The HDP is useful in inferring the number of states that are supported by observations, due to its clustering properties that encourage sparsity of the vector  $\pi_i$  [5]. The sticky parameter  $\kappa \in [0, 1]$  is used to increase the probabilities of self-transitions over those of inter-state switches.

Given the sticky HDP-HMM prior defined in (4)–(8), a Gibbs sampler is used to infer the HMM transition probabilities  $\{\pi_i\}$ , the state sequence  $z_{1:T}$ , and the AR parameters  $\{a_1^{(i)}, \dots, a_R^{(i)}, b^{(i)}\}$  from the observations  $n_{1:T}$ , where  $T$  is the number of samples in half the AC cycle. In particular, the Gibbs sampler iterates between the following two steps [5]:

- *Sampling AR parameters.* Conditioned on the state sequence  $z_{1:T}$  and the observations  $n_{1:T}$ , the estimation of AR parameters can be converted to  $M$  different linear regression problems, where  $M$  is the cardinality of  $z_{1:T}$ . Therefore, the AR parameters corresponding to the  $M$  states can be sampled from the posterior densities  $P(\mathbf{a}^{(i)}, b^{(i)} | n_{S_i})$ , which can be derived taking into account of the MNIW prior.
- *Block sampling  $z_{1:T}$ .* A truncated approximation to the HDP is used and  $z_{1:T}$  is jointly sampled using a variant of the forward-backward message passing algorithm.

#### IV. CYCLOSTATIONARY NOISE WHITENING

Upon estimating the periodically switching AR model, two adaptive filters can be designed and inserted at the OFDM transmitter and receiver, respectively, to whiten the cyclostationary noise during data transmission (Fig. 3). For simplicity, we assume that the data is transmitted through a flat channel.

The filter at the receiver side (RX filter in Fig. 3) is a periodically switching moving average (MA) noise whitening filter,

$$y_k = r_k - \sum_{\tau=1}^R a_{\tau}^{(z_k)} r_{k-\tau}, \quad (9)$$

where  $r_k$  denotes the received signal in time domain before cyclic prefix (CP) removal. Note that  $r_k = s_k + n_k$ . The RX

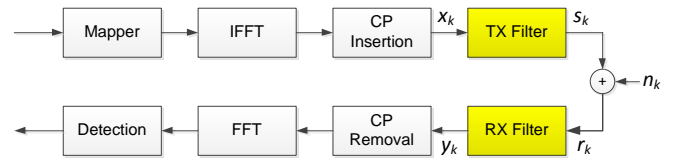


Fig. 3. An OFDM transmitter and receiver block diagram, with two adaptive cyclostationary noise mitigation filters.

filter effectively transforms the cyclostationary noise component  $n_k$  in the received signal to an AWGN with time-varying standard deviation  $b^{(z_k)}$ . On the other hand, the RX filter introduces unwanted distortion to the transmitted signal  $s_k$ . To compensate for this, the same periodically switching AR filter as in (2) is added at the transmitter side,

$$s_k = \sum_{\tau=1}^R a_{\tau}^{(z_k)} s_{k-\tau} + x_k, \quad (11)$$

where  $x_k$  is the time domain OFDM signal after CP insertion.

With the additional TX and RX filters, and assuming unit channel gain, the received signal before CP removal can be expressed as

$$y_k = x_k + b^{(z_k)} v_k, \quad (12)$$

where  $v_k$  is AWGN. The OFDM receiver then continues with standard detection stages as if only AWGN were present.

#### V. SIMULATION RESULTS

The application of the proposed methods to OFDM-based NB PLC systems highly depends on the accuracy of the estimated switching AR model, which further decomposes into two sub-problems: how well the switching AR model can approximate the original LPTV system model, and how accurately the switching AR model can be inferred by non-parametric Bayesian learning. The latter problem has been addressed by the capability of nonparametric Bayesian learning as demonstrated in [5]. To illustrate the former problem, we generate multiple periods of cyclostationary noise from an AWGN sequence using the model in (1) and the spectral shapes in [2]. A switching AR model is identified from one period of the noise trace by nonparametric Bayesian learning, where an order-6 AR filter is employed in all states. Based on the estimated switching AR model, we synthesize a cyclostationary noise trace from the same AWGN input sequence. The synthesized noise is compared to the original noise in both time domain and frequency domain in Fig. 4 and Fig. 5. The spectral and temporal traces of synthesized noise samples resemble those of the original noise, showing the closeness of the estimated switching AR model to the original LPTV system model. To further illustrate the accuracy of the estimated switching AR model, we apply the noise whitening filter, i.e. a filter as in (9), to the original noise. The Lilliefors test for normality over the whitened noise shows that it fits a normal distribution at a significance level of 0.01. Moreover, the autocorrelation of the whitened noise (Fig. 6)

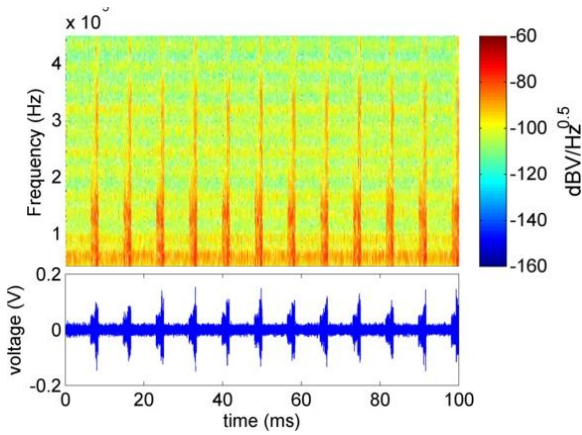


Fig. 4. Cyclostationary noise generated from the LPTV system model.

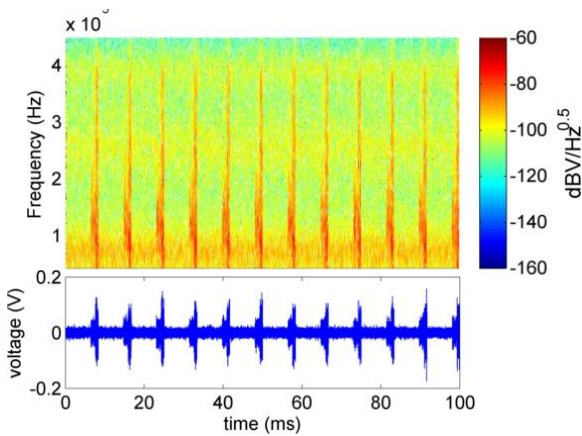


Fig. 5. Cyclostationary noise synthesized from the switching AR model.

highly concentrates around the origin, indicating the whiteness of the noise.

To evaluate the performance of the proposed noise whitening filters, we simulate an OFDM system with 256 subcarriers. The data are QPSK modulated, and a length-30 cyclic prefix is added to each OFDM symbol. For simplicity purposes, we assume a frequency-flat channel, and a trace of cyclostationary noise generated from the LPTV system model is added to the transmitted signal. We simulate the conventional OFDM system and the one with our proposed noise whitening filters. The performance (in terms of symbol error rate) of both systems are plotted and compared in Fig. 7, where minimum SNR is defined as signal to maximum noise power ratio. We observe that our proposed method outperforms the conventional system at SNRs larger than -3dB. Within this range, our noise whitening approach is able to achieves up to 3dB SNR gains. The performance of the proposed noise whitening filter, however, deteriorates at lower SNRs. In such regimes, advanced detection algorithms that take into account of the estimated noise statistics may need to be invoked.

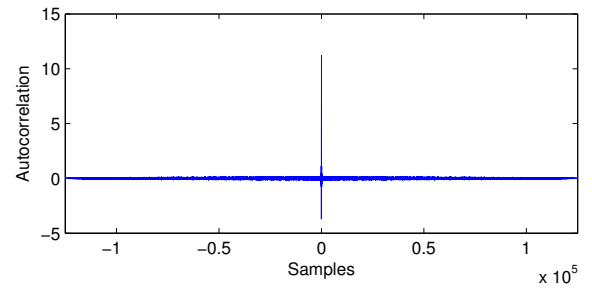


Fig. 6. Autocorrelation of the whitened noise.

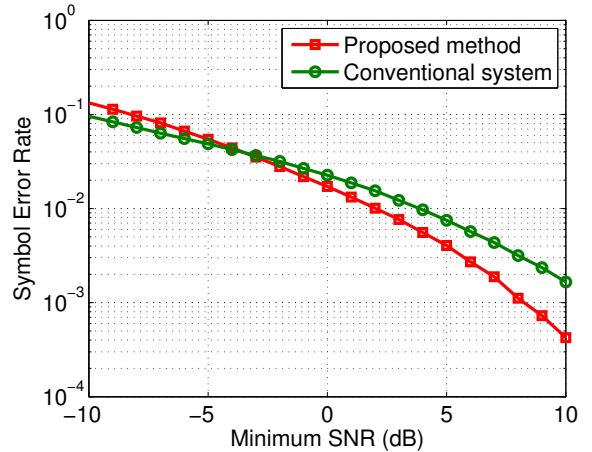


Fig. 7. Symbol error rate of OFDM systems with/without proposed noise mitigation method in the presence of cyclostationary noise.

## VI. CONCLUSION

This paper proposes a periodically switching AR model for simplifying the estimation and mitigation of cyclostationary noise in narrowband powerline communication systems. A nonparametric Bayesian learning algorithm is applied to estimate the switching AR model from observed noise sequence. Based on the estimated model, we present a simple adaptive noise whitening approach that can be immediately integrated into conventional OFDM systems.

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