Equalizer Design to Maximize Bit Rate in ADSL Transceivers

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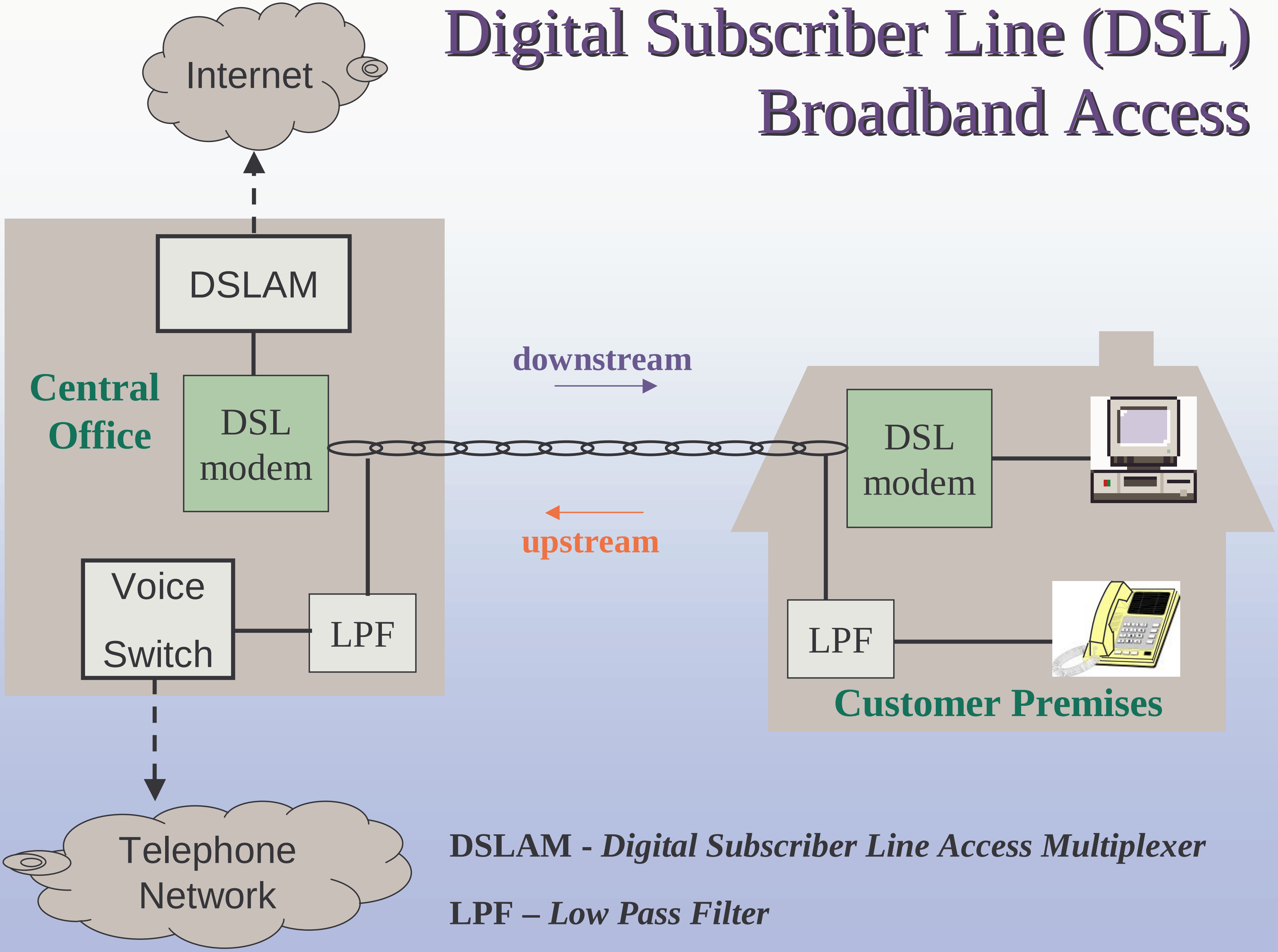
Digital Subscriber Line (DSL) Broadband Access



DSLAM - ***Digital Subscriber Line Access Multiplexer***

LPF – ***Low Pass Filter***

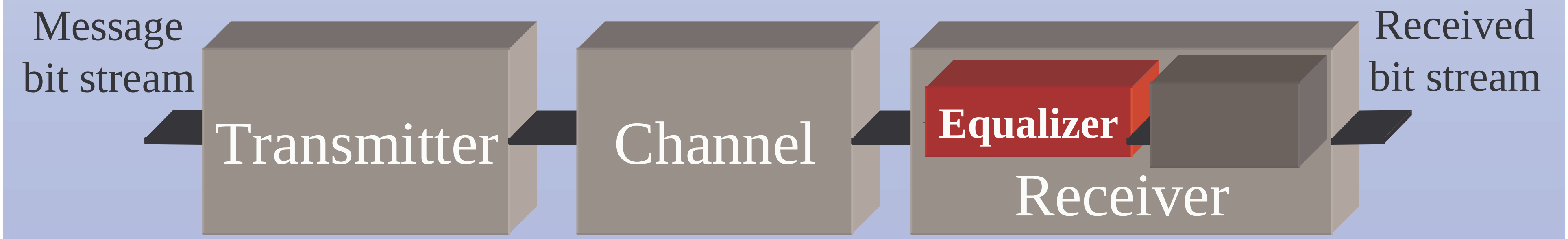
Discrete Multitone (DMT) DSL Standards

- **ADSL – Asymmetric DSL (G.DMT Standard)**
  - Maximum data rates supported (*ideal case*)
    - Echo cancelled: 14.94 Mbps downstream, 1.56 Mbps upstream
    - Frequency division multiplexing: 13.38 Mbps downstream, 1.56 Mbps up
  - Widespread deployment in US, Canada, Western Europe, Hong Kong
    - Central office providers only installing frequency-division multiplexed ADSL
    - ADSL:cable modem market 1:2 in US & 5:1 worldwide
- **VDSL – Very High Rate DSL (Proposed Standard)**
  - Also has symmetric mode: 13, 9, or 6 Mbps
  - Single carrier and DMT
  - DMT VDSL
    - Higher speed G.DMT ADSL
    - Frequency division multiplex
    - 2^m subcarriers $m \in [8, 12]$

	G.DMT ADSL	*Asymmetric DMT VDSL*
Data band	25 kHz – 1.1 MHz	1 MHz – 12 MHz
Upstream subcarriers	32	256
Downstream subcarriers	256	2048/4096
Target up-stream rate	1 Mbps	3 Mbps
Target down-stream rate	8 Mbps	13/22 Mbps

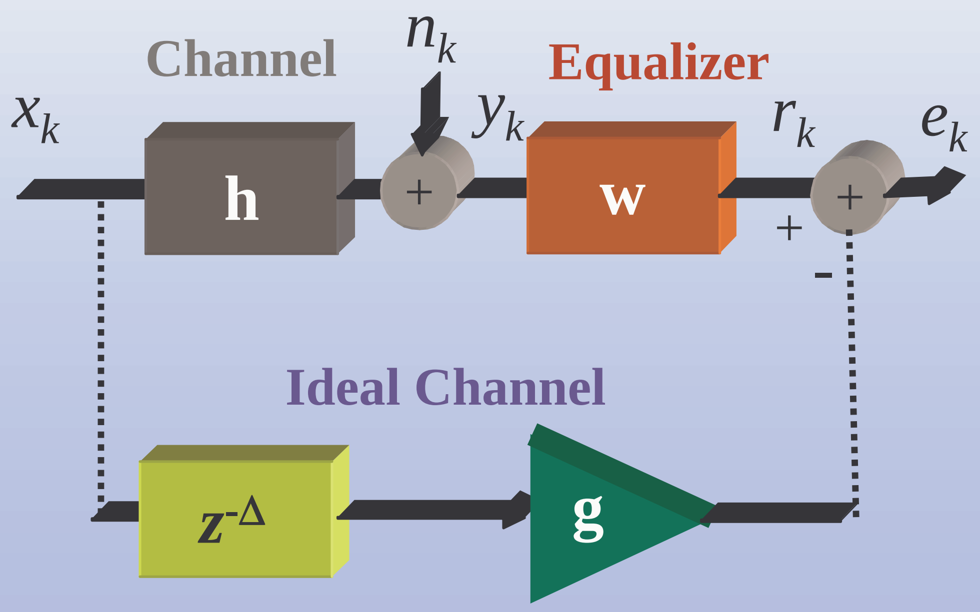
Outline

- **Multicarrier modulation**
- **Conventional equalizer**
  - Minimum Mean Squared Error design [Stanford]
  - Maximum Shortening Signal-to-Noise Ratio design [Tellabs]
  - Maximum Bit Rate design (*optimal*) [UT Austin]
  - Minimum Inter-symbol Interference design (*near-optimal*) [UT Austin]
- **Per-tone equalizer** [Catholic University, Leuven, Belgium]
- **Dual-path equalizer** [UT Austin]
- **Conclusion**



Single Carrier Modulation

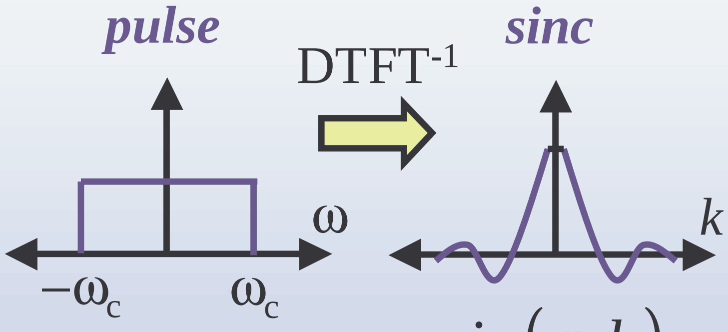
- **Ideal (non-distorting) channel over transmission band**
  - Flat magnitude response
  - Linear phase response: delay is constant for all spectral components
  - No intersymbol interference
- **Impulse response for ideal channel over all frequencies**
  - Continuous time: $g\,\delta(t-T)$
  - Discrete time: $g\,\delta[k-\Delta]$
- **Equalizer**
  - Shortens channel impulse response (*time domain*)
  - Compensates for frequency distortion (*frequency domain*)



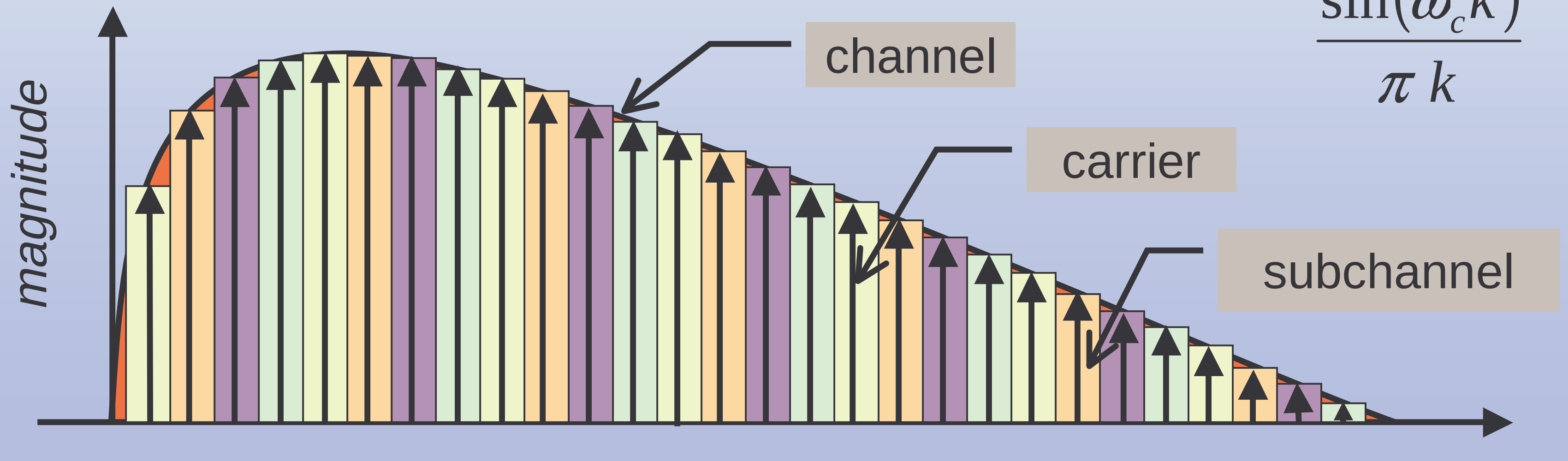
Discretized Baseband System

Multicarrier Modulation

- **Divide channel into narrowband subchannels**
  - No inter-symbol interference (ISI) in subchannels if constant gain within every subchannel and if ideal sampling
- **Discrete multitone modulation**
  - Based on fast Fourier transform (FFT)
  - Standardized for ADSL
  - Proposed for VDSL

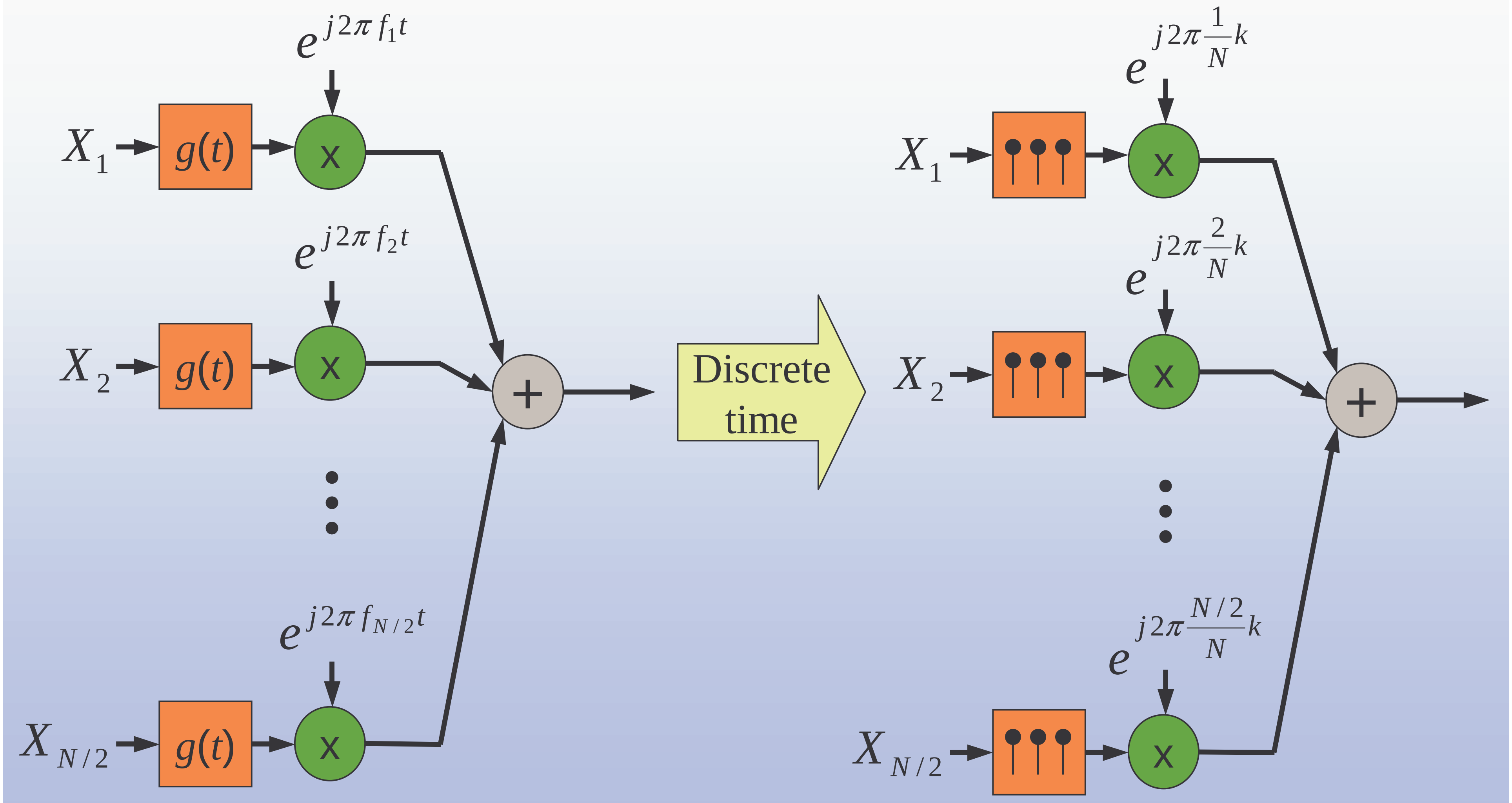


$$\frac{\sin(\omega_c k)}{\pi k}$$



Subchannels are 4.3 kHz wide in ADSL and DMT VDSL

Multicarrier Modulation by Inverse FFT Filter Bank

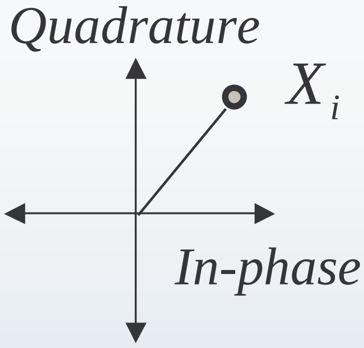


$g(t)$: pulse shaping filter

X_i : i^{th} subsymbol from encoder

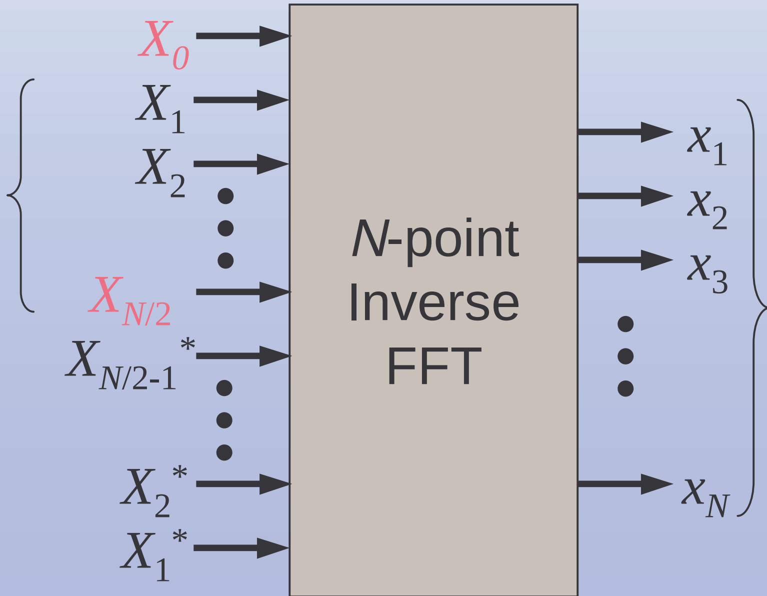
Discrete Multitone Modulation Symbol

- **Subsymbols are in general complex-valued**
  - ADSL uses 4-level Quadrature Amplitude Modulation (QAM) during training
  - ADSL uses QAM of 2^2, 2^3, 2^4, …, 2^{15} levels during data transmission



QAM

- **Mirror and conjugate subsymbols before multicarrier modulation using inverse FFT**

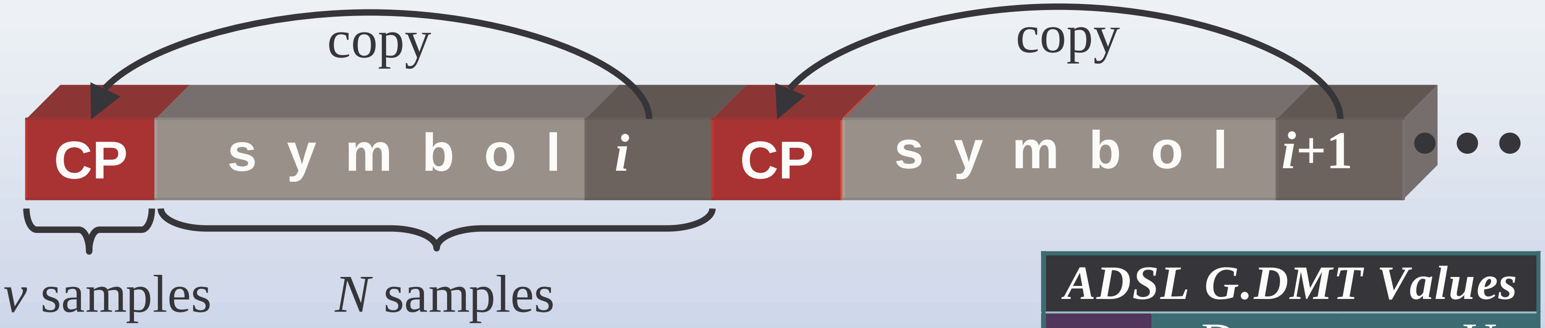


N/2 subsymbols (one subsymbol per carrier)

one symbol of *N* real-valued samples

Discrete Multitone Modulation Frame

- **Frame is sent through D/A converter and transmitted**
  - Frame is the symbol with cyclic prefix prepended
  - Cyclic prefix (CP) consists of last ν samples of the symbol



  - CP reduces throughput by factor of $\frac{N}{N+\nu} = \frac{16}{17}$

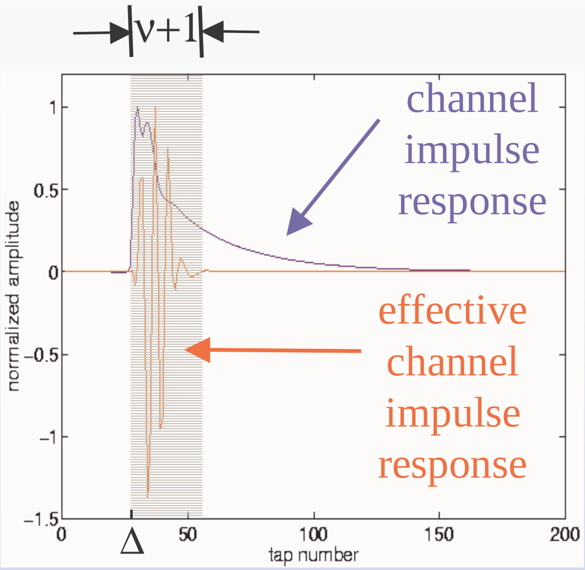
ADSL G.DMT Values

	Down stream	Up stream
ν	32	4
N	512	64

- **Linear convolution of frame with channel impulse response**
  - Is circular convolution if channel length is CP length plus one or shorter
  - Circular convolution ⇨ frequency-domain equalization in FFT domain
  - Time-domain equalization to reduce effective channel length and ISI

Eliminating ISI in Discrete Multitone Modulation

- **Time domain equalizer (TEQ)**
  - Finite impulse response (FIR) filter
  - *Effective channel impulse response*: convolution of TEQ impulse response with channel impulse response
- **Frequency domain equalizer (FEQ)**
  - Compensates magnitude/phase distortion of equalized channel by dividing each FFT coefficient by complex number
  - Generally updated during data transmission
- **ADSL G.DMT equalizer training**
  - *Reverb*: same symbol sent 1,024 to 1,536 times
  - *Medley*: aperiodic sequence of 16,384 symbols
  - At 0.25 s after medley, receiver returns number of bits on each subcarrier that can be supported



Δ: *transmission delay*

ν: *cyclic prefix length*

ADSL G.DMT Values

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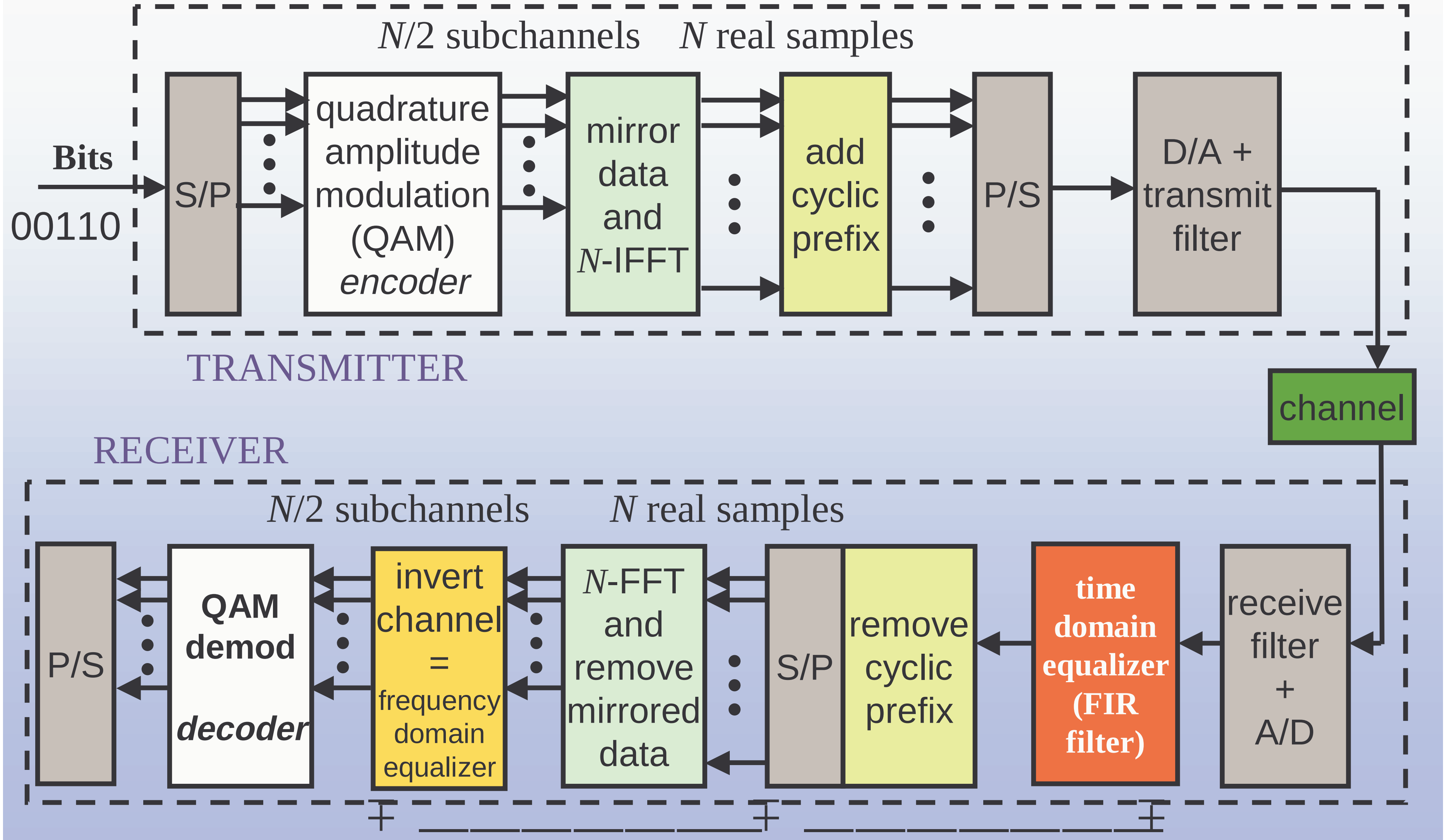
ν	32	4
N	512	64

ADSL Transceiver: Data Transmission



conventional ADSL equalizer structure

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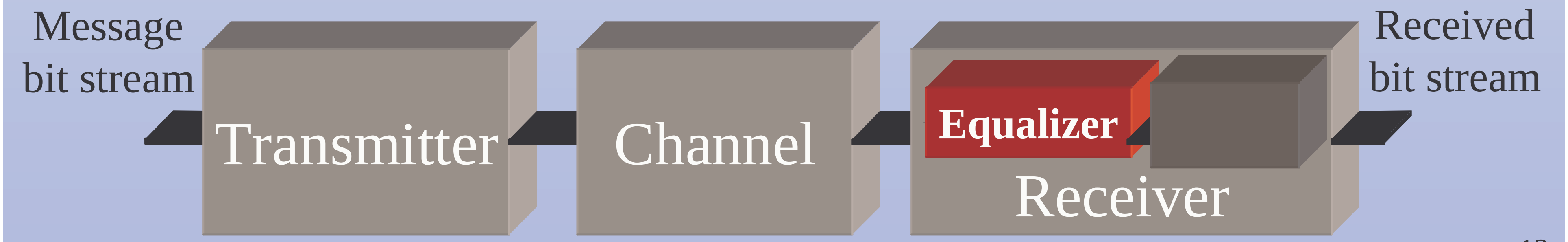
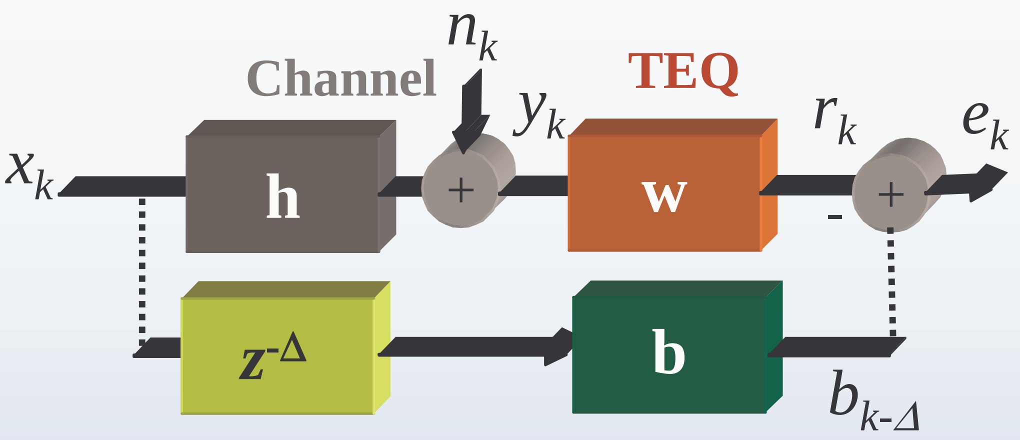
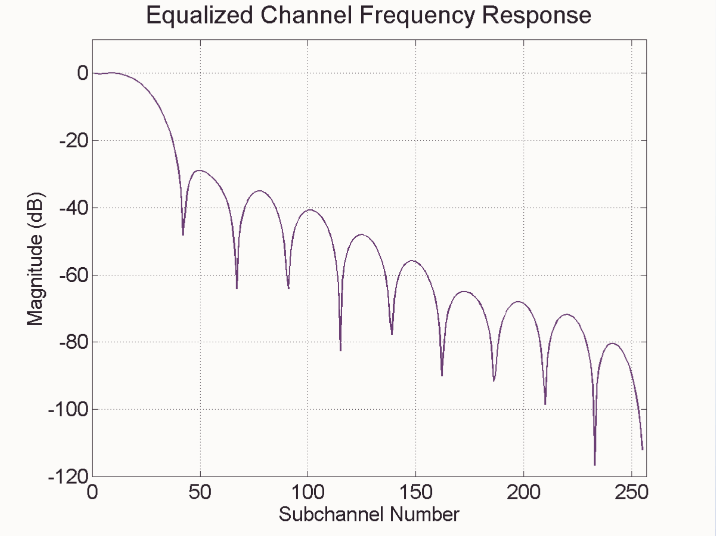


Minimum Mean Squared Error (MMSE) TEQ Design



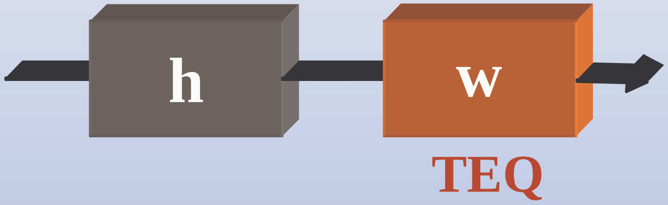


- **Minimize** $E\{e_k^2\}$ [Chow & Cioffi, 1992]
  - Chose length of $\mathbf{b}$ (e.g. ν+1 in ADSL) to shorten length of $\mathbf{h} * \mathbf{w}$
  - $\mathbf{b}$ is eigenvector of minimum eigenvalue of channel-dependent matrix
  - Minimum MSE achieved when $\mathbf{w}^T = \mathbf{b}^T \mathbf{R}_{\mathbf{xy}} \mathbf{R}_{\mathbf{yy}}^{-1}$ where $\mathbf{w} \neq \mathbf{0}$
- **Disadvantages**
  - Does not consider *bit rate*
  - Deep notches in equalizer frequency response (zeros out low SNR bands)
  - Infinite length TEQ: zeros of **b** lock onto unit circle (kills ν subchannels)

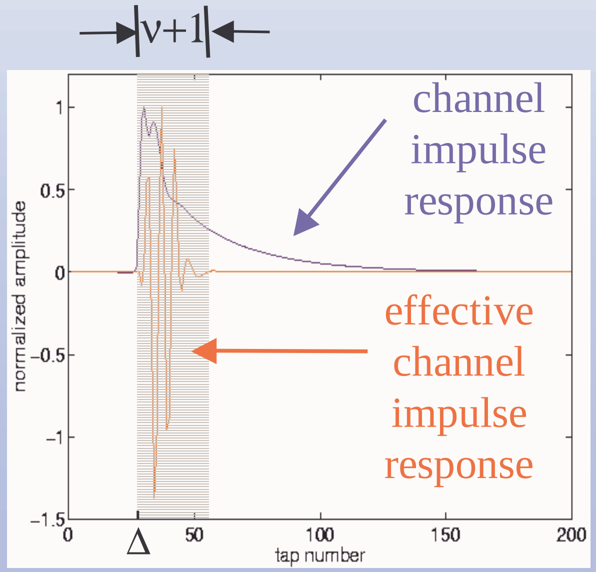
Maximum Shortening SNR (MSSNR) TEQ Design

- **Minimize energy in effective channel impulse response outside of window of $\nu+1$ samples, which causes ISI**
  [Melsa, Younce & Rohrs, 1996]
- **For each possible start position Δ of window of $\nu+1$ samples,**

$$\max_{w}\left(\text{SSNR in dB}\right) = \max_{w}\, 10\log_{10}\frac{\text{energy inside window after TEQ}}{\text{energy outside window after TEQ}}$$



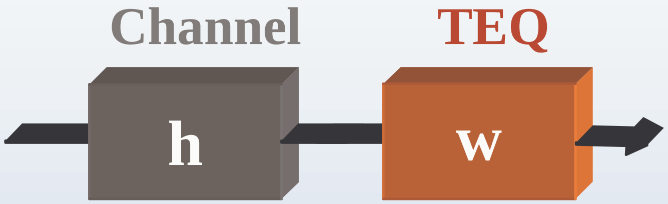
- **Disadvantages**
  - Does not consider channel noise
  - Does not consider *bit rate*
  - Requires Cholesky decomposition
  - Equivalent to MMSE for additive white Gaussian channel noise



Maximum Shortening SNR (MSSNR) TEQ Design

- **Choose w to minimize energy outside window of desired length**

  Locate window to capture maximum channel impulse response energy



$$\mathbf{h}_{wall}^T \mathbf{h}_{wall} = \mathbf{w}^T \mathbf{H}_{wall}^T \mathbf{H}_{wall} \mathbf{w} = \mathbf{w}^T \mathbf{A} \mathbf{w}$$

$$\mathbf{h}_{win}^T \mathbf{h}_{win} = \mathbf{w}^T \mathbf{H}_{win}^T \mathbf{H}_{win} \mathbf{w} = \mathbf{w}^T \mathbf{B} \mathbf{w}$$

h_{win}, h_{wall} **: effective channel within and outside window**

- **Objective function is shortening SNR (SSNR)**

$$\max_{\mathbf{w}} \left(\text{SSNR in dB}\right) = \max_{\mathbf{w}} \; 10 \log_{10} \frac{\mathbf{w}^T \mathbf{B} \mathbf{w}}{\mathbf{w}^T \mathbf{A} \mathbf{w}} \quad \text{subject to} \quad \mathbf{w}^T \mathbf{B} \mathbf{w} = 1$$

$$\mathbf{C} = \left(\sqrt{\mathbf{B}}\right)^{-1} \mathbf{A} \left(\sqrt{\mathbf{B}^T}\right)^{-1}$$

$$\mathbf{w}_{opt} = \left(\sqrt{\mathbf{B}^T}\right)^{-1} \mathbf{q}_{\min} \qquad \mathbf{q}_{\min} : \text{eigenvector of minimum eigenvalue of } \mathbf{C}$$

Modeling Signal, ISI, and Noise at Receiver

- **Receive**

  $\mathbf{y} = \mathbf{x} * \tilde{\mathbf{h}} + \tilde{\mathbf{n}}$

  $\tilde{\mathbf{h}} = \mathbf{w} * \mathbf{h}$

  x is transmitted signal
- **Symbols a b**
- **Symbol length**

  N = 4
- **Length of $\tilde{\mathbf{h}}$**

  L = 4
- **Cyclic prefix**

  v = 1
- **Delay**

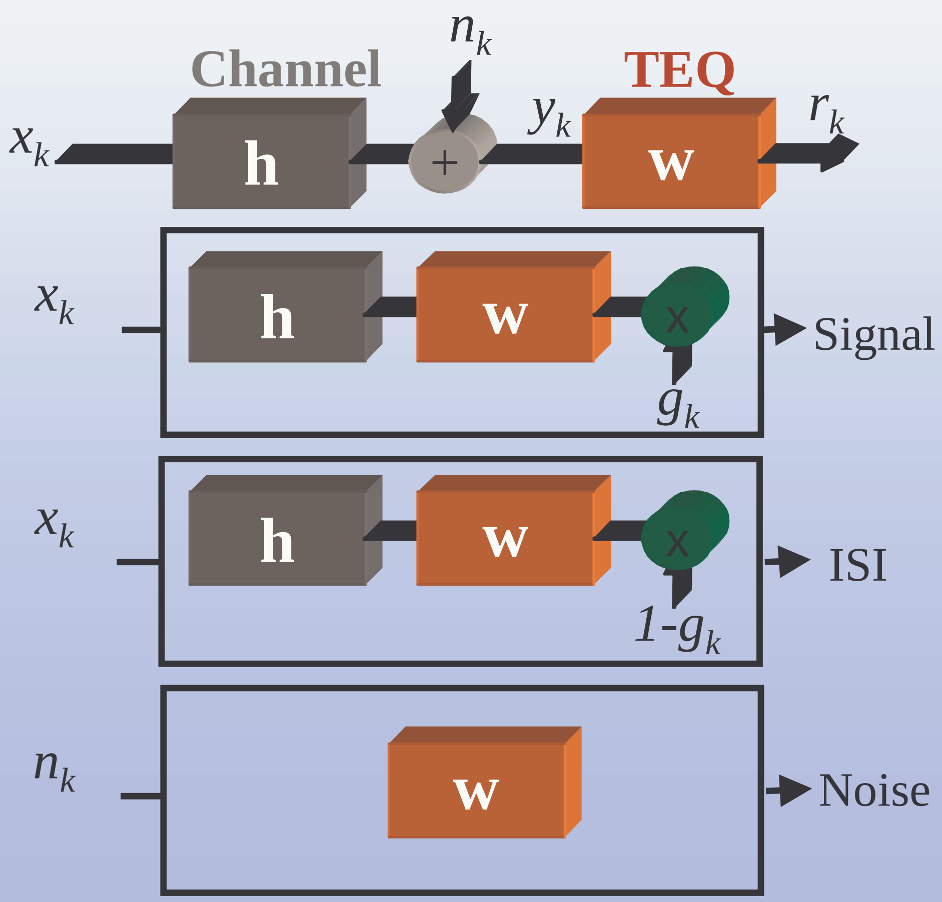
  $\Delta = 1$

$$\begin{array}{lccccccccccc}
\text{Delay} & y_1 & & \tilde{h}_1 a_4 & & & & & & & & \tilde{n}_1 \\
\text{CP} & y_2 & & \tilde{h}_1 a_1 & + & \tilde{h}_2 a_4 & & & & & & \tilde{n}_2 \\
& y_3 & & \tilde{h}_1 a_2 & + & \tilde{h}_2 a_1 & + & \tilde{h}_3 a_4 & & & & \tilde{n}_3 \\
& y_4 & & \tilde{h}_1 a_3 & + & \tilde{h}_2 a_2 & + & \tilde{h}_3 a_1 & + & \tilde{h}_4 a_4 & & \tilde{n}_4 \\
& y_5 & & \tilde{h}_1 a_4 & + & \tilde{h}_2 a_3 & + & \tilde{h}_3 a_2 & + & \tilde{h}_4 a_1 & & \tilde{n}_5 \\
& y_6 & & \tilde{h}_1 b_4 & + & \tilde{h}_2 a_4 & + & \tilde{h}_3 a_3 & + & \tilde{h}_4 a_2 & & \tilde{n}_6 \\
\text{CP} & y_7 & = & \tilde{h}_1 b_1 & + & \tilde{h}_2 b_4 & + & \tilde{h}_3 a_4 & + & \tilde{h}_4 a_3 & + & \tilde{n}_7 \\
& y_8 & & \tilde{h}_1 b_2 & + & \tilde{h}_2 b_1 & + & \tilde{h}_3 b_4 & + & \tilde{h}_4 a_4 & & \tilde{n}_8 \\
& y_9 & & \tilde{h}_1 b_3 & + & \tilde{h}_2 b_2 & + & \tilde{h}_3 b_1 & + & \tilde{h}_4 b_4 & & \tilde{n}_9 \\
& y_{10} & & \tilde{h}_1 b_4 & + & \tilde{h}_2 b_3 & + & \tilde{h}_3 b_2 & + & \tilde{h}_4 b_1 & & \tilde{n}_{10} \\
& y_{11} & & & & \tilde{h}_2 b_4 & + & \tilde{h}_3 b_3 & + & \tilde{h}_4 b_2 & & \tilde{n}_{11} \\
\text{Tail} & y_{12} & & & & & & \tilde{h}_3 b_4 & + & \tilde{h}_4 b_3 & & \tilde{n}_{12} \\
& y_{13} & & & & & & & & \tilde{h}_4 b_4 & & \tilde{n}_{13} \\
& & & \text{ISI} & & & \text{signal} & & & \text{ISI} & & \text{noise}
\end{array}$$

[Arslan, Evans & Kiaei, 2001]

Proposed Subchannel SNR Model

- **Partition equalized channel into signal path, ISI path, noise path** [Arslan, Evans & Kiaei, 2001]



- **Equalized channel impulse response** $\tilde{h}_k = h_k * w_k$

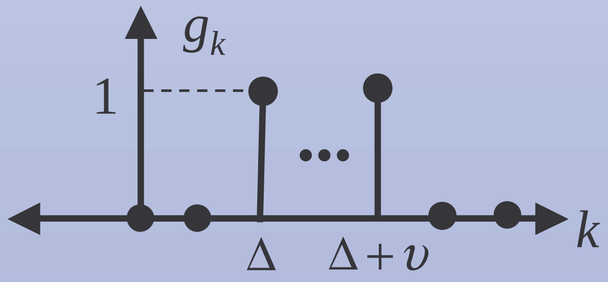
$$h_k^{signal} = \tilde{h}_k g_k$$

$$h_k^{ISI} = \tilde{h}_k (1 - g_k)$$

$$h_k^{noise} = w_k$$

- **Target window**

$$g_k = \begin{cases} 1 & \Delta \le k \le \Delta + \upsilon \\ 0 & \text{otherwise} \end{cases}$$



Proposed Subchannel SNR Definition

- **SNR in *i*th subchannel (leads to maximum bit rate method)**

  [Arslan, Evans & Kiaei, 2001]

$$\mathrm{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}} = \frac{S_{x,i}\left|H_i^{signal}\right|^2}{S_{n,i}\left|H_i^{noise}\right|^2 + S_{x,i}\left|H_i^{ISI}\right|^2}$$

$\left|H_i^{signal}\right|$ gain of h_k^{signal} in subchannel i

$\left|H_i^{ISI}\right|$ gain of h_k^{ISI} in subchannel i

$\left|H_i^{noise}\right|$ gain of h_k^{noise} in subchannel i

$S_{x,i}$: transmitted signal power in subchannel i

$S_{n,i}$: channel noise power in subchannel i

- **Divide SNR$_i$ numerator and denominator by noise power spectral density $S_{n,i}$ (leads to minimum ISI method)**

$$\mathrm{SNR}_i = \frac{\frac{S_{x,i}}{S_{n,i}}\left|H_i^{signal}\right|^2}{\left|H_i^{noise}\right|^2 + \frac{S_{x,i}}{S_{n,i}}\left|H_i^{ISI}\right|^2}$$

Conventional subchannel SNR is $S_{x,i}$ / $S_{n,i}$ →

Maximum Bit Rate (MBR) TEQ Design

- Subchannel SNR as nonlinear function of equalizer taps **w**

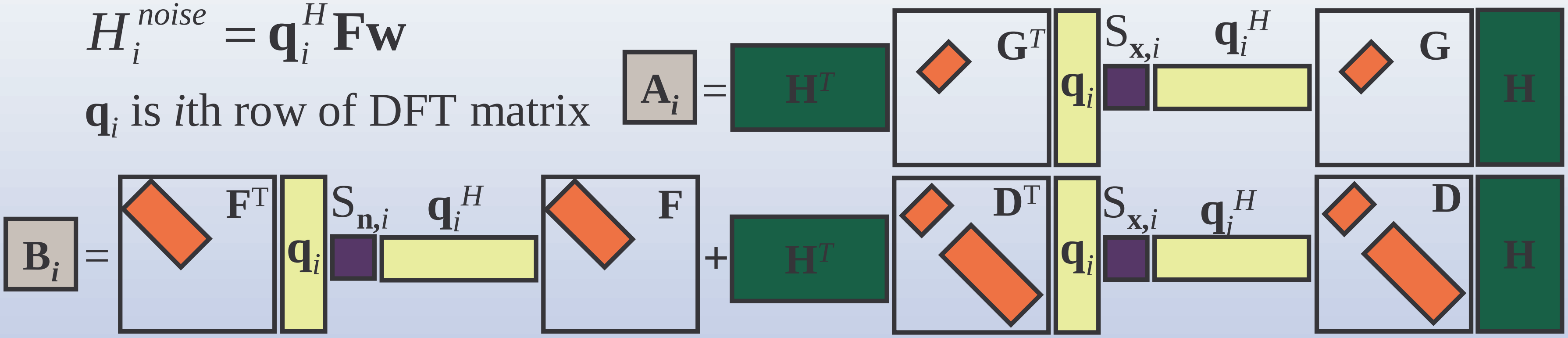
$$H_i^{signal} = \mathbf{q}_i^H \mathbf{G}\mathbf{H}\mathbf{w}$$

$$H_i^{ISI} = \mathbf{q}_i^H \mathbf{D}\mathbf{H}\mathbf{w}$$

$$H_i^{noise} = \mathbf{q}_i^H \mathbf{F}\mathbf{w}$$

$$\mathrm{SNR}_i = \frac{S_{x,i}\left|\mathbf{q}_i^H \mathbf{G}\mathbf{H}\mathbf{w}\right|^2}{S_{n,i}\left|\mathbf{q}_i^H \mathbf{F}\mathbf{w}\right|^2 + S_{x,i}\left|\mathbf{q}_i^H \mathbf{D}\mathbf{H}\mathbf{w}\right|^2} = \frac{\mathbf{w}^T \mathbf{A}_i \mathbf{w}}{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}$$

$\mathbf{q}_i$ is *i*th row of DFT matrix



- Maximize nonlinear function of bits/symbol with respect to **w**

$$b_{DMT} = \sum_{i=1}^{N/2} \log_2\left(1 + \frac{1}{\Gamma}\frac{\mathbf{w}^T \mathbf{A}_i \mathbf{w}}{\mathbf{w}^T \mathbf{B}_i \mathbf{w}}\right)$$

  - Good performance measure for comparison of TEQ design methods
  - Not an efficient TEQ design method in computational sense

Minimum-ISI (Min-ISI) TEQ Design

- **Rewrite proposed subchannel SNR** [Arslan, Evans & Kiaei, 2001]

$$\mathrm{SNR}_i = \frac{\frac{S_{x,i}}{S_{n,i}}\left|H_i^{signal}\right|^2}{\left|H_i^{noise}\right|^2 + \frac{S_{x,i}}{S_{n,i}}\left|H_i^{ISI}\right|^2}$$

ISI power weighted in frequency domain by inverse of noise spectrum

- **Generalize MSSNR method by weighting ISI in frequency**
  - ISI power in *i*th subchannel is $\mathrm{ISI}_i = S_{x,i}\left|\mathbf{q}_i^H \mathbf{DHw}\right|^2$
  - Minimize frequency weighted sum of subchannel ISI power

$$\sum_i \mathrm{ISI}_i = \sum_i K_i\left|\mathbf{q}_i^H \mathbf{DHw}\right|^2 = \mathbf{w}^T \mathbf{Xw}$$

  - Penalize ISI power in high conventional SNR subchannels: $K_i = \frac{S_{x,i}}{S_{n,i}}$
  - Constrain signal path gain to one to prevent all-zero solution for **w**

$$|\, h^{signal}\,|^2 = |\,\mathbf{GHw}\,|^2 = \mathbf{w}^T \mathbf{Yw} = 1$$

  - Solution is generalized eigenvector of **X** and **Y**

Simulation Results for 17-Tap TEQ

Achievable percentage of upper bound on bit rate

ADSL CSA Loop	Minimum MSE	Maximum Geometric SNR	Maximum Shortening SNR	Minimum ISI	Maximum Bit Rate	Upper Bound (Mbps)
1	43%	84%	62%	99%	99%	9.059
2	70%	73%	75%	98%	99%	10.344
3	64%	94%	82%	99%	99%	8.698
4	70%	68%	61%	98%	99%	8.695
5	61%	84%	72%	98%	99%	9.184
6	62%	93%	80%	99%	99%	8.407
7	57%	78%	74%	99%	99%	8.362
8	66%	90%	71%	99%	100%	7.394

Cyclic prefix length	32	Input power	23 dBm
FFT size (N)	512	Noise power	-140 dBm/Hz
Coding gain	4.2 dB	Crosstalk noise	8 ADSL disturbers
Margin	6 dB	POTS splitter	5th order Chebyshev

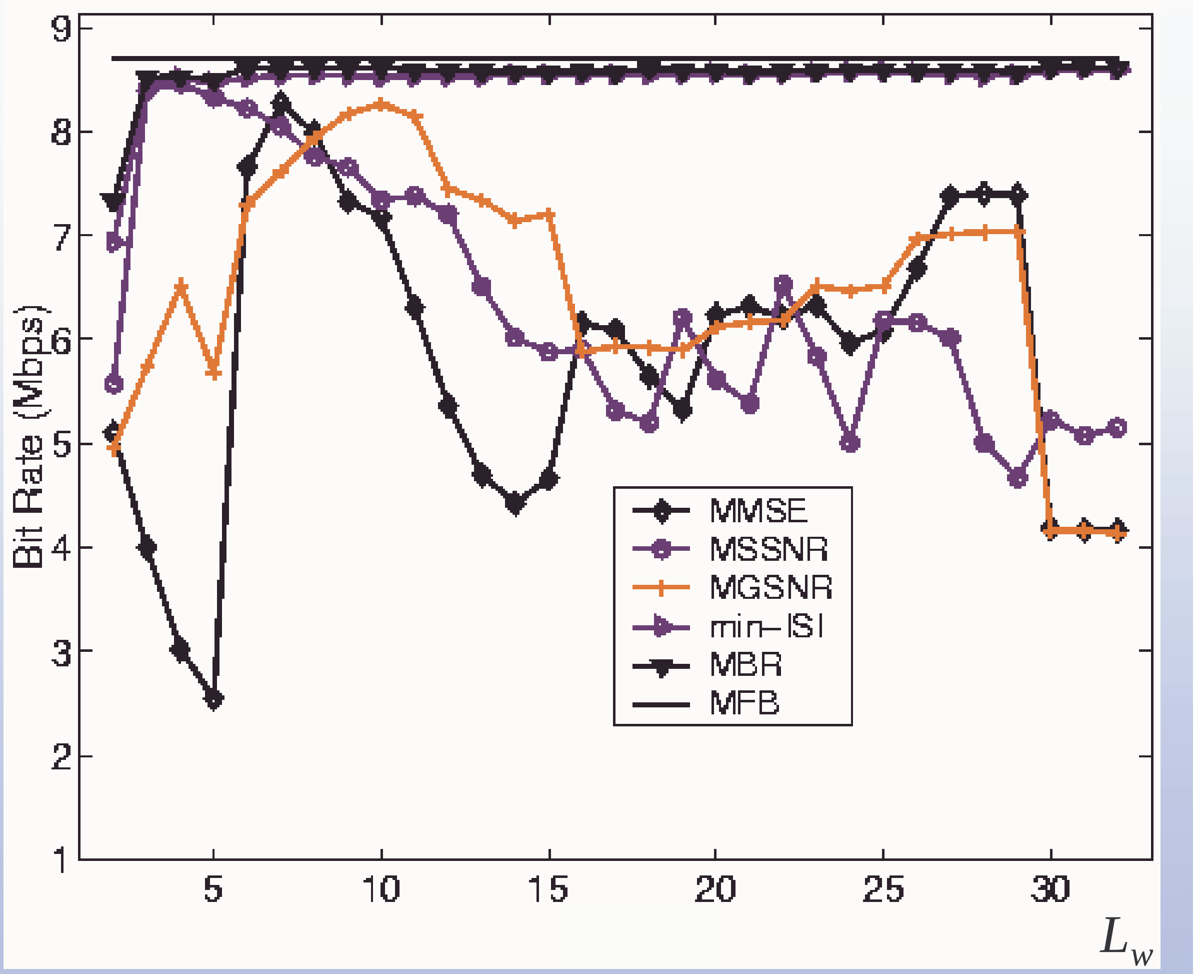
Simulation Results for Three-Tap TEQ

	Achievable percentage of matched filter bound on bit rate					
ADSL CSA Loop	Minimum MSE	Maximum Geometric SNR	Maximum Shortening SNR	Minimum ISI	Maximum Bit Rate	Upper Bound (Mbps)
1	54%	70%	96%	97%	98%	9.059
2	47%	71%	96%	96%	97%	10.344
3	57%	69%	92%	98%	99%	8.698
4	46%	66%	97%	97%	98%	8.695
5	52%	65%	96%	97%	98%	9.184
6	60%	71%	95%	98%	99%	8.407
7	46%	63%	93%	96%	97%	8.362
8	55%	61%	94%	98%	99%	7.394

Cyclic prefix length	32	Input power	23 dBm
FFT size (N)	512	Noise power	-140 dBm/Hz
Coding gain	4.2 dB	Crosstalk noise	8 ADSL disturbers
Margin	6 dB	POTS splitter	5th order Chebyshev

Bit Rate vs. Number of TEQ Taps

- Min-ISI and MBR give similar bit rate
- Three-tap Min-ISI, MBR, and MSSNR achieve matched filter bound (MFB)
- Beyond three taps, MSSNR bit rate falls
- 3-tap Min-ISI beats 21-tap MMSE
- Maximum Geometric SNR close to MMSE



cyclic prefix (ν)	32		
FFT size (*N*)	512	**input power**	23 dBm
coding gain	4.2 dB	**noise power**	-140 dBm/Hz
margin	6 dB	**crosstalk noise**	8 ADSL disturbers

Drawbacks to Minimum ISI Method

- **High complexity to compute X and Y matrices**
- **Sensitivity to transmission delay parameter Δ**
  - Requires computationally intensive search
- **Does not work for all TEQ lengths**
  - Formulation does not work for TEQ lengths longer than ν
  - Also sensitivity to fixed-point implementation due to Cholesky decomposition

- **Recursively calculate diagonal elements of X and Y from first column**
  [Wu, Arslan, Evans, 2000]
- **Reformulate Minimum ISI objective function**
- **Develop iterative method for reformulated objective**
  - Works for any TEQ length
  - Does not require a Cholesky decomposition
  - Works well under fixed-point arithmetic

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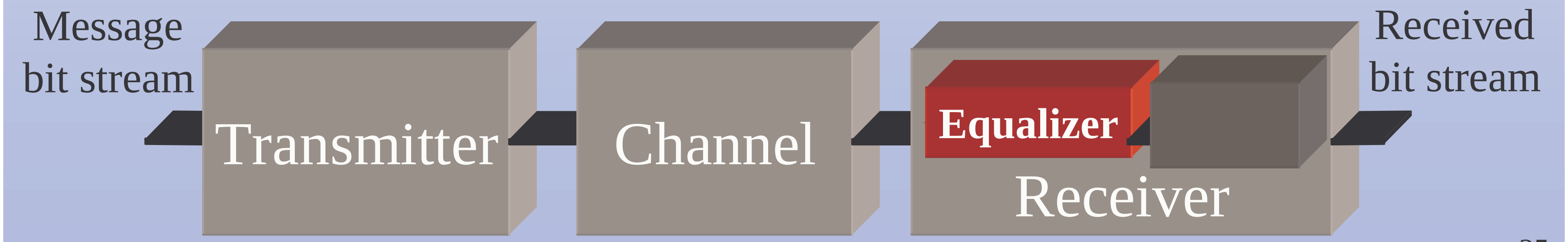
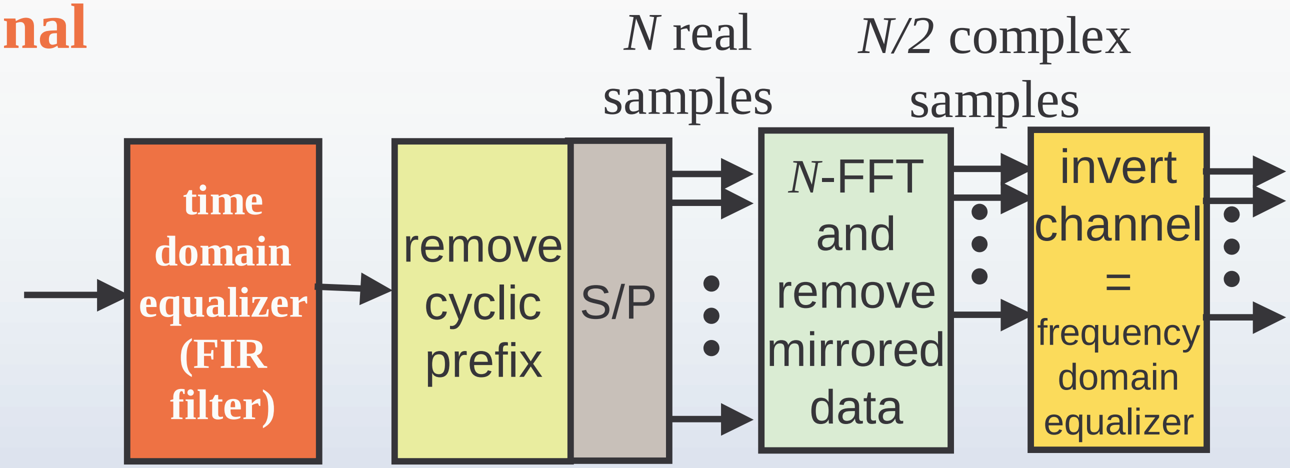
- **Per-tone equalizer** [Catholic University, Leuven, Belgium]
- **Dual-path equalizer**
- **Conclusion**



Drawbacks to Using Single FIR Filter for TEQ

- **Conventional equalizer**



- **Equalizes all tones in combined fashion: may limit bit rate**
- **Output of conventional equalizer for tone *i* computed using sequence of linear operations**

$$Z_i = D_i \,\mathrm{row}_i(\mathbf{Q}_N)\ \mathbf{Y}\ \mathbf{w}$$

Y w *represents convolution*

D_i is the complex scalar value of one-tap FEQ for tone i

$\mathbf{Q}_N$ is the $N \times N$ complex-valued FFT matrix

$\mathbf{Y}$ is an $N \times L_w$ real-valued Toeplitz matrix of received samples

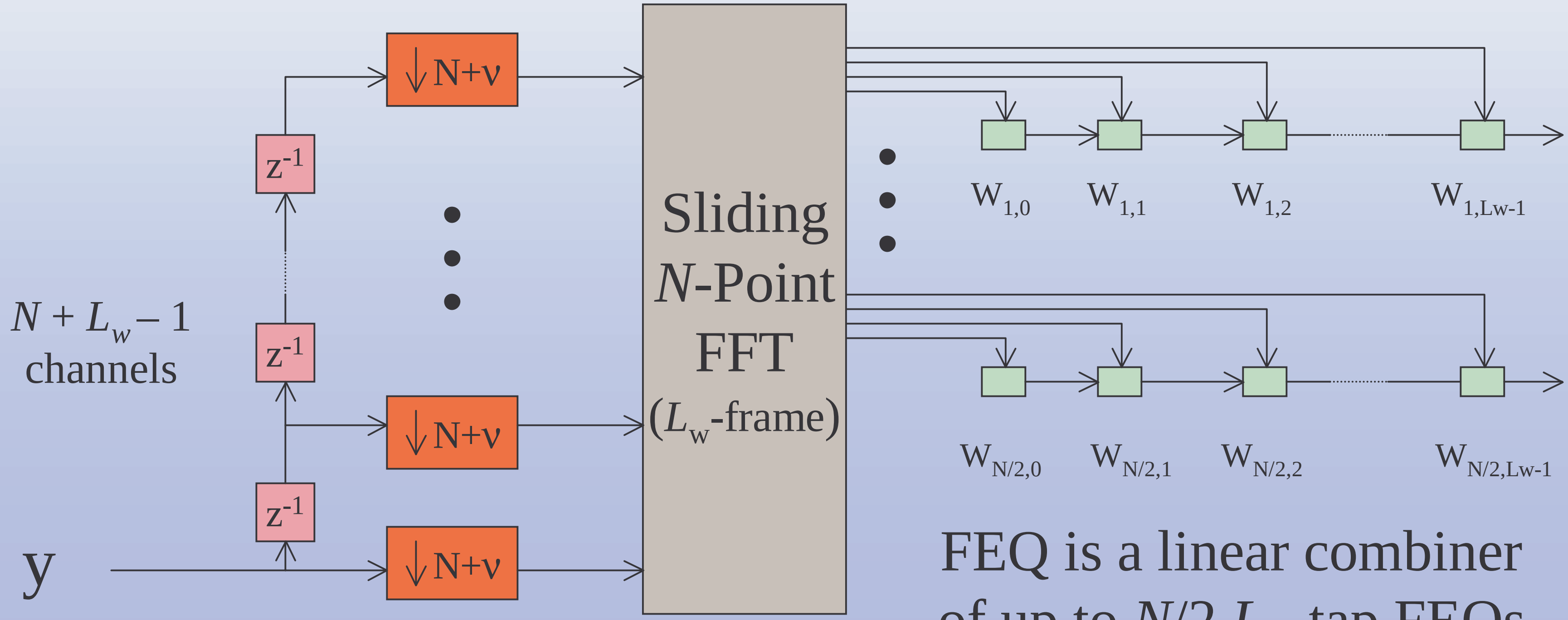
$\mathbf{w}$ is a $L_w \times 1$ column vector of real-valued TEQ taps

Frequency-Domain Per Tone Equalizer

- **Rewrite equalized FFT coefficient for each of *N*/2 tones**
  [Van Acker, Leus, Moonen, van de Wiel, Pollet, 2001]

$$Z_i = D_i \, \mathrm{row}_i(\mathbf{Q}_N) \, \mathbf{Y} \, \mathbf{w} = \mathrm{row}_i(\mathbf{Q}_N \, \mathbf{Y}) \, (\, \mathbf{w} \, D_i \,) = \mathrm{row}_i(\mathbf{Q}_N \, \mathbf{Y}) \, \mathbf{w}_i$$

  - Take sliding FFT to produce $N \times L_w$ matrix product Q_N Y
  - Design $\mathbf{w}_i$ for each tone



FEQ is a linear combiner of up to *N*/2 L_w-tap FEQs

Simulation Results

CSA Loop	MMSE UEC	Maximum SSNR	Minimum ISI	Data Rate Maximum	Least Sq. Per Tone	Filter Bank Bound (Mbps)
1	86.3%	95.0%	97.5%	99.6%	99.5%	11.417
2	87.2%	96.5%	97.3%	99.6%	99.5%	12.680
3	83.9%	97.0%	97.3%	99.5%	99.6%	10.995
4	81.9%	95.4%	98.2%	99.3%	99.1%	11.288
5	88.6%	97.1%	97.2%	99.6%	99.5%	11.470
6	82.7%	96.4%	98.3%	99.5%	99.4%	10.861
7	75.8%	96.7%	96.3%	98.8%	99.6%	10.752
8	82.6%	97.5%	97.5%	98.7%	99.2%	9.615
Average	83.6%	96.4%	97.5%	99.3%	99.4%	11.135

Cyclic prefix length	**32**	**Input power**	**23.93 dBm**
FFT size (*N*)	**512**	**Noise power**	**–140 dBm/Hz**
Coding gain	**0 dB**	**Crosstalk noise**	**49 ADSL disturbers**
Margin	**0 dB**	**Tx/Rx filters**	**2nd order Chebyshev**

Bit rates averaged over 2-32 tap equalizers

1,000 symbols transmitted (accuracy of ± 60 kbps or ± 0.5%)

Implementation Complexity Comparison

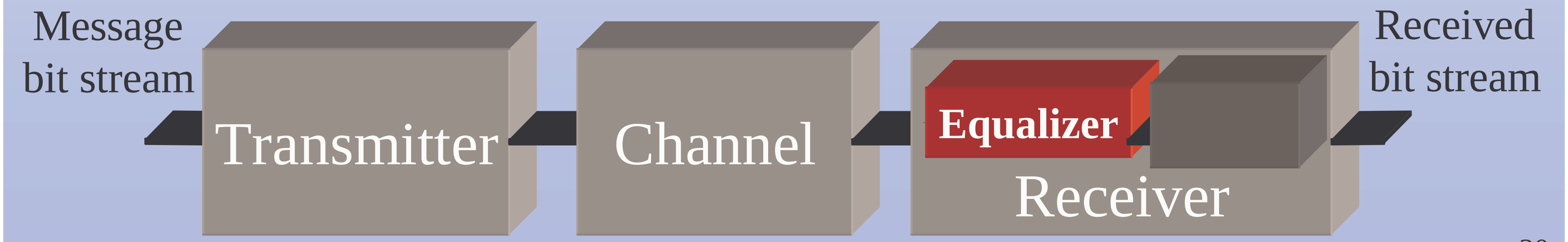
- **Data transmission**
  - Modified per tone equalizer has similar arithmetic complexity as a conventional equalizer but much higher memory usage and memory I/O
  - Memory I/O is larger bottleneck on programmable DSP

Equalizer	*Million Real MACS (8 taps)*	*Word of Memory (8 taps)*	*Million Real MACs (32 taps)*	*Words of Memory (32 taps)*
Per Tone (sliding FFT)	98	7,232	295	19,520
Modified Per Tone	55	6,151	105	18,463
Conventional	59	3,188	112	3,236

- **Training**
  - Conventional: design/adapt L_w real FIR filter coefficients. $O(L_w^3)$
  - Per-tone equalizer: design/adapt ½ $N L_w$ complex taps. $O(N L_w^3)$
  - Per-tone equalizer can train for groups of tones to reduce complexity

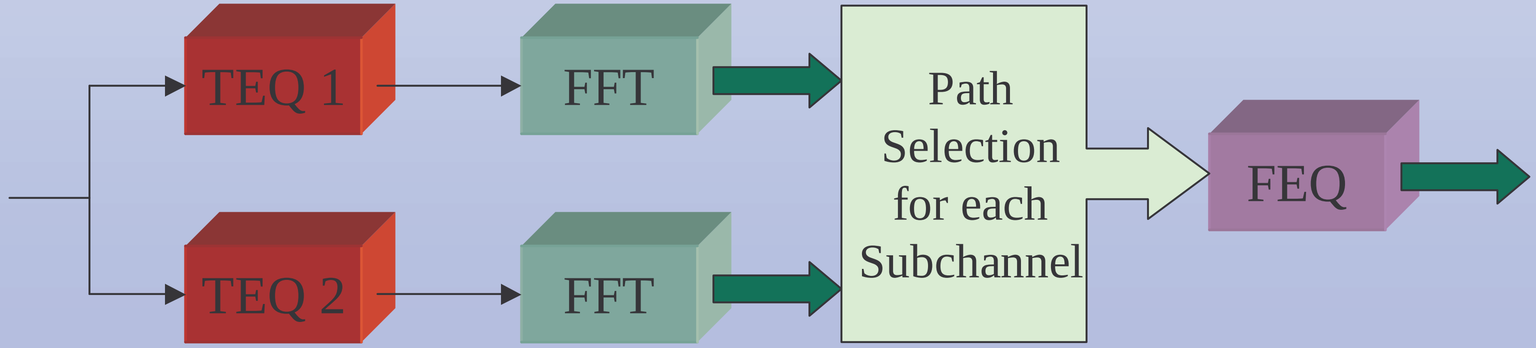
Outline

- **Multicarrier modulation**
- **Conventional equalizer**
  - Minimum Mean Squared Error design
  - Maximum Shortening Signal-to-Noise Ratio design
  - Maximum Bit Rate design (*optimal*)
  - Minimum Inter-symbol Interference design (*near-optimal*)
- **Per-tone equalizer**
- **Dual-path equalizer** [UT Austin]
- **Conclusion**



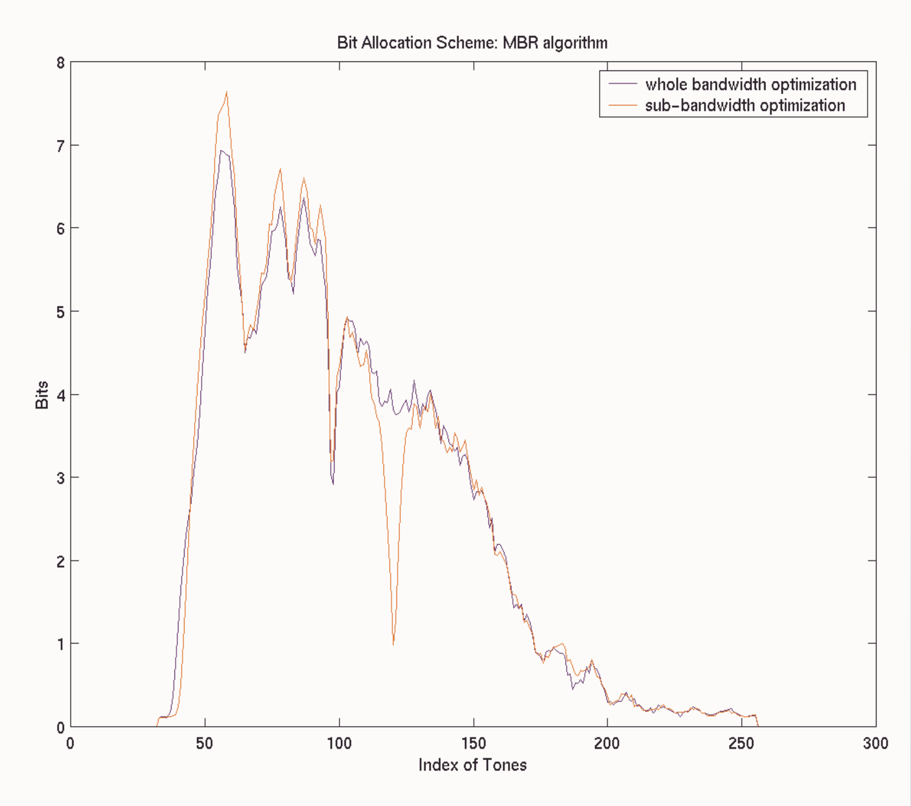
Dual-Path Time Domain Equalizer

- **Per tone equalizer**
  - Achieves higher bit rate than single-FIR TEQ
  - Has significantly more implementation complexity to train equalizer than MMSE, MSSNR, and Min-ISI single-FIR TEQs
- **Dual-path TEQ** [Ding, Redfern & Evans, 2002]
  - First FIR TEQ equalizes entire available bandwidth
  - Second FIR TEQ tailored for subchannels with higher SNR
  - Path selection for each subchannel is fixed during training
  - Enables reuse of previous ASIC designs of conventional equalizers



Simulation Results

- **ANSI-13 Loop**
  - Crosstalk: 24 DSL disturbers
  - Additive white Gaussian noise
- **Dual-Path TEQ**
  - Both paths use tones 33-255
  - Second path only optimizes tones 55-85
- **Achieved Bit Rate**
  - Path 1: 2.5080 Mbps
  - Dual Path: 2.6020 Mbps
  - 4% improvement in bit rate



Results for ANSI-13 channel

[Ding, Redfern & Evans, 2002]

Contributions by Research Group

- **New methods for single-path time-domain equalizer design**
  - Maximum Bit Rate method maximizes bit rate (*upper bound*)
  - Minimum Inter-Symbol Interference method (*real-time*, *fixed-point*)
- **Minimum Inter-Symbol Interference TEQ design method**
  - Generalizes Maximum Shortening SNR by frequency weighting ISI
  - Improve bit rate in an ADSL transceiver by change of software only
  - Implemented in real-time on three fixed-point digital signal processors: Motorola 56000, TI TMS320C6200 and TI TMS320C5000

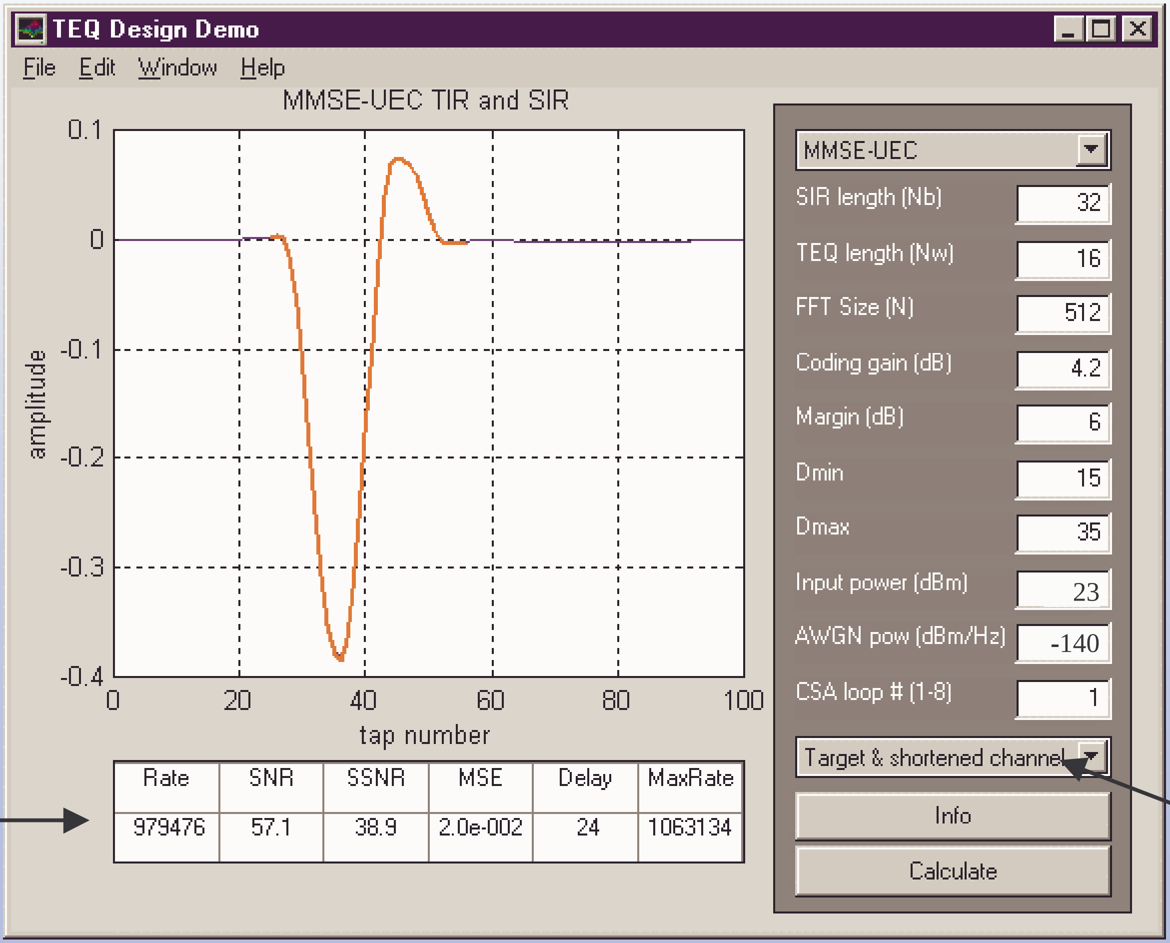
http://www.ece.utexas.edu/~bevans/projects/adsl

- **New dual-path time-domain equalizer**
- **Comparison to frequency-domain per-tone equalizer**
  - Competitive bit rates
  - Lower implementation complexity in training and data transmission

Matlab DMTTEQ Toolbox 3.1

- **Single-path, dual-path, per-tone & TEQ filter bank equalizers**

  Available at http://www.ece.utexas.edu/~bevans/projects/adsl/dmtteq/



MMSE-UEC

SIR length (Nb) 32

TEQ length (Nw) 16

FFT Size (N) 512

Coding gain (dB) 4.2

Margin (dB) 6

Dmin 15

Dmax 35

Input power (dBm) 23

AWGN pow (dBm/Hz) -140

CSA loop # (1-8) 1

Target & shortened channel

Rate	SNR	SSNR	MSE	Delay	MaxRate
979476	57.1	38.9	2.0e-002	24	1063134

default parameters from G.DMT ADSL standard

various performance measures

different graphical views

Backup Slides

Applications of Broadband Access

Residential

Application	*Downstream rate (kb/s)*	*Upstream rate (kb/s)*	*Willing to pay*	*Demand Potential*
Database Access	384	9	High	Medium
On-line directory; yellow pages	384	9	Low	High
Video Phone	1,500	1,500	High	Medium
Home Shopping	1,500	64	Low	Medium
Video Games	1,500	1,500	Medium	Medium
Internet	3,000	384	High	Medium
Broadcast Video	6,000	0	Low	High
High definition TV	24,000	0	High	Medium

Business

Application	*Downstream rate (kb/s)*	*Upstream rate (kb/s)*	*Willing to pay*	*Demand Potential*
On-line directory; yellow pages	384	9	Medium	High
Financial news	1,500	9	Medium	Low
Video phone	1,500	1,500	High	Low
Internet	3,000	384	High	High
Video conference	3,000	3,000	High	Low
Remote office	6,000	1,500	High	Medium
LAN interconnection	10,000	10,000	Medium	Medium
Supercomputing, CAD	45,000	45,000	High	Low

Selected DSL Standards

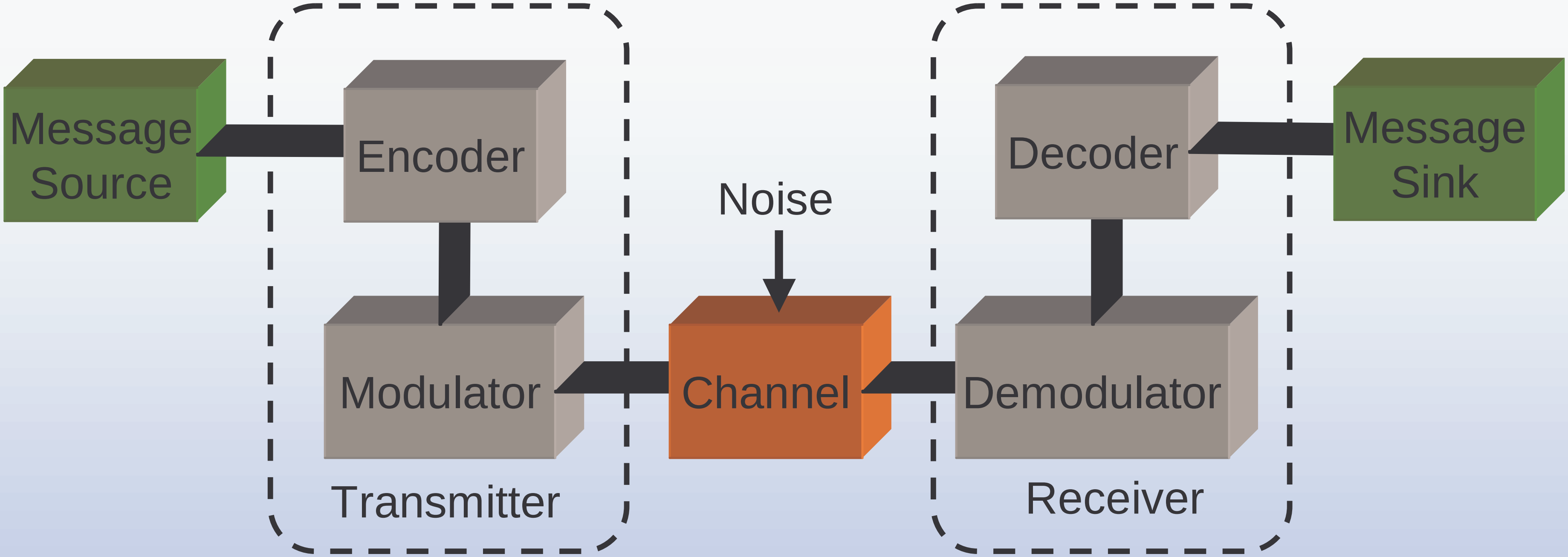
Standard	*Meaning*	*Data Rate*	*Mode*	*Applications*
ISDN	Integrated Services Digital Network	144 kbps	Symmetric	Internet Access, Voice, Pair Gain (2 channels)
T1	T-Carrier One (requires two pairs)	1.544 Mbps	Symmetric	Enterprise, Expansion, Internet Service
HDSL	High-Speed Digital Subscriber Line (requires two pairs)	1.544 Mbps	Symmetric	Pair Gain (12 channels), Internet Access, T1/E1 replacement
HDSL2	Single Line HDSL	1.544 Mbps	Symmetric	Same as HDSL except pair gain is 24 channels
G.Lite ADSL	Splitterless Asymmetric Digital Subscriber Line	up to 1.5 Mbps up to 512 kbps	Downstream Upstream	Internet Access, Digital Video
G.DMT ADSL	Asymmetric Digital Subscriber Line	up to 10 Mbps up to 1 Mbps	Downstream Upstream	Internet Access, Digital Video
VDSL	Very High-Speed Digital Subscriber Line (proposed)	up to 22 Mbps up to 3 Mbps up to 13 Mbps	Downstream Upstream Symmetric	Internet Access, Digital Video, Broadcast Video

Courtesy of Shawn McCaslin (Cicada Semiconductor, Austin, TX)

Discrete Multitone DSL Standards

- **Discrete multitone (DMT) modulation uses multiple carriers**
- **ADSL – Asymmetric DSL (G.DMT)**
  - *Asymmetric*: 8 Mbps downstream and 1 Mbps upstream
  - *Data band*: 25 kHz – 1.1 MHz
  - Maximum data rates possible in standard (ideal case)
    - Echo cancelled: 14.94 Mbps downstream, 1.56 Mbps upstream
    - Frequency division multiplexing: 13.38 Mbps downstream, 1.56 Mbps up
  - Widespread deployment in US, Canada, Western Europe, Hong Kong
    - Central office providers only installing frequency-division ADSL
    - ADSL modems have about 1/3 of market, and cable modems have 2/3
- **VDSL – Very High Rate DSL**
  - *Asymmetric*: either 22/3 or 13/3 Mbps downstream/upstream
  - *Symmetric*: 13, 9, or 6 Mbps each direction
  - *Data band*: 1 – 12 MHz
  - DMT and single carrier modulation supported
  - DMT VDSL essentially higher speed version of G.DMT ADSL

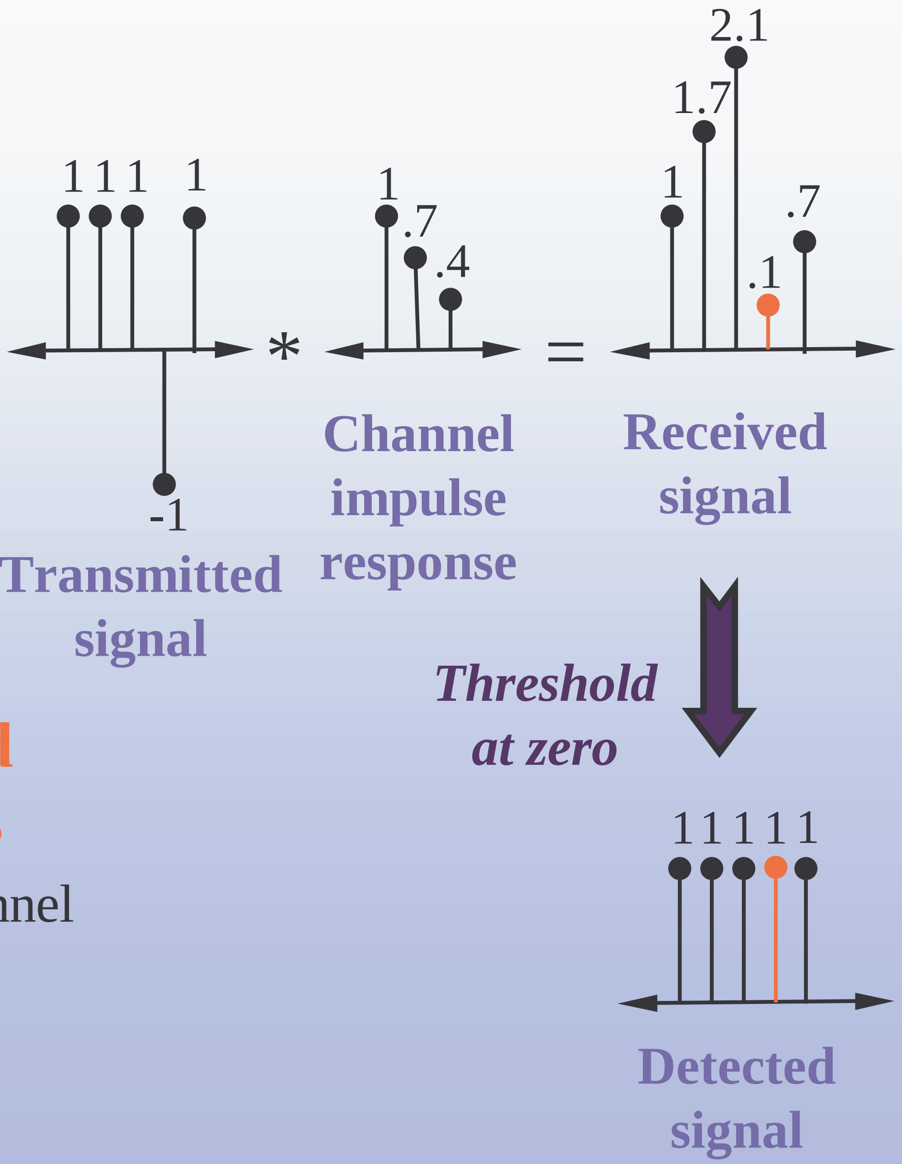
A Digital Communications System



- **Encoder maps a group of message bits to data symbols**
- **Modulator maps these symbols to analog waveforms**
- **Demodulator maps received waveforms back to symbols**
- **Decoder maps the symbols back to binary message bits**

Intersymbol Interference (ISI)

- **Ideal channel**
  - Impulse response is impulse
  - Flat frequency response
- **Non-ideal channel**
  - Causes ISI
  - Channel memory
  - Magnitude and phase variation
- **Received symbol is weighted sum of neighboring symbols**
  - Weights are determined by channel impulse response



Transmitted signal

Channel impulse response

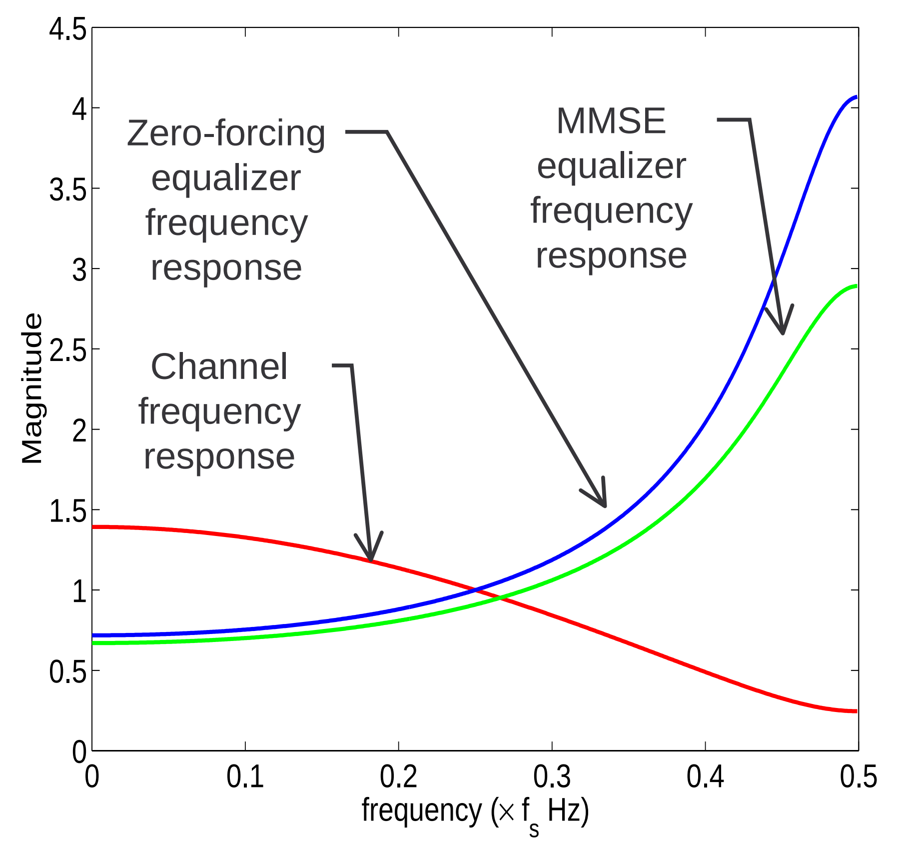
Received signal

Threshold at zero

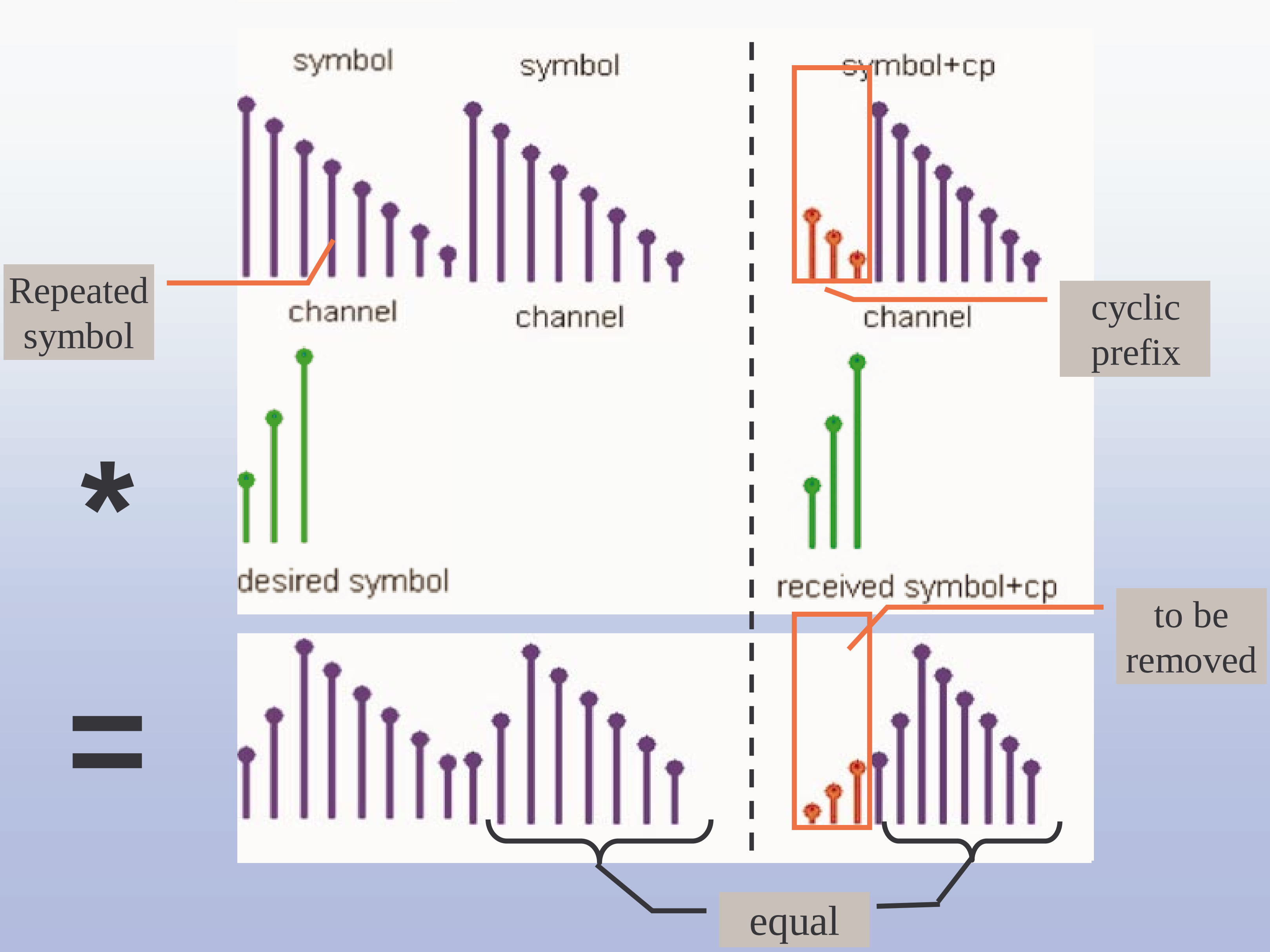
Detected signal

Combat ISI with Equalization

- **Equalization because channel response is not flat**
- **Zero-forcing equalizer**
  - Inverts channel
  - Flattens freq. response
  - Amplifies noise
- **MMSE equalizer**
  - Optimizes trade-off between noise amplification and ISI
- **Decision-feedback equalizer**
  - Increases complexity
  - Propagates error



Cyclic Prefix



Repeated symbol

cyclic prefix

*

to be removed

=

equal

Open Issues for Multicarrier Modulation

- **Advantages**
  - Efficient use of bandwidth without full channel equalization
  - Robust against impulsive noise and narrowband interference
  - Dynamic rate adaptation
- **Disadvantages**
  - ***Transmitter:*** High signal peak-to-average power ratio
  - ***Receiver:*** Sensitive to frequency and phase offset in carriers
- **Open issues**
  - Pulse shapes of subchannels (*orthogonal, efficient realization*)
  - Channel equalizer design (*increase bit rate, reduce complexity*)
  - Synchronization (*timing recovery, symbol synchronization*)
  - Bit loading (*allocation of bits in each subchannel*)
  - Echo cancellation

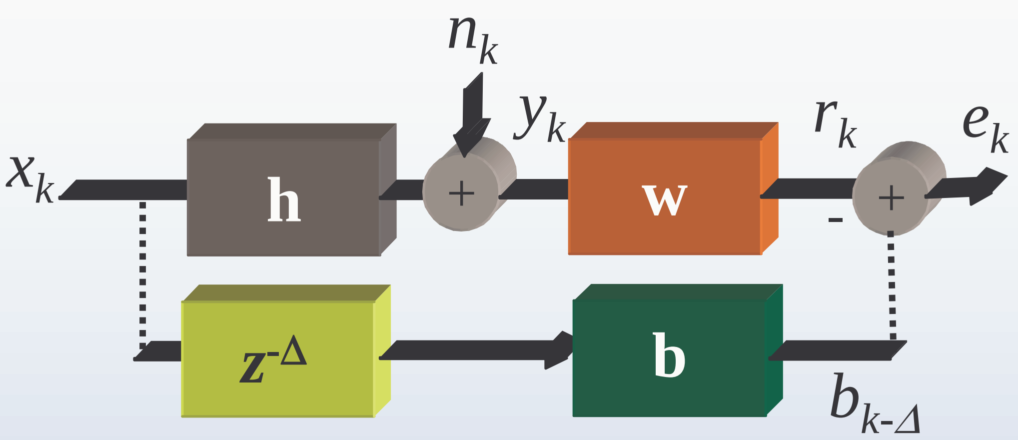
TEQ Algorithm

- **ADSL standards**
  - Set aside 1024 frames (~.25s) for TEQ estimation
  - Reserved ~16,000 frames for channel and noise estimation for the purpose of SNR calculation
- **TEQ is estimated before the SNR calculations**
- **Noise power and channel impulse response can be estimated before time slot reserved for TEQ if the TEQ algorithm needs that information**

Single-FIR Time-Domain Equalizer Design Methods

- ***All methods below perform optimization at TEQ output***
- **Minimizing the mean squared error**
  - Minimize mean squared error (MMSE) method [Chow & Cioffi, 1992]
  - Geometric SNR method [Al-Dhahir & Cioffi, 1996]
- **Minimizing energy outside of shortened (equalized) channel impulse response**
  - Maximum Shortening SNR method [Melsa, Younce & Rohrs, 1996]
  - Divide-and-conquer methods [Lu, Evans, Clark, 2000]
  - Minimum ISI method [Arslan, Evans & Kiaei, 2000]
- **Maximizing bit rate** [Arslan, Evans & Kiaei, 2000]
- **Implementation**
  - Geometric SNR is difficult to automate (requires human intervention)
  - Maximum bit rate method needs nonlinear optimization solver
  - Other methods implemented on fixed-point digital signal processors

Minimum Mean Squared Error (MMSE) TEQ



$$\mathbf{w} = \left[w_0 \quad w_1 \quad \mp \quad w_{L_w-1}\right]^T$$

$$\mathbf{b} = \left[b_0 \;\; b_1 \;\; \mp \;\; b_\nu\right]^T$$

$$\hat{\mathbf{b}} = \left[\mathbf{0}_\Delta \mid \mathbf{b}^T \mid \mathbf{0}_{L_h-\Delta-\nu-1}\right]^T$$

$$\text{MSE} = \varepsilon\{e_k^2\} = \hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{xx}} \hat{\mathbf{b}} - 2\hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{xy}} \mathbf{w} + \mathbf{w}^T \mathbf{R}_{\mathbf{yy}} \mathbf{w}$$

minimum MSE is achieved only if $\mathbf{b}^T \mathbf{R}_{\mathbf{xy}} = \mathbf{w}^T \mathbf{R}_{\mathbf{yy}}$

$$\text{MSE} = \hat{\mathbf{b}}^T \left[\mathbf{R}_{\mathbf{xx}} - \mathbf{R}_{\mathbf{xy}} \mathbf{R}_{\mathbf{yy}}^{-1} \mathbf{R}_{\mathbf{yx}}\right] \hat{\mathbf{b}} = \hat{\mathbf{b}}^T \mathbf{R}_{\mathbf{x|y}} \hat{\mathbf{b}}$$

Define $\mathbf{R}_\Delta = \mathbf{O}^T \mathbf{R}_{\mathbf{x|y}} \mathbf{O}$ then $\text{MSE} = \mathbf{b}^T \mathbf{R}_\Delta \mathbf{b}$

O **selects the proper part out of** $R_{x|y}$ **corresponding to the delay Δ**

Near-optimal Minimum-ISI (Min-ISI) TEQ Design

- **Generalizes MSSNR method by frequency weighting ISI**
  - ISI power in *i*th subchannel is $\mathrm{ISI}_i = S_{x,i} \left| \mathbf{q}_i^H \mathbf{D}\mathbf{H}\mathbf{w} \right|^2$
  - Minimize ISI power as a frequency weighted sum of subchannel ISI

$$\sum_i \mathrm{ISI}_i = \sum_i K_i \left| \mathbf{q}_i^H \mathbf{D}\mathbf{H}\mathbf{w} \right|^2 = \mathbf{w}^T \mathbf{X}\mathbf{w}$$

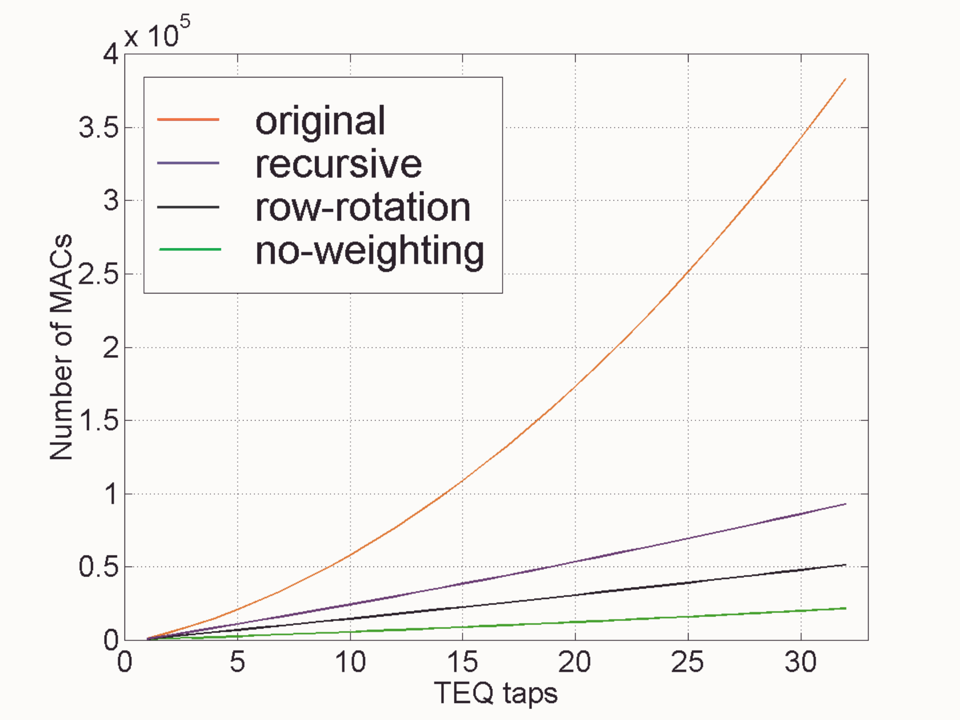
  - Constrain signal path gain to one to prevent all-zero solution

$$| h^{signal} |^2 = | \mathbf{G}\mathbf{H}\mathbf{w} |^2 = \mathbf{w}^T \mathbf{Y}\mathbf{w} = 1$$

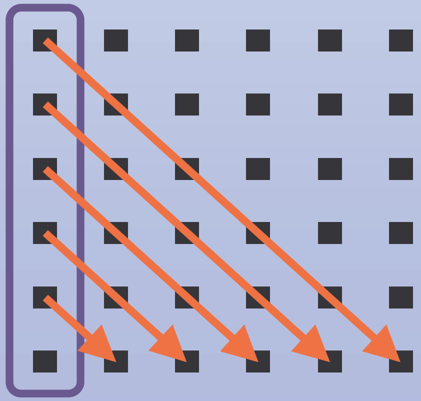
  - Solution is a generalized eigenvector of **X** and **Y**
- **Possible weightings**
  - Amplify ISI objective function in subchannels with low noise power (high SNR) to put ISI in low SNR bins: $K_i = \dfrac{S_{x,i}}{S_{n,i}}$
  - Set weighting equal to input power spectrum: $K_i = S_{x,i}$
  - Set weighting to be constant in all subchannels (MSSNR): $K_i = 1$
- **Performance virtually equal to MBR (optimal) method**

Efficient Implementations of Min-ISI Method

- **Generalized eigenvalue problem can solved with generalized power iteration:** $\mathbf{X}\mathbf{w}^{k+1} = \mathbf{Y}\mathbf{w}^{k}$
- **Recursively calculate diagonal elements of X and Y from first column** [Wu, Arslan, Evans, 2000]



Method	*Bit Rate*	*MACs*
Original	99.6%	132,896
Recursive	99.5%	44,432
Row-rotation	99.5%	25,872
No-weighting	97.8%	10,064



Motivation for Divide-and-Conquer Methods

- **Fast methods for implementing Maximum SSNR method**
- **Maximum SSNR Method**
  - For each Δ, maximum SSNR method requires
    - Multiplications: $(L_h + \frac{7}{6})L_w + \frac{5}{2}L_w^2 + \frac{25}{3}L_w^3$
    - Additions: $(L_h - \frac{5}{6})\,L_w - \frac{3}{2}L_w^2 + \frac{25}{3}L_w^3$
    - Divisions: L_w^2
  - Exhaustive search for the optimal delay Δ

$$0 \le \Delta \le L_h + L_w - \nu - 2 \, (\quad 0 \le \Delta \le 499$$

- **Divide L_w TEQ taps into (L_w - 1) two-tap filters in cascade**
  - Design first two-tap filter then second and so forth (greedy approach)
- **Develop heuristic to estimate the optimal delay**

Divide-and-Conquer Approach

- **The i^{th} two-tap filter is initialized as either**
  - Unit tap constraint (UTC) $\mathbf{w}_i = \begin{bmatrix} 1 \\ g_i \end{bmatrix}$
  - Unit norm constraint (UNC) $\mathbf{w}_i = \begin{bmatrix} \sin \theta_i \\ \cos \theta_i \end{bmatrix}$
- **Calculate best $\boldsymbol{g_i}$ or $\boldsymbol{\theta_i}$ by using a greedy approach either by**
  - Minimizing $\frac{1}{\text{SSNR}}$ (Divide-and-conquer TEQ minimization)
  - Minimizing energy in $\mathbf{h}_{\text{wall}}$ (Divide-and conquer TEQ cancellation)
- **Convolve two-tap filters to obtain TEQ**

Divide-and-Conquer TEQ Minimization (UTC)

- **At i^{th} iteration, minimize J_i over g_i**

$$J_i = \frac{\mathbf{w}_i^T \mathbf{A} \mathbf{w}_i}{\mathbf{w}_i^T \mathbf{B} \mathbf{w}_i} = \frac{\begin{bmatrix} 1 & g_i \end{bmatrix} \begin{pmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{pmatrix} \begin{pmatrix} 1 \\ g_i \end{pmatrix}}{\begin{bmatrix} 1 & g_i \end{bmatrix} \begin{pmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{pmatrix} \begin{pmatrix} 1 \\ g_i \end{pmatrix}} = \frac{a_{1,i} + 2a_{2,i} g_i + a_{3,i} g_i^2}{b_{1,i} + 2b_{2,i} g_i + b_{3,i} g_i^2}$$

- **Closed-form solution**

$$g_{i(1,2)} = \frac{-\left(a_{3,i} b_{1,i} - a_{1,i} b_{3,i}\right)}{2\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)} \pm \frac{\sqrt{D}}{2\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)}$$

$$D = \left(a_{3,i} b_{1,i} - a_{1,i} b_{3,i}\right)^2 - 4\left(a_{3,i} b_{2,i} - a_{2,i} b_{3,i}\right)\left(a_{2,i} b_{1,i} - a_{1,i} b_{2,i}\right)$$

Divide-and-Conquer TEQ Minimization (UNC)

- **At i^{th} iteration, minimize J_i over η_i**

$$J_i = \frac{\mathbf{w}_i^T \mathbf{A} \mathbf{w}_i}{\mathbf{w}_i^T \mathbf{B} \mathbf{w}_i} = \frac{\left(\sin\theta_i \begin{bmatrix} 1 & \eta_i \end{bmatrix}\right) \begin{pmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{pmatrix} \left(\sin\theta_i \begin{pmatrix} 1 \\ \eta_i \end{pmatrix}\right)}{\left(\sin\theta_i \begin{bmatrix} 1 & \eta_i \end{bmatrix}\right) \begin{pmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{pmatrix} \left(\sin\theta_i \begin{pmatrix} 1 \\ \eta_i \end{pmatrix}\right)}$$

$$= \frac{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{pmatrix} a_{1,i} & a_{2,i} \\ a_{2,i} & a_{3,i} \end{pmatrix} \begin{pmatrix} 1 \\ \eta_i \end{pmatrix}}{\begin{bmatrix} 1 & \eta_i \end{bmatrix} \begin{pmatrix} b_{1,i} & b_{2,i} \\ b_{2,i} & b_{3,i} \end{pmatrix} \begin{pmatrix} 1 \\ \eta_i \end{pmatrix}}$$

Calculate η_i in the same way as g_i for UTC version of this method

- **where** $\mathbf{w}_i = \begin{pmatrix} \sin\theta_i \\ \cos\theta_i \end{pmatrix} = \sin\theta_i \begin{pmatrix} 1 \\ \cos\theta_i / \sin\theta_i \end{pmatrix} = \sin\theta_i \begin{pmatrix} 1 \\ \eta_i \end{pmatrix}$

Divide-and-Conquer TEQ Cancellation (UTC)

- **At i^{th} iteration, minimize J_i over g_i**

$$J_i = \tilde{\mathbf{h}}_{\text{wall}}^T \tilde{\mathbf{h}}_{\text{wall}} = \sum_{k \in S} \left(\tilde{h}_{i-1}(k) + g_i \tilde{h}_{i-1}(k-1) \right)^2,$$

$$S = \left\{ 1, 2, \ldots, \Delta, \Delta + \nu + 2, \ldots, L_{\tilde{h}_{i-1}} \right\}$$

- **Closed-form solution for the i^{th} two-tap FIR filter**

$$g_i = -\frac{\sum_{k \in S} \tilde{h}_{i-1}(k-1) \tilde{h}_{i-1}(k)}{\sum_{k \in S} \tilde{h}_{i-1}^2(k-1)}$$

Divide-and-Conquer TEQ Cancellation (UNC)

- **At i^{th} iteration, minimize J_i over θ_I**

$$J_i = \tilde{\mathbf{h}}_{\text{wall}}^T \tilde{\mathbf{h}}_{\text{wall}} = \sum_{k \in S} \left(\tilde{h}_{i-1}(k) \sin\theta_i + \tilde{h}_{i-1}(k-1) \cos\theta_i \right)^2,$$

$$S = \left\{ 1, 2, \mp\ , \Delta, \Delta + \nu + 2, \mp\ , L_{\tilde{h}_{i-1}} \right\}$$

- **Closed-form solution**

$$\sin\theta_i = \pm \sqrt{0.5 \left(1 \pm \sqrt{\frac{a^2}{a^2 + 4b^2}} \right)}, \ \cos\theta_i = \pm \sqrt{0.5 \left(1 \mp \sqrt{\frac{a^2}{a^2 + 4b^2}} \right)}$$

$$a = \sum_{k \in S} \left(\tilde{h}_{i-1}^2(k) - \tilde{h}_{i-1}^2(k-1) \right), \ b = \sum_{k \in S} \tilde{h}_{i-1}(k-1) \tilde{h}_{i-1}(k)$$

Computational Complexity

- **Computational complexity for each candidate Δ**

Method	$\times$	$+$	$\div$	*Memory (words)*
Maximum SSNR	120379	118552	441	1899
DC-TEQ-minimization (UTC)	53240	52980	60	563
DC-TEQ-cancellation (UNC)	42280	42160	20	555
DC-TEQ-cancellation (UTC)	41000	40880	20	554

G.DMT
ADSL
$L_h = 512$
$\nu = 32$
$L_w = 21$

- **Divide-and-conquer methods vs. maximum SSNR method**
  - Reduces multiplications, additions, divisions, and memory
  - No matrix calculations (saves on memory accesses)
  - Avoids matrix inversion, and eigenvalue and Cholesky decompositions

Heuristic Search for the Optimal Delay

- **Estimate optimal delay Δ before computing TEQ taps**

$$\Delta_{\text{ratio}} = \arg\max_{\Delta} \frac{\text{energy inside a window of original } \mathbf{h}}{\text{energy outside a window of original } \mathbf{h}}$$

- **Total computational cost**
  - Multiplications: L_h
  - Additions: $3L_h - 3$
  - Divisions: L_h
- **Performance of heuristic vs. exhaustive search**
  - Reduce computational complexity by factor of 500
  - 2% loss in SSNR for TEQ with four taps or more
  - 8% loss in SSNR for two-tap TEQ

Comparison of Earlier Methods

Method	***MMSE***	***MSSNR***	***Geometric***
Advantages			
Maximize bit rate			✓
Minimize ISI		✓	
Bit Rate	Low-medium	High	Low-medium
Disadvantages			
Nonlinear optimization			✓
Computational complexity	Low	Medium	High
Artificial constraints	✓		✓
Ad-hoc parameters			✓
Lowpass frequency response	✓		✓
Unrealistic assumptions			✓

MBR TEQ vs. Geometric TEQ

Method	***MBR***	***Geometric***
Advantages		
Maximize channel capacity	✓	✓
Minimize ISI	✓	
Bit rate	optimal	Low-medium
Disadvantages		
Low-pass frequency response		✓
Computationally complex	✓	✓
Artificial constraints		✓
Ad-hoc parameters		✓
Nonlinear optimization	✓	✓
Unrealistic assumptions		✓

Min-ISI TEQ vs. MSSNR TEQ

Method	***Min-ISI***	***MSSNR***
Advantages		
Maximize channel capacity		
Minimize ISI	✓	✓
Frequency domain weighting	✓	
Bit rate	high	high
Disadvantages		
Computationally complex	very high	high

- **Min-ISI weights ISI power with the SNR**
  - Residual ISI power should be placed in high noise frequency bands

$$\mathrm{SNR}_i = \frac{\text{signal power}}{\text{noise power} + \text{ISI power}}$$

$$\mathrm{SNR}_{50} = \frac{1}{10} = 0.1$$

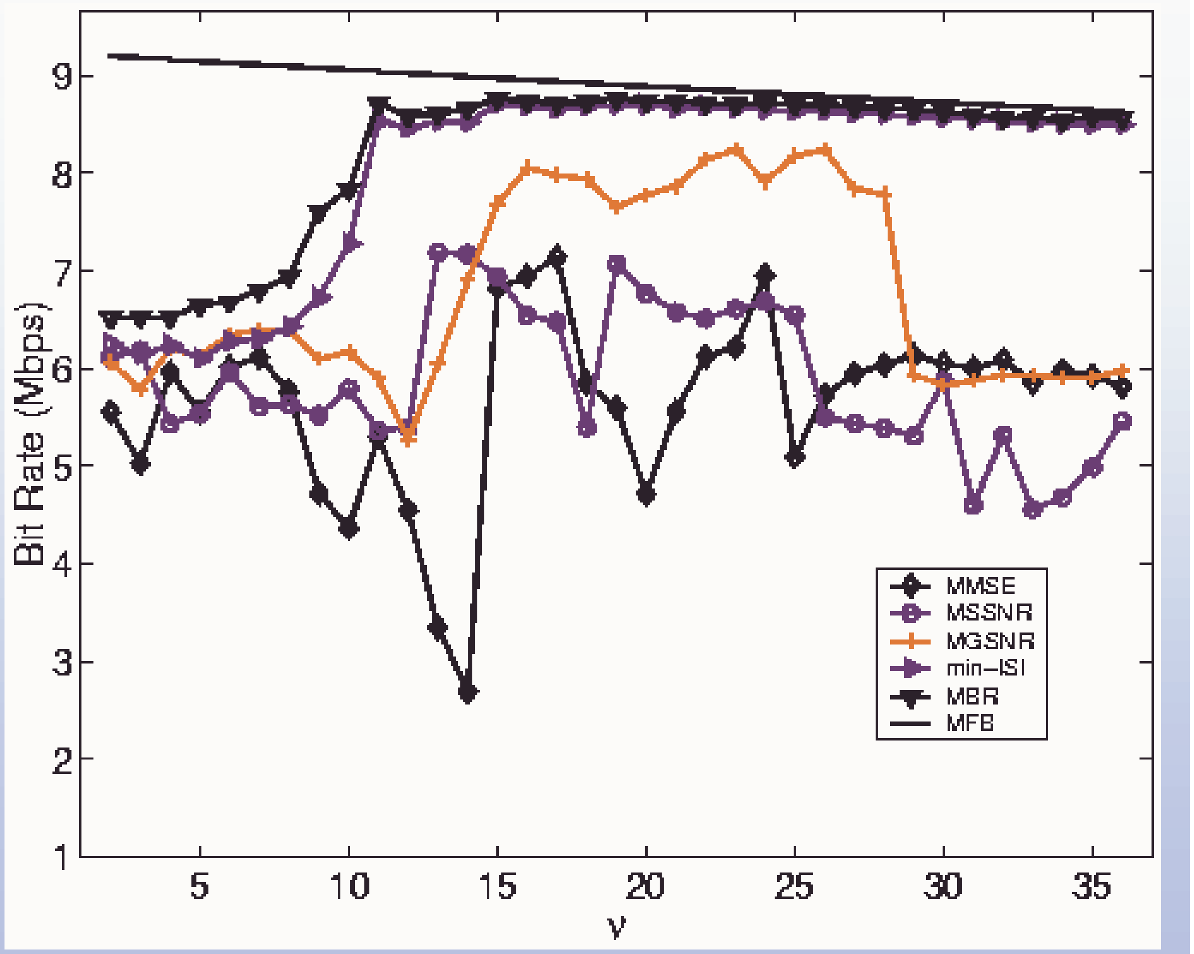
$$\mathrm{SNR}_{2} = \frac{1}{0.1} = 10$$

$$\mathrm{SNR}_{50} = \frac{1}{10+1} = 0.09$$

$$\mathrm{SNR}_{2} = \frac{1}{0.1+1} = 0.9$$

Bit Rate vs. Cyclic Prefix (CP) Size

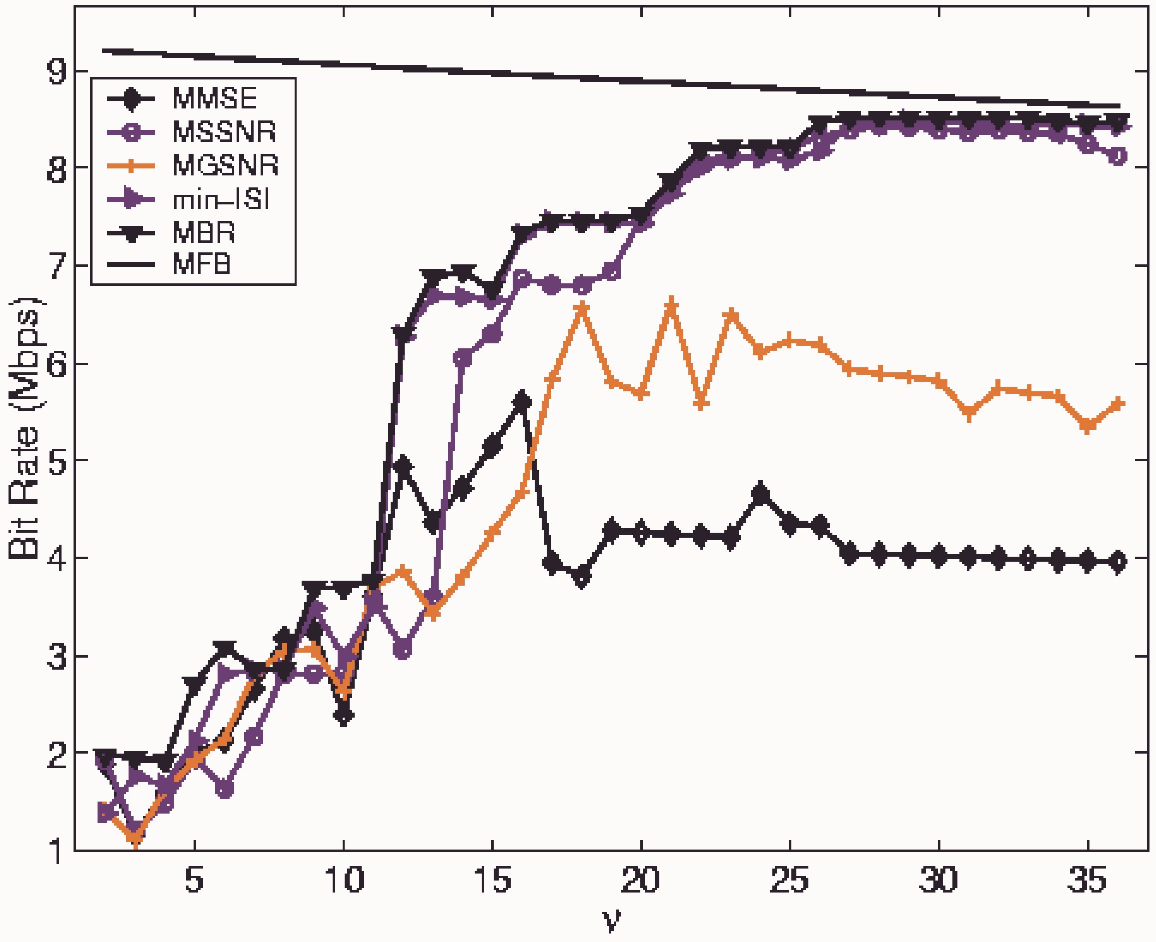
- Matched filter bound decreases because CP has no new information
- Min-ISI and MBR achieve bound with 16-sample CP
- Other design methods are erratic
- MGSNR better for 15-28 sample CPs



TEQ taps (L_w)	17		
FFT size (N)	512	**input power**	23 dBm
coding gain	4.2 dB	**noise power**	-140 dBm/Hz
margin	6 dB	**crosstalk noise**	8 ADSL disturbers

Simulation Results

- Min-ISI, MBR, and MSSNR achieve matched filter bound owith CP of 27 samples
- Min-ISI with 13-sample CP beats MMSE with 32-sample CP
- MMSE is worst

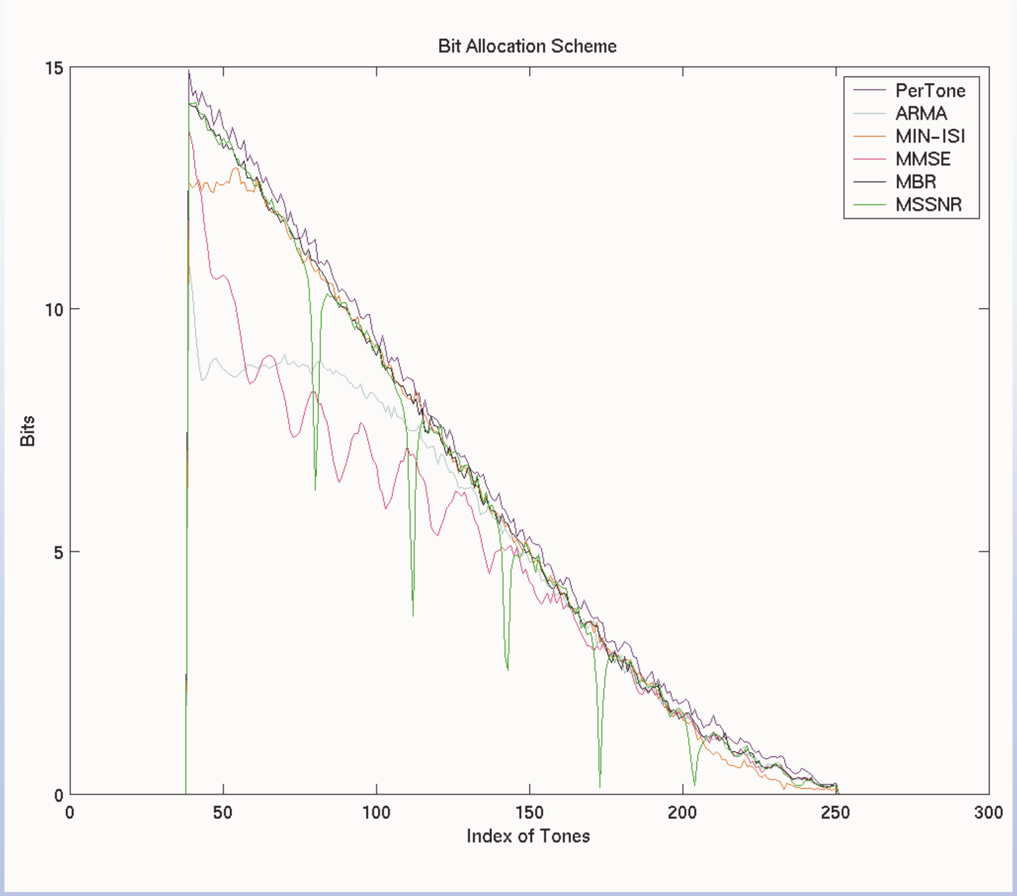


TEQ taps (L_w)	3		
FFT size (N)	512	**input power**	23 dBm
coding gain	4.2 dB	**noise power**	-140 dBm/Hz
margin	6 dB	**crosstalk noise**	8 ADSL disturbers

Bit Allocation Comparison

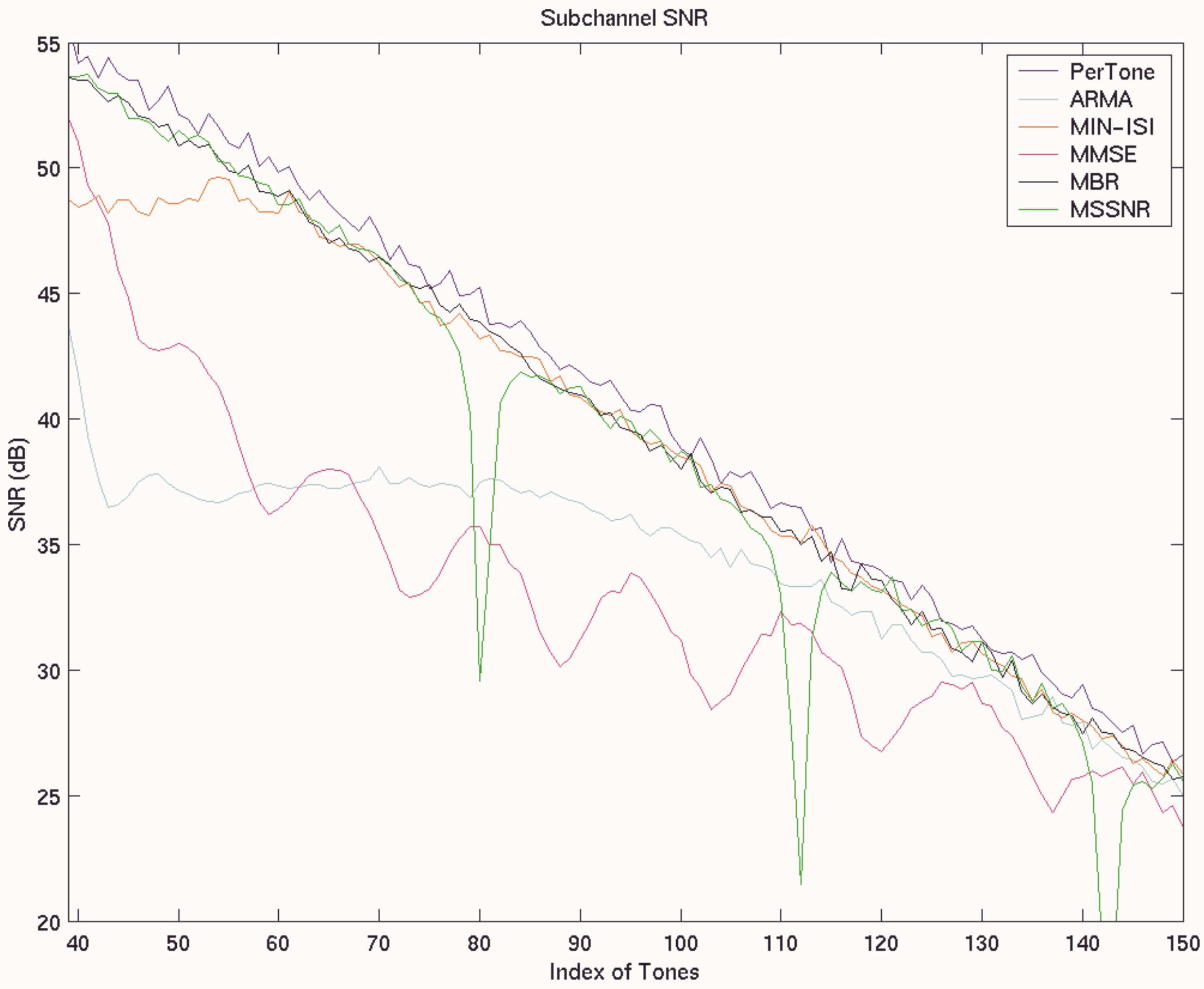
- **AWG 26 Loop: 12000 ft + AWGN**

Equalizer	*Bit Rate*
Per Tone	5.7134 Mbps
MBR	5.4666 Mbps
MSSNR	5.2903 Mbps
Min ISI	5.2586 Mbps
ARMA	4.5479 Mbps
MMSE	4.4052 Mbps



- **Simulation**
  - NEXT from 24 DSL disturbers
  - 32-tap equalizers: least squares training used for per-tone equalizer

Subchannel SNR



Frequency-Domain Per-Tone Equalizer

- **Rearrange computation of FFT coefficient for tone *i***

  [Van Acker, Leus, Moonen, van de Wiel, Pollet, 2001]

  $$Z_i = D_i \, \mathrm{row}_i(\mathbf{Q}_N) \, \mathbf{Y} \, \mathbf{w} = \mathrm{row}_i(\mathbf{Q}_N \, \mathbf{Y}) \, (\mathbf{w} \, D_i)$$

  $\mathbf{Q}_N \, \mathbf{Y}$ produces $N \times L_w$ complex-valued matrix produced by sliding FFT

  Z_i is inner product of *i*th row of $\mathbf{Q}_N \, \mathbf{Y}$ (complex) and $\mathbf{w} \, D_i$ (complex)

  TEQ has been moved into FEQ to create multi-tap FEQ as linear combiner

- **After FFT demodulation, each tone equalized separately**

  Equalize each carrier independently of other carriers ($N/2$ carriers)

  Maximize bit rate at *output of FEQ* by maximizing subchannel SNR

- **Sliding FFT to produce $N \times L_w$ matrix product $\mathbf{Q}_N \, \mathbf{Y}$**

  Receive one ADSL frame (symbol + cyclic prefix) of $N + \nu$ samples

  Take FFT of first N samples to form the first column

  Advance one sample

  Take FFT of N samples to form the second column, etc.

Per-Tone Equalizer: Implementation Complexity

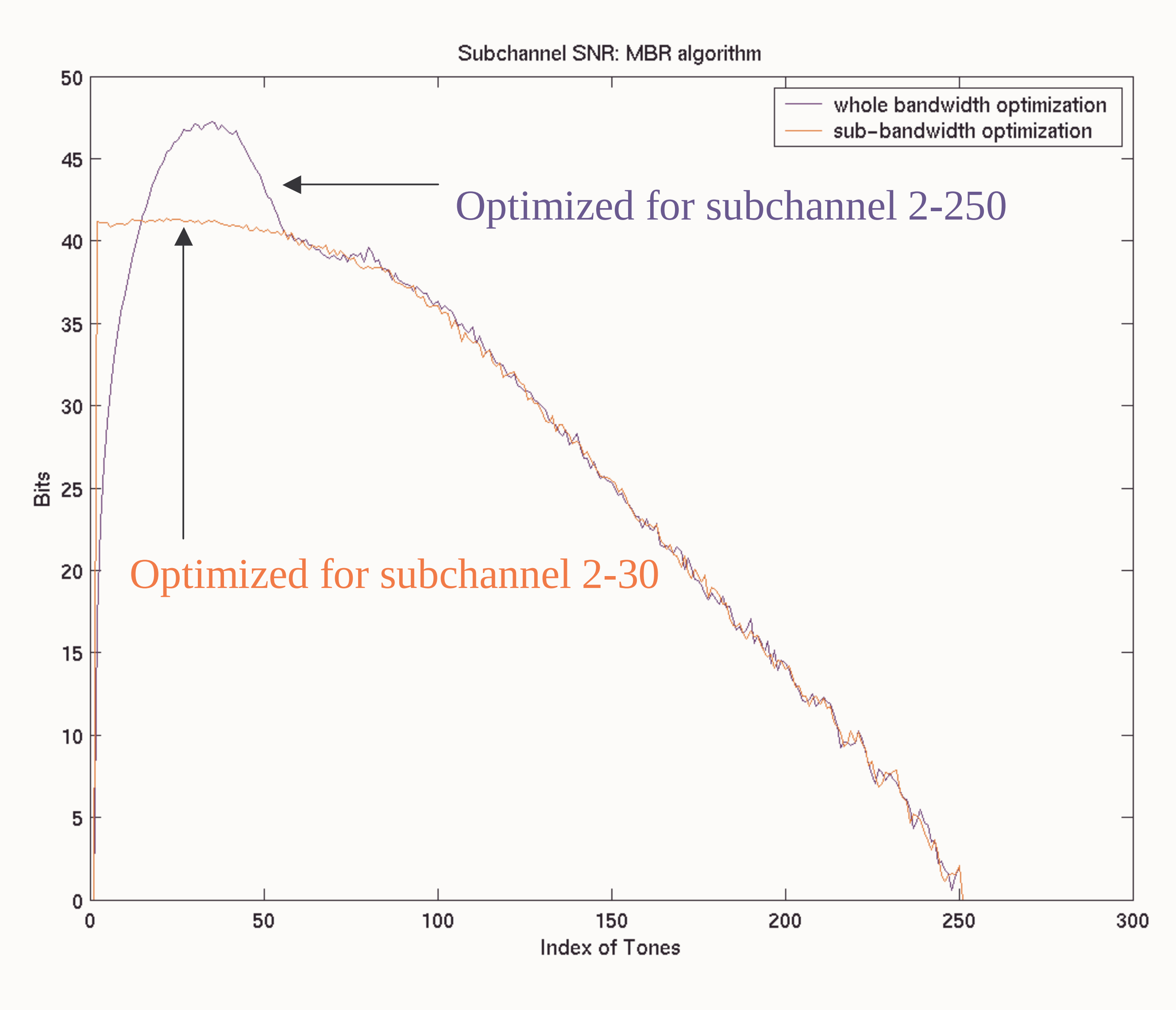
Conventional	Real MACs	Words
TEQ	$L_w f_s$	$2 L_w$
FFT	$2 N \log_2(N) f_{sym}$	$4 N$
FEQ	$4 N_u f_{sym}$	$4 N_u$

Parameter	Symbol	Value
Sampling rate	f_s	2.208 MHz
Symbol rate	f_{sym}	4 kHz
TEQ length	L_w	3-32
Symbol length	N	512
Subchannels used	N_u	256
Cyclic prefix length	ν	32

Per Tone	Real MACs	Words
FFT	$2 N \log_2(N) f_{sym}$	$4 N + 2 \nu$
Sliding FFT	$2 (L_w - 1) N f_{sym}$	N
Combiner	$4 L_w N_u f_{sym}$	$2 (L_w + 1) N_u$

Modified. Per Tone	Real MACs	Adds	Words
FFT	$2 N \log_2(N) f_{sym}$		$4 N$
Differencing		$(L_w - 1) f_{sym}$	$L_w - 1$
Combiner	$2 (L_w + 1) N_u f_{sym}$		$2 L_w N_u$

Dual-Path TEQ (Simulated Channel)



Optimized for subchannel 2-250

Optimized for subchannel 2-30

Motorola CopperGold ADSL Chip

- **Announced in March 1998**
- **5 million transistors, 144 pins, clocked at 55 MHz**
- **1.5 W power consumption**
- **DMT processor consists**
  - Motorola MC56300 DSP core
  - Several application specific ICs
    - 512-point FFT
    - 17-tap FIR filter for time-domain channel equalization based on MMSE method (20 bits precision per tap)

- **DSP core and memory occupies about 1/3 of chip area**