Receivers for Digital Communication in Symmetric Alpha Stable Noise

Marcel Nassar and Marcus DeYoung

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Prof. Robert W. Heath

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Abstract

Wireless communication devices in embedded computational platforms have become increasingly common. As these device become more compact, radio frequency interference (RFI) from other subsystems, such as clocks and buses, becomes more severe and degrades the performance of the communication systems. The interference from these components is impulsive in nature, therefore standard symbol detection techniques based on Gaussian noise statistics are not optimal. In this paper, we investigate methods of mitigating this interference, modeled as symmetric α-stable noise. We propose an approximation to the optimal Bayesian detector and then compare it to suboptimum, but lower-complexity, pre-filtering techniques. These approaches are seen to provide great benefits over the standard matched filter correlation receiver.

I. Introduction

As embedded computational platforms with wireless communication capabilities become smaller and more compact, the radio frequency interference (RFI) from other components becomes an increasingly important factor in the wireless communication system performance. This interference is caused by near-field coupling with the electromagnetic radiation of clocks and busses at both their operating frequencies as well as their harmonic frequencies.

The RFI interference is an independent combination of non-Gaussian events, and is often accurately modeled as impulsive non-Gaussian noise. Common models include Middleton [1] and symmetric α-stable ($S\alpha S$) models [2]. In this project, we will focus on the symmetric α-stable model. This can be justified by the Generalized Central Limit Theorem that states that the sum of many random variables with identical distributions, not necessarily with finite variances, converges to a stable distribution. In addition, recent work has experimentally verified that this measure provides an accurate model of interference in laptop embedded transceivers [3]. As our simulations show, when attempting to demodulate a signal corrupted by impulsive noise with a receiver designed for Gaussian noise, the communication performance is severely degraded. Therefore, methods to mitigate the effects of impulsive noise can be very beneficial to embedded receivers. First, we derive an approximation to the optimum Bayesian detector in the presence of $S\alpha S$ impulsive noise. We then compare suboptimum nonlinear approaches of myriad filtering and hole punching with the performance of the near-optimal MAP approximation and discuss the performance tradeoffs.

II. Background

We begin by reviewing the noise and communication system models used in this paper.

A. Symmetric α-stable Noise

The symmetric α-stable distribution is described by its characteristic function $\phi(\omega) = e^{j\delta\omega-\gamma|\omega|^{\alpha}}$ where $0 < \alpha \leq 2$ is the characteristic exponent, $\delta \in \Re$ is the location parameter, and $\gamma > 0$ is the dispersion of the distribution. The α parameter describes the impulsiveness of the random variable; ranging from very impulsive near zero to Gaussian at 2. From the above characteristic function, it is observed that there is no closed form pdf for $S\alpha S$ distributions except in two special cases: $\alpha = 1$ which is the Cauchy distribution, and $\alpha = 2$ which is the Gaussian distribution [4].

B. System Model

We employ a simple communications model with additive white gaussian noise replaced by symmetric α-stable noise. The discrete time received signal is

$$y(n) = \sum_{k} s(k) g_{tx}(n - kT) + v(n) \tag{1}$$

where $s(n)$ is a sequence of symbols, $g_{tx}(\cdot)$ is the sampled pulse shape, and $v(n)$ is symmetric α-stable noise. Two basic receiver structures are used. For the nonlinear pre-filtering, we first pass the received signal through a nonlinear filter $h_{nl}(\cdot)$, then through a matched filter $g_{rx}(\cdot)$ and into a decision rule $\Lambda(\cdot)$. This receiver model has been studied in the case of zero-memory nonlinearity by Miller and Thomas who motivate it by the fact that it is the structure of a locally optimum detector in additive white noise [5], [6]. The other receiver structure is the standard matched filter $g_{rx}(\cdot)$ followed by a MAP decision rule $\Lambda(\cdot)$ designed for $S\alpha S$ noise distribution.

For the maximum a posteriori (MAP) detector approximation, both Miller [5] and Middleton [6] mention that multiple samples of the received signal provide a method to obtain large performance gains in impulsive noise. Miller assumes a N-path diversity; thus he has N independent versions of the signal. On the other hand, Middleton assumes a fractional sampling approach with N samples per symbol. In this work, we adopt the approach used by Middleton, although the results can be easily extended to the framework proposed by Miller.

We assume that the parameters of the impulsive noise are known. This is a reasonable assumption, since it has been shown that reliable estimates can be computed from samples of noisy data [2]. An implementation of this processing would then require a noise parameter estimator followed by either nonlinear filtering and a standard receiver or the more complex MAP approximation detection rule.

III. Impulsive Noise Mitigation Methods

In this section, we describe three suboptimal methods to aid receiver performance in impulsive noise. The first algorithm derived provides an approximation to the optimal receiver. Myriad filtering, typically used for image processing, is applied to our communication problem. Finally, hole punching provides an low complexity algorithm for reducing the impulsivness of the noise by acting as a digital hard limiter for the received signal.

A. Maximum a Posteriori Approximation

The MAP detector is derived using the Bayesian formulation for hypothesis testing. The detected symbol is the symbol that maximizes the probability of it being sent given the received signal. In the case of binary digital signaling (two hypothesis H_1 and H_2)in additive symmetric α-stable noise, the decision rule is given by

$$\Lambda(\mathbf{Y}) = \frac{p(H_2)p(\mathbf{Y}|H_2)}{p(H_1)p(\mathbf{Y}|H_1)} \lessgtr^{H_1}_{H_2} 1 \tag{2}$$

where $\mathbf{Y}$ is the received signal, and $p(\cdot)$ is the pdf of $S\alpha S$ distribution. However as Section II-A mentions, $S\alpha S$ distributions don't have a closed form pdf, leading us to find an approximation in closed form.

Several approximations have been proposed in the literature. An infinite power series expansion is derived in [4], but does not behave well when shortened. A better approximation to the power series expansion was then proposed, but proves to be numerically unstable and ill-behaved when the random variable is not very large or close to zero [7]. A polynomial approximation proposed in [8], provides an alternative fitting of this pdf, but also suffers from numerical instability .

Fortunately, a well-behaved and computationally tractable approximation is proposed by Kuruoglu [9]. The author shows that we can write a symmetric α-stable random variable as the product of a Gaussian random variable and a positive stable random variable. If we define a normal random variable $X \sim N(0, 2\gamma_x)$ and another, positive stable random variable Y which is independent of X, such that $Y \sim S_{\alpha_z/2}(-1, cos(\frac{\pi * \alpha_z}{4})^{\frac{2}{\alpha_z}}, 0)$, we get a symmetric α-stable random variable $Z = Y^{\frac{1}{2}}X$. Letting $V = Y^{\frac{1}{2}}$, the author eventually shows that the pdf can be approximated by

$$p_{\alpha,0,\gamma,\mu}(z) = \frac{\sum_{i=1}^{N} v_i^{-1} e^{-\frac{(z-\mu)^2}{2\gamma v_i^2}} h(v_i)}{\sum_{i=1}^{N} h(v_i)} \tag{3}$$

To generate this pdf, we can take the characteristic function of the Y and evaluate it at N equally spaced points by taking the fast fourier transform (FFT). Finally we compute the finite pdf using the fact that the mixing function $h(v) = 2vf_Y(v^2)$ and plugging in to Equation 3 yielding

$$p_{\alpha,0,\gamma,\mu}(z) = \frac{\sum_{i=1}^{N} 2e^{-\frac{(z-\mu)^2}{2\gamma v_i^2}} f_Y(v_i^2)}{\sum_{i=1}^{N} f_Y(v_i^2)} \tag{4}$$

This pdf approximation forms the basis for an approximation is used in the MAP detector in Equation 2. A better approximation can be obtained by replacing the uniform sampling by a adaptive optimal sampler that would minimize the squared error.

B. Myriad Filtering

Myriad filtering is a sliding-window based algorithm that exhibits high statistical efficiency in bell-shaped impulsive distributions like $S\alpha S$ distributions. Gonzalez and Arce [10] have shown that myriad filters posses important optimality properties along the α-stable distributions. Myriad filters have been used as both edge enhancers and smoothers in image processing applications

[11]. In the communication domain, they have been applied in the design of robust matched detection [12]. In the latter approach, Gonzalez uses myriad filtering to estimate a sent number over a channel using a known pulse corrupted by additive noise. As a result, the myriad filter is used as a soft estimator of the sent number. In this work, we propose to use the myriad filter as an impulse reduction window for the received signal in a similar way it is applied in image processing applications.

The myriad filter is a sliding window algorithm that outputs the myriad of the sample window. The myriad of order k of a set of samples $x_1, x_2, \ldots, x_N$ is defined as

$$g_M(x_1, x_2, \ldots, x_N) = \hat{\beta}_k = \arg\min_{\beta} \sum_{i=1}^{N} \log[k^2 + (x_i - \beta)^2] = \arg\min_{\beta} \prod_{i=1}^{N} [k^2 + (x_i - \beta)^2] \quad (5)$$

The robustness of this filtering stems from the free-tunable linearity parameter k. This parameter determines the behavior of the myriad filter: for large k the myriad follows the behavior of a linear estimator, as the value of k decreases the estimator becomes more resilient to impulsive noise. The choice for k can be determined by the following empirical formula $k(\alpha) = \sqrt{\frac{\alpha}{2-\alpha}}\gamma^{\frac{1}{\alpha}}$ where α and γ are the parameters of the $S\alpha S$ noise [10].

As can be seen from Equation 5, the myriad filter is a constrained optimization problem where the output $\beta \in [x_{min}, x_{max}]$, where x_{min} and x_{max} are the minimum and maximum value of the data samples in the filtered window. To implement this minimization problem, notice that the objective function under consideration is a polynomial function that is differentiable. Thus the following algorithm can be used to implement the myriad function:

1) Expand the polynomial in Equation 5 in β.
2) Take its derivative by multiplying the coefficients by the appropriate constant according to the corresponding power of β.
3) Root the obtained polynomial and retain the real roots.
4) Evaluate the objective function at the roots and the extremeties and output the minimum.

A less complex algorithm, known as the selection myriad filter, is obtained by restricting our search space just to the samples in the window, ie. $\beta \in \mathcal{S} = \{x_1, x_2, \ldots, x_N\}$ [12].

C. Hole Punching

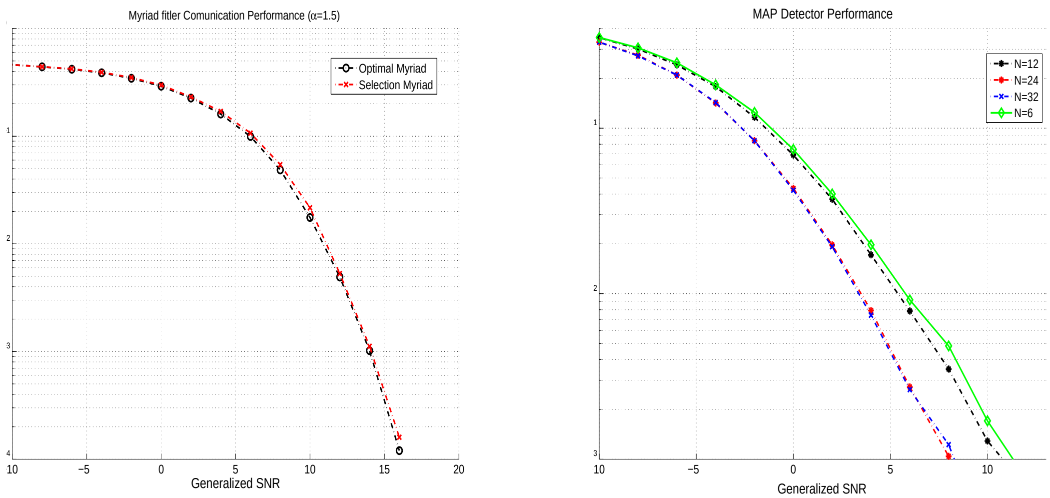
The complexity of the previous algorithms is rather high. Another approach, which is rather ad-hoc, is hole-punching [13]. The algorithm is a nonlinear filter that emulates the functionality of a hard limiter by setting a received sample to zero when it exceeds some threshhold value T_{HP}. Its functionality can be represented as

$$h_{hp}[n] = \begin{cases} x[n] & x[n] \leq T_{hp} \\ 0 & x[n] > T_{hp} \end{cases} \tag{6}$$

The intuition is that when a large value is received, we assume it is an impulse and cannot be sure what the true value is. Hole punching works well in impulsive noise, but provides no advantage in Gaussian noise. We will see in simulations following in Section IV that hole punching performs better than the matched filter receiver. The main advantage in hole punching over the previous two algorithms is the significantly reduced computational complexity.

D. Complexity Analysis

The computational complexity per symbol of the simple hole puncher is $\mathcal{O}(M)$, where M is the oversampling ratio. This follows from the fact that every sample must be evaluated and a decision made about it. The complexity of the myriad filter depends on the specific implementation. The polynomial rooting approach has an $\mathcal{O}(M\,(W^2 + W + W^3))$ complexity, where W is the window size, since polynomial expansion (equivalent to a convolution) is needed followed by W multiplications and root finding which is equivalent to eigenvalue decomposition. On the other hand the selection myriad requires $\mathcal{O}(M\,(W^2 + W))$ since we need to multiply W values for each value and compare them. The complexity of the approximate MAP is $\mathcal{O}(MNS)$ where N is the number of Gaussian components in the Gaussian mixture used to approximate the



(a) Myriad filter performance

(b) MAP receiver performance

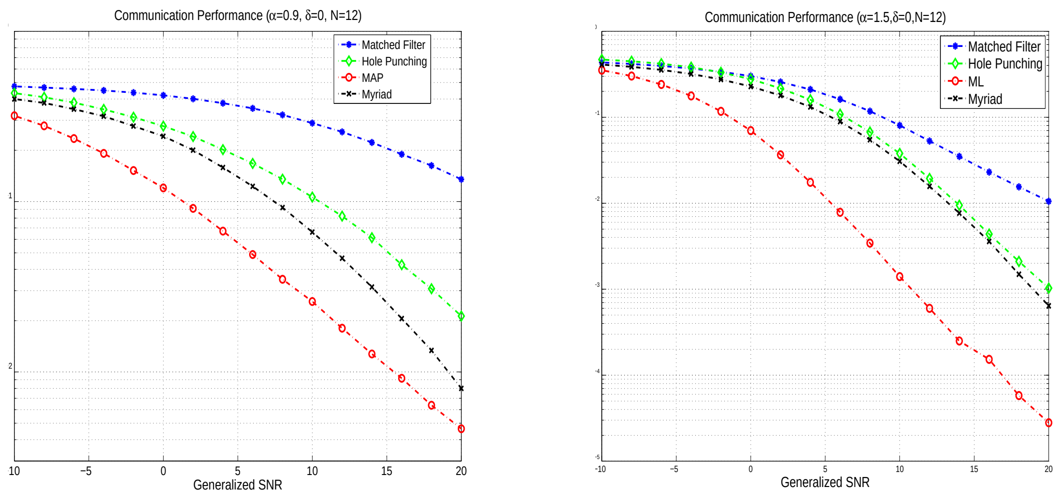
Fig. 1. Communication Performance of the myriad filter and MAP receiver with varying parameters. The selection myriad filter compares favorably to the optimal myriad filter. For the MAP receiver, we see that increasing the oversampling up to around $N = 24$ provides performance improvement

symmetric α-stable distribution, and S is the constellation size. This follows from the fact the we need to sum N weighted exponentials for S times.

IV. Simulation Results

We have developed a set of MATLAB functions to test the RFI mitigation methods discussed in Section III. This work is based upon, and had been added to, the RFI mitigation toolbox developed by our research group (www.ece.utexas.edu/~bevans/rfi/software). We have used the symmetric α-stable noise generator from this toolbox, and contributed the MAP detector, myriad filter, and hole punching filter as well as bit error rate (BER) curve simulation software. Note that second moments do not exist for symmetric α-stable noise, so the curves are plotted as a function of the generalized SNR, defined as a function of the dispersion $GSNR = 10log(\frac{\mathcal{E}_s}{\gamma})$.

We first compare different paramters for the MAP approximation and myriad filtering. First



(a) $\alpha = 0.95$

(b) $\alpha = 1.5$

Fig. 2. Communication performance under different values of α. When α is lower, indicating more impulsiveness, the performance is significantly worse than for higher values of α.

we examine the Myriad filter in Figure IV(a) using both the polynomial rooting approach, and the signficantly lower complexity selection myriad filter. The polynomial rooting method only provides marginal benefits, so it is most likely worth the reduction in complexity to use the simpler selection myriad. Figure IVb) shows how the performance of the approximate MAP varies as we change the time-bandwidth product N. As N is reduced the performance of the MAP degrades, which agrees with the results obtained by Middleton for the case of Class A noise [6]. However, it should be noted that performance variations are relatively small which will prompt us to use smaller values of N which are more practical to use. As N increases, the sampling rate of the analog to digital converter must increase as well. For large bandwidth signals, this may prove intractable. Along with the increased computational complexity, this fact leads us to the pre-filtering methods.

We see in Figure IV the effectiveness of the different receiver algorithms for $\alpha = 0.9$ and

$\alpha = 1.5$. In both cases, the performance of the MAP detector is the best, followed by the myriad filter, hole punching, and then the standard correlation receiver. At $\alpha = 0.9$, the distribution is extremely impulsive and the performance of all four algorithms suffers. Compared to $\alpha = 1.5$, we notice that the difference between the myriad filter and hole punching performance is less than 1dB. Therefore, when α is higher, the complexity versus performance tradeoff between the hole puncher and myriad filter may push us towards the significantly simpler hole punching. Recent work has shown that α around 1.5 is a typical impulsive index in computational platforms [3], implying that hole punching may be a viable alternative to the more complex algorithms.

V. Conclusions and Future Work

From the simulation results, it is clear that there is a large impact of impulsive noise on communications receivers designed for Gaussian noise. Using Kuruloglu's symmetric α-stable noise pdf approximation, we have derived an approximation to the maximimum a posteriori detector. We have also used the myriad filter, traditionally seen in image processing applications, to pre-filter the received signal. Both of these algorithms have been compared to a hole punching filter and the standard matched filter receiver. The three algorithms provide significant communication performance benefits, albeit at potentially high computational cost. A real benefit to the myriad and hole punching filter is that these algorithms may be applied to obtain a signal of reduced impulsiveness that can be used to improve the performance of many other communication algorithms such as channel estimation. Thus a potential future research is to study the effect of these filters on the performance of these algorithms. The hole punching filter, although admittedly ad-hoc, provides a reasonable benefit at very low computational cost. There may be ways to further increase the benefits of hole punching type receivers, such as reducing the signal power instead of zeroing it out when the received signal is between some range. This remains an area for further research. Additionally, the model we have used is quite simple and does not take into account the wireless channel. Channel estimation and equalization in impulsive interference remains another promising area for further research. Initial results have shown benefit in using the hole punching and myriad filters before channel estimation to achieve more accurate estimates. We may use the same pdf approximation to derive a maximum likelihood sequence detector (MLSD) in a frequency selective channel. Overall, large performance gains can be obtained by tailoring the receiver to the impulsive noise in embedded transceivers.

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