LETTER

Space-Time Block Codes with Limited Feedback Using Antenna Grouping

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SUMMARY We propose an antenna grouping method that improves the error rate performance of space-time codes in a wide range of mobility environments. The idea is to group symbols to antennas based on limited feedback from the mobile station to utilize all antennas. Our approach requires only two bits of feedback information to achieve better link performance and full rate for a certain four transmit antenna system. Numerical results confirm the bit/frame error gains over the Alamouti-based space-time block code and antenna subset selection strategies.

key words: *space-time block code, full-diversity full-rate system, antenna selection, antenna grouping*

1. Introduction

Orthogonal space-time block codes (STBC) provide spatial diversity gain from multiple antennas even without feedback information from the mobile station (MS) [1]–[3], it has thus been adopted by various standards [4], [5]. Limited feedback can be used to further improve diversity performance [6]–[8]. The methods proposed in [6]–[8], however, have degraded performance in high mobility scenarios due to outdated feedback information and to delay in the feedback channel. In high mobility scenarios, the system is not affected by the feedback and works like an open loop system. In this case, utilizing all transmit antennas increases the link reliability [2]. This creates a new constraint in the limited feedback problem.

Several studies have been conducted on the limited feedback problem in low mobility scenarios [6]–[10]. In [6], [8], a criterion for selecting the optimal antenna selection for linear, coherent receivers over a slowly varying channel was proposed. An optimal antenna subset selection strategy with STBC to achieve full diversity was proposed in [7], [8]. Other closed-loop systems that use double STBC were proposed in [9]. They use transmit antenna shuffling, which obtains full diversity in spatially correlated channels with low rate feedback. Grassmannian beamforming was proposed in [10] as an alternative to STBC. It requires more feedback (usually more than three bits) and is also sensitive to feedback errors as is [6]–[9].

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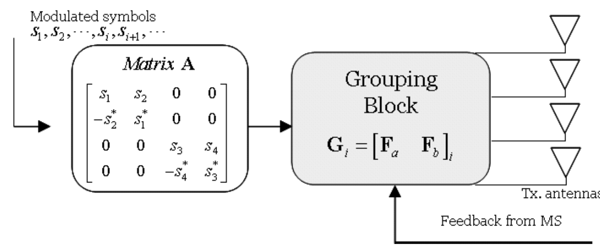

Fig. 1 A block diagram of the proposed system.

The closed-loop solutions [6]–[10] exhibit better bit error rate (BER) performance than the open-loop solutions [1]–[5] in low mobility environments, but their performance deteriorates in high mobility environments due to outdated feedback information. This motivates us to develop robust techniques that have good link level performance in low and high mobility settings.

To reduce the feedback overhead and improve the link reliability in a wide range of mobility environments, we propose in this paper a simple feedback strategy that offers better link level performance for a particular space time code known as *Matrix* **A** (used in IEEE 802.16 [4] and 3GPP2 UMB [5]) for four transmit antenna systems as shown in Fig. 1*. The key idea is to group transmit symbols to antennas based on limited feedback from the MS to utilize all antennas while the conventional antenna (subset) selection strategies in [6]–[8] employ only a few selected antennas. Simulation results show that our approach improves the coded bit/frame error rates in low mobility environments, compared to *Matrix* **A** [4], [5]. The proposed method also outperforms conventional antenna (subset) selection strategies [6]–[8] in high mobility environments while the feedback overhead of the proposed method is smaller than that of the selection-based strategies.

2. System Model and Antenna Grouping

Let us assume that four transmit antennas and one receive antenna are available at the base station (BS) and the MS, respectively. In the precoded system with an Alamouti encoder, the data symbols are multiplied by a precoding matrix **F** of size 4×2 and sent over the selected transmit antennas by the precoder **F**. Let the transmission symbol matrices $\mathbf{S}_a$ and $\mathbf{S}_b$ be

*This matrix will be explained in Sect. 2 in detail.

$$\mathbf{S}_a = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}, \ \mathbf{S}_b = \begin{bmatrix} s_3 & s_4 \\ -s_4^* & s_3^* \end{bmatrix} \tag{1}$$

where s_i, $(i = 1, 2, 3, 4)$ are the modulated input signals and $(\cdot)^*$ denotes the complex conjugate. The received symbol vectors $\mathbf{y}_a$ for $\mathbf{S}_a$ and $\mathbf{y}_b$ for $\mathbf{S}_b$ at the MS are

$$\mathbf{y}_p^T = \mathbf{h}_p^T \mathbf{F}_p \mathbf{S}_p + \mathbf{n}_p^T \tag{2}$$

where, $(\cdot)^T$ denotes the transpose, p is a or b, and $\mathbf{F}_a$ and $\mathbf{F}_b$ are the precoding matrices for $\mathbf{S}_a$ and $\mathbf{S}_b$, respectively. In (2), $\mathbf{h}_p = [h_1 \cdots h_4]^T$ and $\mathbf{n}_p = [n_1 \ n_2]^T$ denote the channel and the additive Gaussian noise vectors where h_k $(k = 1, \cdots, 4)$ is the channel coefficient associated with the kth transmit antenna. The conventional precoder design criterion for orthogonal space-time block codes is to maximize the received signal-to-noise ratio (SNR) at the MS [8]:

$$\hat{\mathbf{F}}_{p,i} = \arg \max_{\mathbf{F}_{p,i} \in \mathcal{F}} \|\mathbf{h}_p^T \mathbf{F}_{p,i}\|_F^2 \tag{3}$$

where $\|\cdot\|_F$ is the Frobenius norm and $\mathcal{F}$ is all possible combinations of two antennas out of four for antenna subset selection; i.e., $\mathbf{F}_{p,i}$=[{1, 2}] (if $i = 1$), [{1, 3}] (if $i = 2$), [{1, 4}] (if $i = 3$), [{2, 3}] (if $i = 4$), [{2, 4}] (if $i = 5$), [{3, 4}], where $[\{m, n\}]$ denotes a 4×2 matrix that has 1 for the $(m, 1)$th and $(n, 2)$th elements while the other elements in the matrix are all zeros. As an example, the precoding matrix $\mathbf{F}_{p,2}$ is given by

$$\mathbf{F}_{p,2} = \{1, 3\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T. \tag{4}$$

Note that link level performance of the selection-based solutions [6], [7] deteriorates in rapidly changing channels due to outdated feedback information.

In this paper, we propose to jointly design the pair of precoding matrices such that each symbol is sent on a different antenna. We call this antenna grouping. It requires only 2 bits of feedback for the four symbols (i.e., $[\mathbf{F}_a\mathbf{F}_b]_i$=[{1, 2}{3, 4}] (if $i = 1$), [{1, 3}{2, 4}] (if $i = 2$), [{1, 4}{2, 3}] (if $i = 3$)) and has robustness even in high mobility scenarios by utilizing all transmit antennas. Note that the precoders $\mathbf{F}_a$ and $\mathbf{F}_b$ are jointly optimized; i.e., $\mathbf{F}_b$ is automatically selected once $\mathbf{F}_a$ is selected to activate all transmit antennas.

On the other hand, in the antenna subset selection strategy [7], the precoders $\mathbf{F}_a$ and $\mathbf{F}_b$ are independently optimized to maximize the average received SNR so only two selected antennas will be used for two symbols for the antenna subset selection [7], [8]. The antenna selection-based solutions, however, are sensitive to feedback channel error because their spatial diversity gain has a stronger dependency on the antenna selection decision, so these solutions cannot be used in fast fading channels. For reliable communication even in high mobility scenarios, we propose to utilize all transmit antennas, and thus the received symbols at the MS are

$$\mathbf{y}^T = \mathbf{h}^T [\mathbf{F}_a\mathbf{F}_b]_i \mathbf{A} + \mathbf{n}^T = \mathbf{h}^T \mathbf{G}_i \mathbf{A} + \mathbf{n}^T, \tag{5}$$

where $\mathbf{G}_i = [\mathbf{F}_a\mathbf{F}_b]_i$ and $\mathbf{0}$ is the zero matrix of size 2×2 and i is the grouping index $(i = 1, 2, 3)$. In (5), $\mathbf{A}$ (also known as *Matrix* $\mathbf{A}$) is defined by[†]

$$\mathbf{A} = \begin{bmatrix} \mathbf{S}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_b \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & 0 & 0 \\ -s_2^* & s_1^* & 0 & 0 \\ 0 & 0 & s_3 & s_4 \\ 0 & 0 & -s_4^* & s_3^* \end{bmatrix}. \tag{6}$$

Note that *Matrix* $\mathbf{A}$ does not achieve full spatial diversity since each symbol experiences only two independent channels. Let $\mathbf{h}_{\text{eff}}^{(i)}$ denote the effective channel gain vector by *Matrix* $\mathbf{A}$ and the combined channel $\mathbf{G}_i^T\mathbf{h}$, and (5) can be expressed as

$$\begin{aligned} \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} &= \begin{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}[1] & \mathbf{h}_{\text{eff}}^{(i)}[2] \\ \mathbf{h}_{\text{eff}}^{(i)}[2]^* & -\mathbf{h}_{\text{eff}}^{(i)}[1]^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \\ \begin{bmatrix} y_3 \\ y_4^* \end{bmatrix} &= \begin{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}[3] & \mathbf{h}_{\text{eff}}^{(i)}[4] \\ \mathbf{h}_{\text{eff}}^{(i)}[4]^* & -\mathbf{h}_{\text{eff}}^{(i)}[3]^* \end{bmatrix} \begin{bmatrix} s_3 \\ s_4 \end{bmatrix} + \begin{bmatrix} n_3 \\ n_4^* \end{bmatrix} \end{aligned} \tag{7}$$

where $\mathbf{h}_{\text{eff}}^{(i)}[k]$ is the kth element of the vector $\mathbf{h}_{\text{eff}}^{(i)}$.

We propose to use the effective minimum distance to choose the optimum *Matrix* $\mathbf{G}_i$. To explain how we choose the grouping *Matrix* $\mathbf{G}_i$, let d_{min} denote the corresponding minimum distance of the normalized unit energy constellation, where $d_{min}^2 = 12/(2^R - 1)$ for 2^R-QAM. We estimate the error probability as

$$P_e \le N_e Q\left(\sqrt{\frac{E_s}{N_0} d_{MIN}^2}\right) \tag{8}$$

where E_s and N_0 are the transmit power and the noise variance, respectively, d_{MIN} is the minimum distance of the diversity constellation at the MS, N_e is the number of nearest neighbors in the constellation, and $Q(x) = \frac{1}{2}\text{erfc}(x/\sqrt{2})$, where $\text{erfc}(\cdot)$ is the complementary error function. For the Alamouti code, the squared minimum distance of the diversity constellation at the receiver, d_{MIN}^2, is bounded by

$$d_{MIN}^2 \le \frac{\min\left(\left\|\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}\right\|_F^2, \left\|\begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}\right\|_F^2\right)}{N_t} d_{min}^2, \tag{9}$$

where $\mathbf{I}$ is the identity matrix of size 2×2 and N_t is the power normalization parameter [11], [12].

We propose to modify the mapping based on feedback from the receiver to the transmitter. Note that the upper bound in (9) depends on the mapping of symbols onto the transmit antennas. To maximize $\min\left(\left\|\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}\right\|_F^2, \left\|\begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{h}_{\text{eff}}^{(i)}\right\|_F^2\right)$ in (9), we propose that the receiver choose $\mathbf{G}_i$ as follows:

$$\arg \max_{\mathbf{G}_i, i \in \{1,2,3\}} \left\{\min\left(\left\|\begin{bmatrix} \mathbf{I} & \mathbf{0} \end{bmatrix} \mathbf{G}_i^T\mathbf{h}\right\|_F^2, \left\|\begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{G}_i^T\mathbf{h}\right\|_F^2\right)\right\}. \tag{10}$$

[†]Hereafter, we use the term *Matrix* **A** for the mapping matrix **A** for consistency.

Since the effective channel matrices in (7) are orthogonal, the selection criterion can be simplified as

$$\arg \max_{\mathbf{h}^i_{\text{eff}}, i\in\{1,2,3\}} \left\{ \min \left(\left|\mathbf{h}^{(i)}_{\text{eff}}[1]\right|^2 + \left|\mathbf{h}^{(i)}_{\text{eff}}[2]\right|^2, \left|\mathbf{h}^{(i)}_{\text{eff}}[3]\right|^2 + \left|\mathbf{h}^{(i)}_{\text{eff}}[4]\right|^2 \right) \right\}. \tag{11}$$

The MS computes (10) or (11), and selects an appropriate *Matrix* $\mathbf{G}_i$ that maximizes the upper bound of d^2_{MIN} in (9). Here, only two bits of feedback information are needed from the MS to select one out of three possible antenna grouping combinations ($\mathbf{G}_1$, $\mathbf{G}_2$, or $\mathbf{G}_3$).

3. Performance Evaluation

Consider the rate one STBC with mapping *Matrix* **A**, used in IEEE 802.16 and 3GPP2 UMB [4], [5]. *Matrix* **A** can be mapped to transmitted symbols using three different methods: i) over four time symbols, ii) over four subcarriers, or iii) over two time symbols and two subcarriers. Because it is the most conservative in the sense of coherent time and frequency, the third approach is used for simulations in this letter in an orthogonal frequency division multiplexing (OFDM) system; i.e. the first two columns are allocated to the first subcarrier (f_1) over two time symbols (t_1, t_2) and the remaining columns are allocated to the second subcarrier (f_2) over two time symbols (t_1, t_2) (see Fig. 4). The mapping methods i) and ii), however, can also be used.

To evaluate the link level performance of the proposed method, we run simulations with parameters selected according to the band adaptive modulation coding (Band AMC) sub-channelization structure, which is equivalent to the clustered subchannel in IEEE 802.16 [4]. The number of bands per symbol is 24, the number of bins per band is 4, and the number of subcarriers per bin is 9. The BS selects the best band based on feedback channel information from the MS. To simulate a more realistic scenario, we impose a two-frame delay on the feedback channel as in the IEEE 802.16 working group [4]. In all the numerical examples, rate 1/2 convolutional turbo codes (CTC) are used.

Figure 2 compares the coded BER and frame error rate (FER) performances of the following approaches: i) the proposed grouping method with *Matrix* $\mathbf{G}_i$, ii) *Matrix* **A** [4], iii) antenna selection [6] and iv) antenna subset selection [7]. The figure compares the performances in a low mobility channel environment (ITU Ped A). For a fair comparison, we assume that all methods have the same total transmit power at the BS.

In the low mobility scenario, the proposed grouping method significantly outperforms the conventional *Matrix* **A** in [4], but better coded BER/FER performances are attained by antenna selection [6] and antenna subset selection [7]. It is noticeable, however, that these selection-based methods require more feedback bits for the same antenna configuration, e.g., maximum 8 bits per 4 time/frequency spans for [6] and maximum 6 bits per 4 time/frequency spans for [7], respectively, while the proposed method requires only 2 bits per 4 time/frequency spans. We summarize the feedback overhead in Table 1. The feedback overhead for the selection-based solutions can be reduced based on coherent time/frequency but we consider optimal feedback described in Table 1 to achieve the best performance in our simulations.

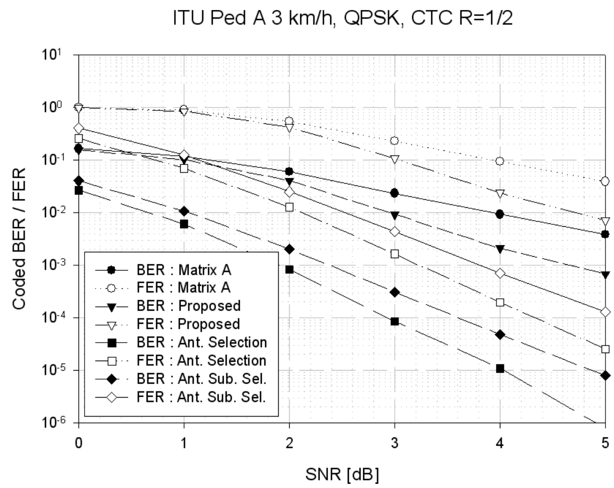

Fig. 2 Coded BER/FER vs. SNR for the proposed antenna grouping, *Matrix* **A** [4], antenna selection [6], and antenna subset selection [7] (ITU Ped A, 3km/h, QPSK, CTC R=1/2).

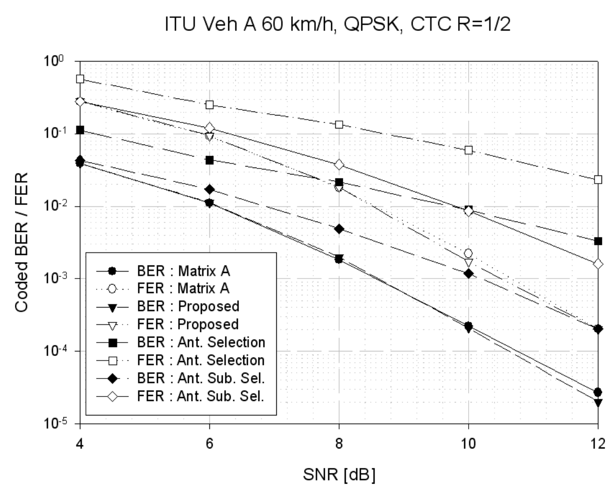

Fig. 3 Coded BER/FER vs. SNR for the proposed antenna grouping, *Matrix* **A** [4], antenna selection [6], and antenna subset selection [7], (ITU Veh A, 60km/h, QPSK, CTC R=1/2).

$$\begin{array}{cccc} \mathbf{t_1} & \mathbf{t_2} & \mathbf{t_1} & \mathbf{t_2} \end{array}$$
$$\begin{bmatrix} s_1 & s_2 & 0 & 0 \\ -s_2^* & s_1^* & 0 & 0 \\ 0 & 0 & s_3 & s_4 \\ 0 & 0 & -s_4^* & s_3^* \end{bmatrix} \begin{array}{l} \textbf{Ant \#1} \\ \textbf{Ant \#2} \\ \textbf{Ant \#3} \\ \textbf{Ant \#4} \end{array}$$
$$\underbrace{\quad\quad}_{\mathbf{f_1}} \quad \underbrace{\quad\quad}_{\mathbf{f_2}}$$

Fig. 4 Resource mapping method used for simulations.

Table 1 Feedback overhead used in simulations.

	Proposed Method	Antenna Selection [6]	Antenna Subset Selection [7]
Feedback (bps=bits/spans)	2 bits/4 spans =0.5 bps	8 bits/4 spans =2 bps	6 bits/4 spans =1.5 bps

The coded performance of the proposed antenna grouping method is superior in scenarios with significant mobility, like the ITU Veh A channel considered in this study as shown in Fig. 3. In such a scenario, the proposed antenna grouping method and conventional *Matrix* **A** outperform the antenna selection-based methods at any SNR value. In particular, the proposed method exhibits 5.2 dB and 1.5 dB SNR gains compared to [6] and [7] respectively at FER=10^{-3}. This is because the proposed method always uses all transmit antennas, while the antenna selection-based solutions only select some of the transmit antennas. Due to outdated feedback information, all methods make incorrect grouping or selection decisions; the antenna selection-based solutions [6], [7] are, however, more sensitive to these errors because their diversity gain has a stronger dependency on the antenna selection decision than that of the antenna grouping strategy.

4. Conclusion

In this letter, we proposed a simple antenna grouping method exploiting conventional space-time block codes. This method can be used for configurations with an even number of transmit antennas and it is particularly attractive for systems with four transmit antennas, where an excellent engineering trade-off between performance and complexity is achieved. Near full diversity and full rate is achieved with an Alamouti-like transmitter using the antenna grouping. Only a two-bit feedback link is required for the four transmit antenna system. Numerical results showed that the proposed method significantly outperforms the conventional *Matrix* **A** strategy in the low mobility scenario and also considerably outperforms antenna selection [6] and subset selection [7], while it attains the same performance as *Matrix* **A** in [4] in the high mobility scenario. Consequently, we can conclude that the proposed system shows good performance with relatively low complexity in a wide range of mobility environments.

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