The University of Texas at Austin
Department of Electrical and Computer Engineering

EE381V-11: Large Scale Optimization — Fall 2012

Problem Set Six

Caramanis/Sanghavi Due: Thursday, November 1, 2012.

Reading Assignments

1. Reading: Boyd & Vandenberghe: Chapters 7 & 8.

Matlab and Computational Assignments. Please provide a printout of the Matlab code you wrote to generate the solutions to the problems below.

1. **MaxCut**. In class we saw the SDP-relaxation for the MaxCut problem. Use CVX to formulate and solve MaxCut on:
   (a) The Petersen Graph: `http://en.wikipedia.org/wiki/Petersen_graph`.
   (b) Any two planar graphs of your choice (with a reasonable number of nodes).

Written Problems

1. **Network Congestion Control** In class, we saw that if the SYSTEM problem was to maximize utility subject to capacity constraints, then looking at the dual of the resulting convex optimization problem allowed us to "decouple" in the following way: there are a set of link prices $\mu_j \geq 0$ for every link j, and a set of user prices λ_s for every user s, such that for every route r that "serves" s, we have that

$$\lambda_s = \sum_{j \in r} \mu_j \tag{1}$$

   and, the resulting user and network problems result in a global optimum.

   Show that a similar reasoning holds when, instead of a capacity for each link, we have a convex *cost function:* a flow of f_j through link j incurs a cost of $C_j(f_j)$, and $C_j(\cdot)$ is a convex function. That is,
   (a) Formulate the overall system problem - to maximize utility minus cost - as a convex optimization problem.
   (b) Show that, even with this more general setting, there is a decoupling of the problem, and there exist μ's and λ's that satisfy (1) and for which the resulting USER and NETWORK problems result in an overall optimum for SYSTEM.

   It will be useful to read the first three sections of the paper "Charging and rate control for elastic traffic" by F. Kelly.

2. Problem 7.12 form Boyd & Vandenberghe.

3. Problem 7.13 from Boyd & Vandenberghe.

4. Problem 8.8 from Boyd & Vandenberghe.
5. Problem 8.9 from Boyd & Vandenberghe.
6. Problem 8.23 from Boyd & Vandenberghe.
7. Problem 8.24 from Boyd & Vandenberghe.
8. Problem 8.25 from Boyd & Vandenberghe.
9. Optional: 8.2 from Boyd & Vandenberghe.
10. Optional: 8.13 from Boyd & Vandenberghe.