The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Fall 2009

Problem Set One

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This problem set is intended to get us started thinking about differential equations and their solution, as well as properties of the solution. In addition, it will give a little practice for some basic linear algebra.

1. Exercise 1.2 from the book.

2. Using the Taylor expansion for sin, cos, and for the exponential, show (i.e., derive the relationship, do not just quote the result) that

$$e^{i\alpha t} = \cos(\alpha t) + i \sin(\alpha t).$$

Then based on this, conclude that for $a \in \mathbb{C}$, the magnitude of

$$x(t) = e^{at},$$

depends on the real part of a, and not on the imaginary part of a.

3. Consider the three-by-three matrix

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

   (a) Show that if vectors v_1 and v_2 both satisfy $Av_1 = 0$ and $Av_2 = 0$, then for any real numbers α and β, the vector $v = \alpha v_1 + \beta v_2$ also satisfies: $Av = 0$.
   (b) Compute the set of vectors v such that $Av = 0$.
   (c) Computer the determinant of the matrix A.
   (d) Now consider the matrix:

$$\begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

   (e) Compute the set of vectors v such that $Av = 0$.
   (f) Computer the determinant of the matrix A.

4. Consider the differential equation:

$$q^{(4)} + 3q^{(3)} - 2\ddot{q} + \dot{q} + 2q = 0.$$

Convert this to standard form (i.e., to the form $\dot{\mathbf{x}} = A\mathbf{x}$), and then use the matlab command `eig(A)` to compute the eigenvalues of A.

5. Read Example 2.9 in the textbook.
   - Convert this (see equation 2.27 in the book) into the form $\dot{x} = f(x, u)$. Note that here x will be a six dimensional vector, and therefore the function f will have six components.
   - Set $u = 0$ (note that u here has two components, u_1 and u_2. Set both to zero.). Show that $(x, y, \theta) = (0, 0, 0)$ is an equilibrium of the resulting system and compute the Jacobian of f at that value. The result should be a six by six matrix.
6. Optional: Exercise 1.4 from the book.