The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Fall 2009

Problem Set Three

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This problem set is intended to get us started thinking about linear algebra, and also to continue giving us some practice with Matlab.

1. Consider the matrices:

$$A_1 = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Compute the Range and NullSpace of both matrices. Draw the Range of the two matrices.

2. Recall in class we discussed the consensus problem in sensor networks. This is also described in the book in Example 2.12, page 58 of the printed edition. The nodes can only see and communicate with neighbors within some distance from them. Once we draw edges representing who can communicate with whom, the resulting structure is a graph: nodes joined by bi-directional edges. How quickly the estimates at the nodes converge to the true average depends on graph properties of the connectivity network. This exercise explores this by looking at a few simple cases.

   Consider three different graphs with 10 vertices, labelled (uniquely) with a number from 1 to 10:

   (a) Graph 1: All nodes connected to all other nodes. This is denoted as K_{10}, and is called the *complete graph* on 10 nodes.

   (b) Graph 2: The Petersen graph. You can find an illustration of this ten-node graph here: `http://en.wikipedia.org/wiki/Petersen_graph`.

   (c) Graph 3: A line. Arrange all 10 nodes in a linear fashion, connect nodes 1 and 2, then 2 and 3, 3 and 4, and so on, up to 9 and 10.

   The task at hand is to find the average of the numbers, 1 to 10, using consensus propagation. So we are assuming here that at the initial iteration, node i broadcasts the value i.

   For each of these three ten-node graphs, implement the recursion in (2.32) for different values of γ. At each step, plot the estimate of each node. You should get a graph similar to (b) in Figure 2.21 in the book, except that you will have 10 lines instead of just 5 as they have there (we have 10 nodes, not 5).

   What do you notice about the convergence times?

3. Let A be an n by n matrix. Consider the set of vectors v which are mapped to a vector b by A:

$$V = \{v \,:\, Av = b\}.$$

Show that V is a vector space if $b = 0$, but is not a vector space if b is any non-zero constant.

4. Let A be an n by n matrix. Suppose there are two nonzero solutions to $Ax = b$, i.e., we have x_1 and x_2, so that $Ax_1 = b$ and $Ax_2 = b$. Show that the columns of A cannot all be independent, and therefore the matrix A cannot be invertible.

5. Let A be an n by n matrix. Suppose that there are no solutions to $Ax = b$. Show that the columns of A cannot all be independent, and therefore the matrix A cannot be invertible.

6. Consider the matrix:

$$A = \begin{bmatrix} -2 & 6 & 1 \\ 6 & 12 & -6 \\ 1 & -6 & 4 \end{bmatrix}.$$

Use Matlab to compute:

- The eigenvalues of A.
- The product of the eigenvalues of A.
- The determinant of A.
- The determinant of $A^\top$.
- Consider the matrix

$$T = \begin{bmatrix} 5 & 5 & 4 \\ 9 & 0 & 6 \\ 8 & 8 & 8 \end{bmatrix}$$

  Check that T is invertible.
- Compute the determinant of $T^{-1}AT$.
- Try this again for some other invertible matrix, T_2. Any conjectures?