The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Fall 2009

Problem Set Three

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This problem set is intended to give some more exercise with linear algebra, and also get us started thinking about trajectories of linear systems, and concepts of stability.

1. We showed in class that if the $n \times n$ matrix T is invertible, then for any $n \times n$ matrix A, A and $T^{-1}AT$ have the same determinant, and the same eigenvalues.
   - Show both of those statements again, here.
   - If v is a eigenvector of A with eigenvalue λ, then λ is also an eigenvalue of $T^{-1}AT$. What is the corresponding eigenvector?

2. Consider the nonlinear differential equation:

$$\dot{x} = f(x,u) = \frac{1}{1+x} - 1 + u.$$

   Suppose that you use feedback control: $u = u(x) = -\alpha x$ for some constant value α. Then the dynamics become:

$$\dot{x} = f(x, -\alpha x) = F(x) = \frac{1}{1+x} - 1 - \alpha x.$$

   (a) Compute the equilibrium points as a function of α.

   (b) Consider the equilibrium point $x_e = 0$. Using the candidate Lyapunov function $V(x) = \frac{1}{2}x^2$, compute the range of the parameter α for which this Lyapunov function shows that the system is stable.

   (c) Now repeat this procedure using linearization: Instead of the non-linear dynamics

$$\dot{x} = F(x),$$

   linearize $F(x)$ around the equilibrium point, $x_e = 0$, to obtain:

$$F_l(x) = F(0) + F'(0)x.$$

   Consider the *linearized dynamics*

$$\dot{x} = F_l(x),$$

   and find the range of the parameter α for which the system is neutrally stable, and the range over which ths system is asymptotically stable, with this linear feedback policy.

3. Exercise 4.4 from the book: Lyapunov functions and stability.

4. Exercise 4.10 from the book: Eigenvalue placement as a function of the control parameters (root locus).
5. Exercise 4.14 from the book.
6. (Optional) Read the section on bifurcation, and do exercise 4.6 from the book.
7. (Optional) Exercise 4.7 from the book.