The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Fall 2009

Problem Set Five

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This problem set focuses on the new concepts introduced in the last two classes: reachability and state feedback. As usual, we also work in some exercise with important concepts from linear algebra.

1. Reachable Canonical Form: Consider the system from above:

$$A = \begin{bmatrix} -1 & 4 & 3 \\ 2 & 6 & -1 \\ -2 & -5 & 2 \end{bmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

   (a) What is the reachable canonical form, $\tilde{A}$, for the matrix A?

   (b) In class we showed that $\tilde{A}$ and A are related by the relationship: $\tilde{A} = TAT^{-1}$, for some invertible matrix T.[1] We showed in class that this kind of transformation preserves eigenvalues, and therefore also determinants. Use Matlab (you can compute by hand, if you wish) to verify that $\tilde{A}$ and A have the same eigenvalues, and the same determinant (equal to the product of the eigenvalues).

   (c) Follow the procedure we outlined in class (and also outlined in the text) to compute the invertible matrix T that can be used to transform from A to $\tilde{A}$. Check your answer to see that this indeed has worked.

2. Show that the change of coordinates we are so freely using is not bogus: Consider the system:

$$\dot{x} = Ax + Bu.$$

   Let T be some invertible matrix, and consider the change of coordinates $z = Tx$.

   - Write down the dynamics for the system in the z coordinates.
   - Show that if the original system is reachable, then so is the new one, and then conversely, if the new system is reachable, then the original one must be reachable. Therefore, you are showing that reachability is a physical property of the system and is not representation-dependent.

3. Eigenvalue Assignment: Consider the system:

$$A = \begin{bmatrix} -1 & 5 & 2 \\ -2 & 4 & -1 \\ -2 & 3 & 2 \end{bmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

[1] This is known as a *similarity transformation*.

(a) What is the reachable canonical form, $\tilde{A}$, for the matrix A?

(b) Compute the invertible matrix T that can be used to transform from A to $\tilde{A}$ (as you did in the problem above, and as we did in class a couple of times).

(c) Is the open-loop system stable?

(d) Design state feedback $u = -Kx$ so that the closed loop system is stable, with closed-loop eigenvalue $\lambda_{cl} = \{-1, -2, -3\}$.

4. Robustness: In class I claimed that if we perturb a matrix by a small perturbation, then the location of the eigenvalues of the matrix will not change much. In particular, if the eigenvalues of a matrix are deep in the OLHP, then under a small perturbation, they will continue to be (somewhere in) the OLHP.

   (a) Using Matlab, compute a random matrix perturbation as follows: let $r = \text{randn}(3)$, and then let $R = (r + r')/2$. The first command generates a 3×3 matrix whose entries are generated in an independent and identically distributed (iid) fashion from a standard Gaussian distribution. The second command generates a symmetric matrix.

   (b) Using the same matrix A from above, and the same closed loop control $u = -Kx$ as above, consider the closed loop system matrix: $A_{cl} = ((A + R/p) - BK)$. Here, p is a normalization parameter. Find the closed-loop eigenvalues of the perturbed system for different values of p, i.e., $p = 10,000,\ 1,000,\ 100,\ 10,\ 5,\ 3,\ 2,\ 1$. When does the closed loop system become unstable?

   (c) Now choose a new feedback K so that the eigenvalues of the nominal (i.e., the unperturbed) closed loop matrix are at $-20, -20, -20$. Now when does the system become unstable? Use the same r as above, i.e., do not generate a new random matrix R.[2]

5. Eigenvalue Assignment: Exercise 6.9 from the book.

6. Second Order Systems and State Feedback Design: Thus far, we have largely focused on stability as our performance metric of choice. Yet once stability is assured, we can focus on finer details. A few classes ago, we discussed other system properties, such as rise time, overshoot, and settling time. In this exercise, we will explore how controlling eigenvalues through Eigenvalue assignment is important beyond just placing them in the OLHP to insure stability.

   Read the beginning of Section 6.3 on second order systems. Replicate the computations in Example 6.6 exploring the tradeoff of overshoot and rise time by plotting the curves for smaller values of ζ (the book shows the curve for $\zeta = 0.9$).

7. (Optional) Cayley-Hamilton Theorem: Exercise 6.10 from the book.

[2]Note that the actual results will depend on the random matrix generated. Most likely a random perturbation will give you some eigenvalues in the RHP, but if not, just try generating a new matrix.