The University of Texas at Austin
Department of Electrical and Computer Engineering

EE362K: Introduction to Automatic Control—Fall 2009

Problem Set Six

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This problem set focuses on the new concepts introduced in the last three classes: reachability and state feedback, and integral action, and also observability and state estimation. As usual, we also work in some exercise with important concepts from linear algebra.

1. Integral Action:

   (a) Consider the system:

   $$A = \begin{bmatrix} 1 & -6 \\ 4 & 0 \end{bmatrix}, \qquad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
   $$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

   Compute a feedback control policy, $u = -Kx$, so that the closed loop eigenvalues are both at -1.

   (b) Let the reference signal r be equal to 1. Compute the feedforward term so that the steady state output is equal to the reference signal, r.

   (c) Plot the trajectory of the output (i.e., plot $y(t)$) to verify that you have chosen K and k_r appropriately so that the system is stable, and so that the steady state output is equal to r.

   (d) As in the previous exercise, compute a perturbation matrix, and choosing a value of p so that the closed loop system is still stable, plot the behavior of the perturbed system.

   (e) Now back to the nominal system: suppose we have a constant disturbance, so that the system dynamics are now:
   $$\dot{x} = Ax + Bu + Fd,$$
   where d is an unknown but constant disturbance, and F is the 2×1 matrix $(1, 1)^\top$. Pick a constant value for d, and plot the output trajectory for the same values of K and k_r.

   (f) Write down the closed loop augmented system, where you add a state z that is the integral of the error, $y - r$, and where you use feedback control with integral action:
   $$u = -Kx - k_i z.$$
   In particular, write down the augmented system matrix (this should be a 3×3 matrix).

   (g) Find values for K and k_i so that the augmented closed loop system matrix is stable. (You can use the same K you computed above, if you like).

   (h) Plot the output trajectory ($(y(t))$) for different values of the feedforward gain, k_r.

(i) Fix your control parameters, and plot the output trajectory for different values of constant disturbance, d. What happens?

2. Answer the following:

   (a) Is the following system observable?

$$A = \begin{bmatrix} -1 & 4 & 3 \\ 2 & 6 & -1 \\ -2 & -5 & 2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

   (b) Find a linear observer (i.e., a matrix L) so that the state estimate, $\hat{\mathbf{x}}$, given by dynamics

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + L(y - \hat{y}),$$

   converges to $\mathbf{x}$. Note that this involves:

   - Converting to OCF.
   - Computing the matrix T that achieves this transformation.
   - Computing $\hat{L}$ in those coordinates.
   - Then computing L from $\hat{L}$.

3. Now consider the following system:

$$A = \begin{bmatrix} 3 & -2 & 4 & 0 \\ 2 & 3 & -1 & 0 \\ 3 & -2 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

   (a) Is this system observable?

   (b) Compute a matrix L so that the error $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ goes to zero asymptotically. That is, do the same thing you did for the last part of the problem above. Note: this **is possible** despite your answer to the observability question just above...