Array as follows:

$$\begin{array}{llllll}
\text{Sequence 1:} & 1 & a_2 & a_4 & a_6 & \dots \\
\text{Sequence 2:} & a_1 & a_3 & a_5 & a_7 & \dots \\
\text{Sequence 3:} & b_1 & b_2 & b_3 & b_4 & \dots \\
\text{Sequence 4:} & c_1 & c_2 & c_3 & c_4 & \dots \\
\text{Sequence 5:} & d_1 & d_2 & d_3 & d_4 & \dots \\
\vdots & & \vdots & & \vdots &
\end{array}$$

The first and second sequences are formed from the even and odd coefficients of the polynomial. All subsequent sequences are formed from the two sequences directly above as illustrated below:

$$\begin{array}{llll}
b_1 = \frac{a_1 a_2 - a_3}{a_1}, & b_2 = \frac{a_1 a_4 - a_5}{a_1}, & b_3 = \frac{a_1 a_6 - a_7}{a_1} & \dots \\
c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}, & c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}, & c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1} & \dots \\
d_1 = \frac{c_1 b_2 - b_1 c_2}{c_1}, & d_2 = \frac{c_1 b_3 - b_1 c_3}{c_1}, & d_3 = \frac{c_1 b_4 - b_1 c_4}{c_1} & \dots
\end{array}$$

The array terminates because later rows have successively fewer entries, and finally we have two successive rows with just a single entry. The Routh-Hurwitz test asserts that the roots are all in the left hand plane, as long as the coefficients in the first column of the Routh array are all positive.

(a) Suppose the denominator of the transfer function has the form:

$$1 + KG = s^4 + 2s^3 + 3s^2 + 8s + 8.$$

Use the test above to show that this system is not stable.

(b) Consider a a transfer function with denominator of the form

$$q(s) = s^3 + 5s^2 + (K - 6)s + K.$$

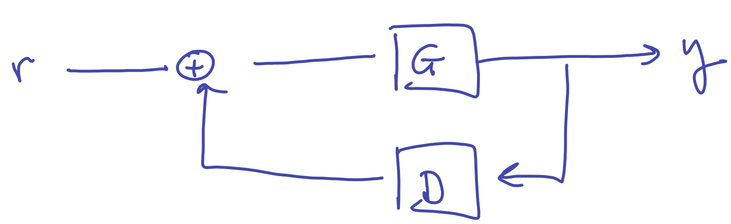
Use the Routh-Hurwitz test to determine the range of K for which the system is stable.

(c) Consider the block diagram below, where the plant G is given by

$$G(s) = \frac{1}{s^2 + 3s - 4},$$

and the controller D is a PID controller given by:

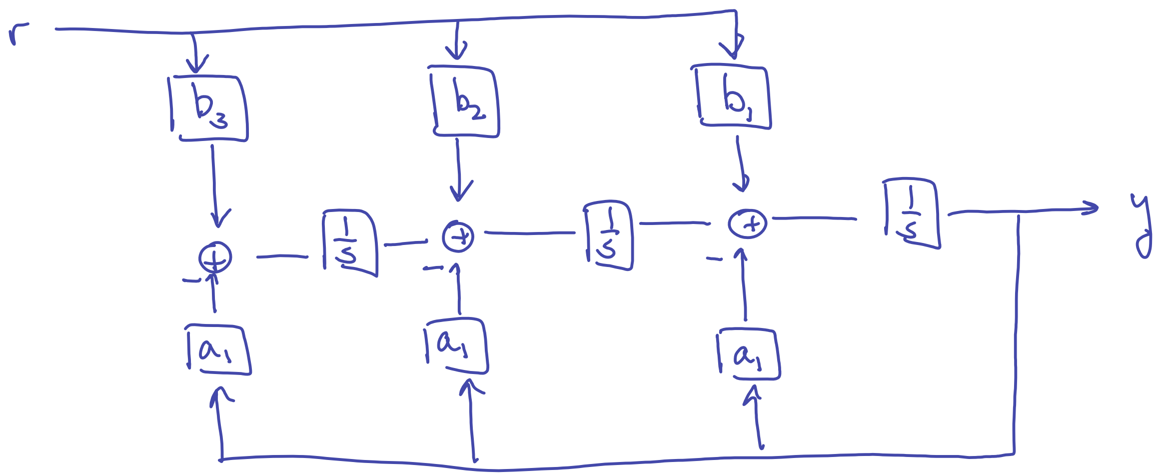
$$D(s) = K_1 + \frac{1}{s}K_2 + sK_3.$$



- Compute the closed loop transfer function from r to y.
- Compute conditions on the constants K_1, K_2, and K_3, so that the resulting closed loop system is stable.

4. Simplify the following block diagrams to determine the single transfer function from the input to output.

   (a) Block Diagram 1:



   (b) Block Diagram 2: