The University of Texas at Austin
Department of Electrical and Computer Engineering

EE380K: Linear Systems Theory—Fall 2008

PROBLEM SET FIVE

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This problem set focuses on the solution to LTI systems, and Lyapunov stability.

1. Consider a continuous time system:
$$\dot{x} = Ax + Bu,$$
where
$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

   (a) Find the solution $x(t)$ as a function of the initial state and the control.

   (b) In a few weeks, we will be talking about controllability, namely, the idea of employing a control, $u(t)$, to drive the system from the initial point, $x(0)$, to a desired final point, $x(T)$ by time $t = T$. It turns out, in this case, that it is possible to drive the system from the intial point $x(0) = x_0$ to a final point $x(T) = x_F$, where
$$x_0 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad x_T = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$
Compute the minimum energy input $u(t)$ that achieves this. In other words, solve the problem:
$$\begin{aligned} \text{min}: &\quad \textstyle\int_0^T u(t)^2 \, dt \\ \text{s.t.}: &\quad x(0) = x_0 \\ &\quad x(T) = x_T. \end{aligned}$$
Hint: This problem can be made to look like a least squares problem in the infinite dimensional space $L^2([0,T])$. Recall that this is the space of signals $f(t)$ such that
$$\int_0^T f(t)^2 \, dt < \infty.$$
This is a vector space, and in fact it has an inner product, given by
$$\langle f(t), g(t) \rangle \triangleq \int_0^T f(t)g(t) \, dt.$$

Recall that during our discussion of least squares, I mentioned that this can be done in much greater generality, using the so-called Gramian. Look back to Chapters 2 and 3 for a refresher on this...

2. Exercise 13.1 from the course notes. Part (*b*) should read: "For what range of α in $[-1, 1]$ can you verify that...".

3. Exercise 13.3 from the course notes.

4. Exercise 13.8 from the course notes.

5. Exercise 14.2 from the course notes, parts (*a*), (*c*), and (*d*). This should look rather familiar. Note that for part (*a*), the characteristic polynomial can be any arbitrary polynomial (with real coefficients). The problem is not asking you to compute general conditions under which arbitrary polynomials have roots in the left half plane, but rather just to write down the characteristic polynomial, and say that its roots must have negative real part.

6. Exercise 7.4 and 14.3 from the course notes. In part (*ii*) of 14.3, do show that there is no constant gain for which the system is asymptotically stable.

7. Exercise 14.7. Do 3 out of the 5 parts of the question, but do at least one of the discrete-time ones.