The University of Texas at Austin
Department of Electrical and Computer Engineering

EE380K: Linear Systems Theory—Fall 2008

SOLUTIONS FOR PROBLEM SET FIVE

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1. Continuous time control minimization

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} x_T = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

   (a) The solution of the continuous time system is:

$$x(T) = e^{AT}x(0) + \int_0^T e^{A(T-\tau)}Bu(\tau)d\tau.$$

   (b)

$$e^{At} = \begin{bmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix};$$

so the input we seek must satisfy

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \int_0^T \begin{bmatrix} 1 & -\tau & \frac{\tau^2}{2} \\ 0 & 1 & -\tau \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(\tau)d\tau$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} T+1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \int_0^T \begin{bmatrix} \frac{\tau^2}{2} \\ -\tau \\ 1 \end{bmatrix} u(\tau)d\tau$$

Looking at the dimensions individually:

$$0 = 0 + 1 \cdot \int_0^T u(\tau)d\tau$$

$$0 = 1 - 1 \cdot \int_0^T \tau u(\tau)d\tau + T \cdot \int_0^T u(\tau)d\tau$$

$$0 = T + 1 + 1 \cdot \int_0^T \frac{\tau^2}{2}u(\tau)d\tau - T \cdot \int_0^T \tau u(\tau)d\tau + \frac{T^2}{2} \cdot \int_0^T u(\tau)d\tau.$$

Combining yields:

$$\int_0^T u(\tau)d\tau = 0;\ \int_0^T \tau u(\tau)d\tau = 1;\ \int_0^T \frac{\tau^2}{2}u(\tau)d\tau = -1.$$

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[1] Many solutions written in whole or in part by Johnson Carroll.

Using these properties, we can write

$$\prec \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix}, u(t) \succ = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Following the least squares solution, we find that the optimal input is

$$u^*(t) = \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix} \prec \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix}, \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix} \succ^{-1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$$

$$= \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix} \begin{pmatrix} \langle 1, 1 \rangle & \langle 1, t \rangle & \langle 1, \frac{t^2}{2} \rangle \\ \langle t, 1 \rangle & \langle t, t \rangle & \langle t, \frac{t^2}{2} \rangle \\ \langle \frac{t^2}{2}, 1 \rangle & \langle \frac{t^2}{2}, t \rangle & \langle \frac{t^2}{2}, \frac{t^2}{2} \rangle \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$$

$$= \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix} \begin{pmatrix} T & \frac{T^2}{2} & \frac{T^3}{6} \\ \frac{T^2}{2} & \frac{T^3}{3} & \frac{T^4}{8} \\ \frac{T^3}{6} & \frac{T^4}{8} & \frac{T^5}{20} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$$

$$= \begin{pmatrix} 1 & t & \frac{t^2}{2} \end{pmatrix} \begin{pmatrix} \frac{9}{T} & \frac{-36}{T^2} & \frac{60}{T^3} \\ \frac{-36}{T^2} & \frac{192}{T^3} & \frac{-360}{T^4} \\ \frac{60}{T^3} & \frac{-360}{T^4} & \frac{720}{T^5} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$$

$$= \frac{-36}{T^2} - \frac{60}{T^3} + (\frac{192}{T^3} + \frac{360}{T^4})t + (\frac{-360}{T^4} - \frac{720}{T^5})\frac{t^2}{2}$$

Sure enough, we can check the integral conditions above and the control achieves the desired state transition.

2. **Exercise 13.1** a) The system

$$\begin{aligned} \dot{x} &= z \\ \dot{z} &= -4x^3 + 2\alpha x = 4x\left(-x^2 + \frac{\alpha}{2}\right) \end{aligned}$$

has an equilibrium point at $(0, 0)$ for any value of α. Also, if $\alpha > 0$ there are two more equilibrium points: $\left(0, \pm\sqrt{\frac{\alpha}{2}}\right)$.

b) Linearizing the system around $(0, 0)$ we get the Jacobian:

$$A = \begin{pmatrix} 0 & 1 \\ 2\alpha & 0 \end{pmatrix}$$

The characteristic polynomial of the system is $det(A - \lambda I) = \lambda^2 - 2\alpha$. If $\alpha > 0$ there is an unstable root, if $\alpha < 0$ both roots are on imaginary axis, and the linearized system is neither asymtotically stable nor unstable (marginally stable). To analyze stability of the original non-linear system in this case we would have to look at the higher order terms. For the two other equilibrium points which exist for $\alpha > 0$ we get the Jacobian:

$$\begin{pmatrix} 0 & 1 \\ -4\alpha & 0 \end{pmatrix}$$

The characteristic polynomial for the system is $det(A - \lambda I) = \lambda^2 + 4\alpha$. Note $\alpha \leq 0$ does not concern us in this case since the equilibrium points $\left(0, \pm\sqrt{\alpha/2}\right)$ are valid when $0 < \alpha \leq 1$. If $\alpha > 0$ both roots lie on $j\omega$ axis and the system is marginally stable.

3. **Exercise 13.3** a) If $\bar{x}$ is a minimum of $P(x)$ then the gradient is equal to zero at that point $\frac{\partial P(\bar{x})}{\partial x} = 0$, therefore $\bar{x}$ is an equilibrium point of the system $\dot{x} = -\frac{\partial P}{\partial x}$.

   b) Since $\bar{x}$ is a minimum $V(x)$ is positive definite: $P(x) - P(\bar{x}) > 0$. Since it is also an isolated minimum, there exists a ball surrounding $\bar{x}$ such that the gradient is non-zero. Therefore the derivative of $V(x)$ is strictly negative in that ball:

$$\dot{V}(x) = -\left\|\frac{\partial P}{\partial x}\right\|^2 < 0$$

4. **Exercise 13.8** For a discrete-time LTI system $x(k+1) = Ax(k)$, let $V(x) = x'Px$, where P is a symmetric, p.d. matrix. What condition will guarantee that $V(x)$ is a Lyapunov function for this system?

   P is already p.d., making V l.p.d.; we must also have V decreasing along trajectories:

$$0 > V(x(k+1)) - V(x(k)) = x(k+1)'Px(k+1) - x(k)'Px(k)$$

$$= x(k)A'PAx(k) - x(k)'Px(k) = x(k)[A'PA - P]x(k).$$

   which is guaranteed if $(A'PA - A)$ is negative semidefinite.

   What condition involving A and P will guarantee asymptotic stability of the system?

   Well, if $(A'PA - A)$ is negative definite, then the system will be asymptotically stable around the origin.

5. **Exercise 14.2** (a) The system is asymptotically stable if all the roots of the characteristic polynomial lie in the left half of the complex plane. Note that characteristic polynomial for matrix A in a control canonical form is given by

$$det(A - \lambda I) = \lambda^N + a_0\lambda^{N-1} + \ldots + a_{N-1}.$$

   b) Use continuity argument to prove that destabilizing perturbation with the smallest Frobenius norm will place an eigenvalue of $A+\Delta$ on the imaginary axis. Suppose that the minimum perturbation is $\bar{\Delta}$. Assume that there is an eigenvalue in the right half plane. Consider a perturbation of the form $c\bar{\Delta}$, where $0 \leq c \leq 1$. As c changes from 0 to 1 at least one eigenvalue had to cross $j\omega$ axis, and the resulting perturbation has a smaller Frobenius norm than $\bar{\Delta}$. This proves contradiction with the original assumption that $A + \bar{\Delta}$ has an eigenvalue in the right half plane.

   c) The characteristic polynomial for the perturbed matrix is

$$det(A - \lambda I) = \lambda^N + (a_0 + \delta_0)\,\lambda^{N-1} + \ldots + (a_{N-1} + \delta_{N-1})$$

   We know that there exists a root $\lambda = j\omega$, where ω is real. If we plug this solution in, and assemble the real and imaginary parts and set them equal to zero, we'll get two linear equations in δ with coefficients dependent on a_k and powers of ω. For example, for a 4^{th} order polynomial:

$$(j\omega)^4 + (a_0+\delta_0)(j\omega)^3 + (a_1+\delta_1)(j\omega)^2 + (a_2+\delta_2)j\omega + a_3 + \delta_3 = 0$$

results in the following two equations:

$$\begin{aligned} \omega^4 - (a_1+\delta_1)\omega^2 + a_3 + \delta_3 &= 0 \\ -(a_0+\delta_0)\omega^3 + (a_2+\delta_2)\omega &= 0 \end{aligned}$$

This equation can be written in matrix form as follows:

$$\begin{pmatrix} 0 & -\omega^2 & 0 & 1 \\ -\omega^3 & 0 & \omega & 0 \end{pmatrix} \begin{pmatrix} \delta_0 \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{pmatrix} = \begin{pmatrix} -\omega^4 + a_1\omega^2 - a_3 \\ a_0\omega^3 - a_2\omega \end{pmatrix}$$

or

$$A(\omega)\delta = B(\omega)$$

Therefore the problem can be formulated as finding a minimal norm solution to an underdetermined system of equations:

$$\min_{A(\omega)\delta = B(\omega)} \|\delta\|$$

By inspection we can see that matrix A has full row rank for any value of ω unequal to zero. If $\omega = 0$ the solution is $\delta_3 = -a_3$, and the rest of the δ_k equal to zeros. For all other values of ω the solution can be expressed as a function of ω:

$$\bar{\delta}(\omega) = A'(\omega)\left[A(\omega)A'(\omega)\right]^{-1} B(\omega)$$

Note that the matrix AA' is diagonal, and can be easily inverted. By minimizing the norm of this expression over ω we can find $\bar{\omega}$ that corresponds to the minimizing perturbation. Then plug this $\bar{\omega}$ in the previous equation to compute minimizing perturbation $\bar{\delta}(\bar{\omega})$ explicitly. Compare the norms of the solutions corresponding to $\omega = \bar{\omega}$ and $\omega = 0$ (i.e. compare $\left\|\bar{\delta}(\bar{\omega})\right\|$ and a_3), and choose the minimum as the solution. This way we converted the problem to minimization of a function of a single variable, which can be easily solved.

d) In case $N = 2$ the characteristic polynomial of the perturbed matrix is

$$\lambda^2 + (a_0+\delta_0)\lambda + (a_1+\delta_1) = 0$$

where $\lambda = j\omega$. For $\omega = 0$ the minimizing solution is $\delta_1 = -a_1$, $\delta_0 = 0$. If $\omega \neq 0$, plug in $\lambda = j\omega$, and the resulting system of equations is

$$\begin{aligned} \delta_1 &= \omega^2 - a_1 \\ \delta_0 &= -a_0 \end{aligned}$$

This is a proper system (number of equations is equal to the number of unknowns), and its solution is given directly by the equations. To minimize the norm of the solution we set $\omega = \sqrt{a_1}$. Note that stability of original matrix A requires that $a_1 > 0, a_0 > 0$ (in fact positivity of all coefficients is always a necessary condition, but not sufficient for $N > 2$ - use Routh criterion for a test in that case!). Next, we have to compare $|a_1|$ and $|a_0|$, and choose the smallest of them to null with δ_1 or δ_0. In our problem $a_0 = a_1 = a$, therefore there are 2 solutions: $(0, a)$ and $(a, 0)$ for the set of δ's.

6. **Exercise 7.4** In this problem, we have $x(k+1) = A(k)x(k)+B(k)u(k)$ where $A(k+N) = A(k)$ and $B(k+N) = B(k)$. That is, $A(kN) = A(0)$, $A(kN+1) = A(1)$, ..., $A(kN+N-1) = A(N-1)$. Similarly, $B(kN) = B(0)$, $B(kN+1) = B(1)$, ..., $B(kN+N-1) = B(N-1)$. Now,

$$z[k+1] = x((k+1)N) = A(kN+N-1)x(kN+N-1) + B(kN+N-1)u(kN+N-1).$$

But,

$$\begin{aligned} x(kN+N-1) &= A(kN+N-2)x(kN+N-2) + B(kN+N-2)u(kN+N-2) \\ x(kN+N-2) &= A(kN+N-3)x(kN+N-3) + B(kN+N-3)u(kN+N-3) \\ &\vdots \\ x(kN+2) &= A(kN+1)x(kN+1) + B(kN+1)u(kN+1) \\ x(kN+1) &= A(kN)x(kN) + B(kN)u(kN) \end{aligned} \tag{1}$$

Subsituting into the equation for $z[k+1]$ we get,

$$\begin{aligned} & x((k+1)N) = A(N-1)A(N-2)\cdots A(0)x(kN) + B(N-1)u(kN+N-1) \\ & +A(N-1)B(N-2)u(kN+N-2) + A(N-1)A(N-2)B(N-3)u(kN+N-3) \\ & +\cdots + A(N-1)A(N-2)\cdots A(1)B(0)u(kN) \\ = \; & A(N-1)A(N-2)\cdots A(0)x(kN) \\ & + \begin{bmatrix} A(N-1)\cdots A(1)B(0) & \dots & A(N-1)B(N-2) & B(N-1) \end{bmatrix} \begin{bmatrix} u(kN) \\ u(kN+1) \\ \vdots \\ u(kN+N-1) \end{bmatrix} \end{aligned}$$

So, $z[k+1] = Fz[k] + Gv[k]$, with

$$F = A(N-1)A(N-2)\cdots A(0),$$

and

$$G = \begin{bmatrix} A(N-1)\cdots A(1)B(0) & \dots & A(N-1)B(N-2) & B(N-1) \end{bmatrix}.$$

Exercise 14.3 In Exercise 7.4 it was shown that the periodic system can be expressed as an LTI system where

$$z(k) = x(kn)\ ,\ v(k) = \begin{pmatrix} u(kn) \\ u(kn+1) \\ \vdots \\ u(kn+n-1) \end{pmatrix},$$

i.e.,

$$z(k+1) = Fz(k) + Gv(k),$$

where

$$F = \begin{pmatrix} A_{N-1} & \cdots & A_0 \end{pmatrix}.$$

First note that a periodic system

$$x(k+1) = A(k)x(k) + B(k)u(k)$$

is asymptotically stable, which is equivalent to say that

$$z(k+1) = Fz(k) + Gv(k)$$

is asymptotically stable at origin.
<u>Proof</u>: ($\rightarrow$) If the periodically varying system is asymptotically stable, then $\|x(k)\| \rightarrow 0$ as $k \rightarrow \infty$, which implies that $\|z(k)\| \rightarrow 0$ so that the system $z(k+1) = Fz(k) + Gv(k)$ is asymptotically stable.
($\leftarrow$) If $z(k+1) = Fz(k) + Gv(k)$ is asymptotically stable, then $\|z(k)\| \rightarrow 0$ as $k \rightarrow \infty$, which implies that $\|x(kN)\| \rightarrow 0$ so that $\|x(k)\| \rightarrow 0$ implying that the periodically varying system is asymptotically stable.
Finally $z(K = 1) = Fz(k) + Gz(k)$ is asymptotically stable, which is equivalent that all the eigenvalues of F are inside of unit disk.
i) Given

$$\begin{aligned} x(k+1) &= \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} x(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k) \\ y(k) &= \begin{pmatrix} 1 & 1 \end{pmatrix} x(k) \end{aligned}$$

with $u(k) = g(k)y(k)$ and the standard notations, we can express the closed system as follows:

$$\begin{aligned} \rightarrow x(k+1) &= Ax(k) + Bg(k)y(k) \\ y(k) &= Cx(k) \\ \rightarrow x(k+1) &= Ax(k) + Bg(k)Cx(k) \\ y(k) &= Cx(k) \\ \rightarrow x(k+1) &= (A + Bg(k)C)x(k) \\ y(k) &= Cx(k) \\ \rightarrow x(k+1) &= A_{CL}x(k) \\ y(k) &= Cx(k). \end{aligned}$$

ii) Suppose $g(k) = g$, then we have

$$\begin{aligned} A_{CL} &= A + BgC \\ &= \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} g \begin{pmatrix} 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ g & g \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 \\ 1+g & g-1 \end{pmatrix}. \end{aligned}$$

Thus the eigenvalues of the matrix A_{CL} are

$$\lambda^2 - \lambda(g-1) - (1+g) = 0$$
$$\rightarrow \lambda_{1,2} = \frac{(g-1) \pm \sqrt{(g-1)^2 + 4(1+g)}}{2}.$$

In order for the system to be stable, both eigenvalues should be within the unit disk. For some g, absolute value of $\frac{(g-1)+\sqrt{(g-1)^2+4(1+g)}}{2}$ becomes less than one, but the absolute value of $\frac{(g-1)-\sqrt{(g-1)^2+4(1+g)}}{2}$ can not be smaller than one.
therefore no constant g can give the asymptotic stability to the system.
Now consider the case where

$$g(k) = \begin{cases} -1 & = & k = \text{even} \\ 3 & = & k = \text{odd} \end{cases}.$$

With this $g(k)$, we have

$$\begin{aligned} A_{CL} &= A + Bg(k)C = A + g(k)BC \\ &= \begin{pmatrix} 0 & 1 \\ 1 + g(k)g(k) - 1 & \end{pmatrix}. \end{aligned}$$

Consider the case when the period is 2. Then

$$F = A_{CL}(0)A_{CL}(1) = \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ -8 & -4 \end{pmatrix}.$$

The eigenvalues of the matrix F are both 0. Hence $z(k+1) = Fz(k)$ is asymptotically stable, which implies that $x(k+1) = A_{CL}(k)x(k)$ is asymptotically stable as well.
It can be seen more clearly, if A_{CL} is decompsed into Jordan form.

$$\begin{aligned} \begin{pmatrix} 4 & 2 \\ -8 & -4 \end{pmatrix} &= \begin{pmatrix} 1 & -0.5 \\ -2 & 0.5 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -4 & -2 \end{pmatrix} \\ &= M \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} M^{-1}. \end{aligned}$$

Thus

$$\begin{aligned} F^2 &= M \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} M^{-1} \\ &= M \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} M^{-1}. \end{aligned}$$

Therefore no matter what initial conditions you choose for $z(0)$, $z(2)$ is always 0. That implies that $x(4)$ is always zero for any $x(0)$.

7. **Exercise 14.7** Use Lyapunov's indirect method to determine stability of the origin for the following systems:

   (a)

$$\begin{aligned} \dot{x_1} &= -x_1 + x_2^2 \\ \dot{x_2} &= -x_2(x_1 + 1) \end{aligned}$$

   Clearly the origin is an equilibrium.

$$\dot{x} \approx \left[\begin{array}{cc} -1 & 2x_2 \\ -x_2 & -x_1 - 1 \end{array} \right]_{(0,0)} = \left[\begin{array}{cc} -1 & 0 \\ 0 & -1 \end{array} \right]$$

   This matrix is clearly stable (eigenvalues $\{-1, -1\}$), so the origin is a locally asymptotically stable equilibrium point.

   (b)

$$\begin{aligned} \dot{x_1} &= -x_1^3 + x_2 \\ \dot{x_2} &= x_1 - x_2 \end{aligned}$$

   Clearly the origin is an equilibrium.

$$\dot{x} \approx \left[\begin{array}{cc} -3x_1^2 & 1 \\ 1 & -1 \end{array} \right]_{(0,0)} = \left[\begin{array}{cc} 0 & 1 \\ 1 & -1 \end{array} \right]$$

   This matrix is unstable (eigenvalues $\{0.618, -1.618\}$), so the origin is an unstable equilibrium point.

   (c)

$$\begin{aligned} \dot{x_1} &= -x_1 + x_2 \\ \dot{x_2} &= x_1^2 - x_2 \end{aligned}$$

   Clearly the origin is an equilibrium.

$$\dot{x} \approx \left[\begin{array}{cc} -1 & 1 \\ 2x_1 & -1 \end{array} \right]_{(0,0)} = \left[\begin{array}{cc} -1 & 1 \\ 0 & -1 \end{array} \right]$$

   This matrix is clearly stable (eigenvalues $\{-1, -1\}$), so the origin is a locally asymptotically stable equilibrium point.

   (d)

$$\begin{aligned} x_1(k+1) &= 2x_1(k) + x_2(k)^2 \\ x_2(k+1) &= x_1(k) + x_2(k) \end{aligned}$$

   Clearly the origin is an equilibrium.

$$x(k+1) \approx \left[\begin{array}{cc} 2 & 2x_2(k) \\ 1 & 1 \end{array} \right]_{(0,0)} = \left[\begin{array}{cc} 2 & 0 \\ 1 & 1 \end{array} \right]$$

   This matrix is unstable (eigenvalues $\{2, 1\}$), so the origin is an unstable equilibrium point.

(e)

$$\begin{aligned} x_1(k+1) &= 1 - e^{x_1(k)x_2(k)} \\ x_2(k+1) &= x_1(k) + 2x_2(k) \end{aligned}$$

Clearly the origin is an equilibrium.

$$x(k+1) \approx \left[\begin{array}{cc} -x_2(k)e^{x_1(k)x_2(k)} & x_1(k) - x_2(k)e^{x_1(k)x_2(k)} \\ 1 & 2 \end{array} \right]_{(0,0)} = \left[\begin{array}{cc} 0 & 0 \\ 1 & 2 \end{array} \right]$$

This matrix is unstable (eigenvalues $\{2, 0\}$), so the origin is an unstable equilibrium point.