The University of Texas at Austin
Department of Electrical and Computer Engineering

EE380K: Linear Systems Theory—Fall 2008

PROBLEM SET 8

C. Caramanis Due **Wednesday**, November 12, 2008.

This problem set focuses on the ideas of optimal control, as introduced in the last three lectures.[1]

1. (Optional) If you want some practice with observability, controllability, and state feedback, have a look at the relevant problem set from the (undergraduate) feedback control class EE362K: `http://users.ece.utexas.edu/~cmcaram/ee362k/problemset6.pdf`.

2. In this exercise, you will use many of the optimal control ideas introduced, pertaining to LTI systems, to compute, among other things, the solution to the ARE by using the Hamiltonian matrix.

   Consider the system with dynamics, and infinite horizon cost function given as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{u} \\ V &= \int_0^\infty (4x_1^2 + 4x_1x_2 + x_2^2 + u_1^2 + u_2^2)\, dt. \end{aligned}$$

   (a) Compute the weighting matrices Q, R, and factor $Q = C^\top C$ for C a 1×2 matrix.

   (b) Before computing the optimal control for this problem, write down the Hamiltonian matrix, and use our results from class relating the eigenvalues of $\mathcal{H}$ to the eigenvalues of A_{cl}, to compute the two eigenvalues of A_{cl}.

   (c) Again using the Hamiltonian matrix, its eigenvectors, and the results from class today, compute the solution $\bar{P}$ to the ARE. Verify that the matrix you obtain indeed satisfies the ARE.

   (d) Compute the optimal control. Then compute the closed loop system matrix A_{cl}, and then verify that its eigenvalues are indeed the ones that you computed in part (b) above.

3. In this exercise, you will compute the solution to the ARE directly, without going through the Hamiltonian. Consider the system with dynamics, and infinite horizon cost function given as follows:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \boldsymbol{u} \\ y &= x_1 \\ V &= \int_0^\infty (x_1^2 + u^2)\, dt. \end{aligned}$$

   (a) Solve the ARE directly, by solving algebraic equations.

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[1] These problems are due to T. Baser, S. Meyn, and W.R. Perkins.

(b) Write down the optimal control.

(c) Compute the closed loop system matrix A_{cl}, and compute its eigenvalues.

4. Consider the scalar LTI system with dynamics and cost given by:

$$\begin{aligned} \dot{x} &= ax + u \\ V &= \int_0^\infty (x(t)^2 + ru(t)^2)\, dt. \end{aligned}$$

The parameters a and r are constants.

(a) Compute the optimal control as a function of the two problem parameters, a and r.

(b) What happens to the closed loop eigenvalues of the system when $r \to \infty$ and when $r \to 0$?